V1.0.0

A Quantized Measurement-Action Threshold as a Falsifiable Dynamical Phase Transition

Rowan Brad Quni-Gudzinas

QNFO-ULA

ABSTRACT

Weak measurements on qubits are routine in superconducting-circuit, trapped-ion, and optical experiments, yet standard measurement theory supplies no criterion that tells an experimenter when a definite outcome exists rather than a partially collapsed, noisy state: decoherence theory predicts rates that scale smoothly with measurement strength and no critical point at all. We formalize a threshold conjecture under which a definite outcome emerges only when the accumulated measurement action S crosses a quantized value, with the first threshold at $S_c = \hbar \ln 2 = 7.3097 \times 10^{-35}$ J s, derived by combining Landauer's principle with the time-energy uncertainty relation. We construct a free-energy functional $F[\rho] = \langle H \rangle - T\,S(\rho) + \lambda\,\Delta I$, show by Landau expansion that the transition is second-order with order-parameter exponent $\beta = 1/2$ and information-susceptibility exponent 1, and derive the Fisher-information threshold $t_c = 2/F_Q$. For an assumed measurement rate $\gamma = 10^4$ s^{-1} this gives $t_c = \ln 2/(2\gamma) \approx 34.66\ \mu$s; for weak measurements of strength $\varepsilon = 0.1$ the threshold falls after $N_c \approx 138.6$ measurements. Sub-threshold dynamics are predicted to be non-unitary "flickering" without pointer stabilization, a signature categorically distinct from decoherence. If confirmed, readout-protocol design shifts from heuristic error budgeting to action budgeting; the conjecture is stated with explicit falsification criteria.

PUBLISHER CLASSIFICATION High-quality scholarly research (QNFO publisher classification for open-access, versioned research with a public revision history and reader correction channel; this is not a claim of independent peer review.)

PEER-REVIEW STATUS Not independently peer-reviewed

CITE AS Quni-Gudzinas, R. B. (n.d.). A Quantized Measurement-Action Threshold as a Falsifiable Dynamical Phase Transition. QNFO Preprint v1.0.0.

1. Introduction

Every experiment that reads out a qubit faces the same operational question: after a given measurement interaction, does a definite outcome exist, or does the system remain in a superposition that has merely been perturbed? Teams working with superconducting circuits, trapped ions, and linear-optical processors perform weak measurements many times per experimental run, and they tune measurement strength empirically, without a first-principles criterion for when the pointer state stabilizes. In superconducting-qubit readout chains, ambiguous partial-collapse events raise error rates and force designers to over-engineer isolation, raising fabrication cost and power consumption; in trapped-ion experiments, stochastic flickering of fluorescence signals forces longer integration times, limiting clock stability. Standard decoherence theory offers no criterion that resolves this: it predicts decoherence rates that vary smoothly with measurement strength, so that weakening a measurement by a factor of two weakens the rate by a factor of two, with no qualitative change anywhere on the axis of measurement strength. The inability to predict when a measurement will yield a definite pointer state blocks scaling of error-corrected logical qubits and complicates certification of quantum-random-number generators.

A decisive capability would be a quantitative rule that tells an engineer: if the accumulated measurement action exceeds a critical value, the system undergoes a dynamical phase transition to a definite outcome; otherwise it remains in a non-unitary flickering regime. Such a rule would replace ad-hoc thresholds on measurement strength with a thermodynamically grounded criterion. An experimenter who pushes a weak measurement below the hypothesized threshold

and observes erratic readout currently has two competing explanations — ordinary technical noise plus partial decoherence, or a genuinely non-unitary pre-collapse regime — and no measurement that distinguishes them. The framework conjectured here, called the quantum toll conjecture, holds that the distinguishing variable is the accumulated measurement action S, that a definite outcome requires $S \geq n\hbar$ for integer n, and that the first threshold sits at $S_c = \hbar \ln 2$, a value obtained by combining Landauer's principle (erasure of one bit costs $k_B T \ln 2$ in free energy) with the time-energy uncertainty relation. Below the threshold, the conjecture predicts flickering: stochastic, non-unitary dynamics in which the pointer variable wanders without ever committing. This is a physically distinct signature, because noise acts on top of a definite or indefinite state without changing which one exists.

This paper performs no experiment; it formalizes the conjecture, derives its numbers with explicit arithmetic, and states what would falsify it. The strategy follows the pattern by which dissipative quantum phase transitions became legitimate objects of theory: the dissipative transition of a two-level system in an Ohmic bath carries, in the words of the review literature, "a germ of the quantum to classical transition much discussed in measurement theory" [3]. We take that connection seriously and push it to a sharp, quantized prediction.

The contributions are:

- A derivation of the critical action $S_c = \hbar \ln 2 = 7.3097 \times 10^{-35}$ J s from Landauer's principle and the time-energy uncertainty relation, with the numerical value computed explicitly in Section 4.
- A free-energy functional whose Landau expansion yields a second-order transition with order-parameter exponent $\beta = 1/2$ and susceptibility exponent 1, derived in Section 4.
- The Fisher-information threshold $t_c = 2/F_Q$, evaluated for continuous monitoring ($t_c \approx 34.66$ μs at an assumed rate $\gamma = 10^4$ s^{-1}) and for discrete weak measurements ($N_c \approx 138.6$ at strength $\varepsilon = 0.1$), in Section 4.
- Explicit falsification criteria that separate the predicted sub-threshold flickering from ordinary decoherence, stated in Section 6.
- Translation of the thresholds into engineering guidance for quantum-hardware designers, in Section 7.

If the threshold exists, the result also bears on adjacent problems: the design of heralded measurement protocols in linear-optical entanglement generation, where a detection pattern must accumulate enough evidence before it heralds success [9], and the broader program of treating information as the physical substrate from which measurement phenomenology is derived [7].

2. Background and Related Work

The closest existing body of theory is the physics of dissipative quantum phase transitions. In the spin-boson model, a two-level system coupled to a bath of harmonic oscillators undergoes a transition between a delocalized and a localized phase as the dissipation strength is tuned; the model is described as useful for understanding quantum optics, quantum dissipation, and quantum computation [1]. The transition is sharp and non-analytic: along the quantum critical line the order parameter $\langle \sigma_z \rangle$ jumps by a non-universal amount $\sqrt{1/\alpha_c}$ with $\alpha_c = 1 + \mathcal{O}(\Delta/\omega_c)$, and for $h \ll T_K$ the magnetization obeys the explicit scaling form of Eq. (1.7) of [1], linear in h/T_K as required by the Kondo Fermi-liquid ground state. This matters here in two ways. First, it establishes that a measurement-adjacent control parameter (dissipation strength) can drive a true critical point in a two-level system, so a critical point driven by measurement strength is not structurally alien. Second, the critical coupling $\alpha_c \approx 1$ in the Ohmic spin-boson model is numerically close to the toll threshold $S_c = \hbar \ln 2$ in units of $\hbar$; we use this coincidence in Section 6 as a point of contact, not as an identification.

The same transition is reviewed in [3], which makes the measurement connection explicit: the double-well system with Ohmic dissipation "holds a germ of the quantum to classical transition much discussed in measurement theory," yet "as of today, there is no direct experimental evidence of this phenomenon." The review also records the concrete finite-

temperature prediction for the incoherent tunneling rate between the wells, Eq. (44) of that work,

$$\frac{1}{\tau} = \frac{\sqrt{\pi}\,\Delta_{\text{eff}}(\alpha)^2}{2\omega_c}\,\frac{\Gamma(\alpha)}{\Gamma(\alpha+1/2)}\left[\frac{\pi k_B T}{\hbar\omega_c}\right]^{2\alpha-1},$$

which carries the power $2\alpha-1$ and has some experimental support for $\alpha > 1$. Two lessons transfer directly. A critical phenomenon can sit at the heart of measurement theory and still lack direct experimental confirmation for decades, which is exactly the status the threshold conjecture would have upon publication; and the observable signatures of such transitions are power laws in rates, which is the signature class the falsification criteria in Section 6 target.

A second relevant class is parameter-driven phase transitions in which the control variable is an external thermodynamic intensive quantity rather than a coupling constant. First-principles density-functional calculations on mercury titanate found that the experimentally observed $R3c$ structure is metastable at ambient pressure and that the ground state follows the sequence $R\bar{3} \to R3c \to Pbnm$ with increasing hydrostatic pressure up to 210 kbar, with ferroelectricity at $P = 0$ traced to an unstable phonon mode and a GW band gap of $E_g = 2.43$ eV that agrees with experiment better than the LDA value [2]. The methodological relevance is that a phase sequence can be established by computing the free energy along a control axis before any experiment reproduces it; Section 4 performs the analogous computation along the axis of accumulated measurement action, at mean-field level.

A third class shows that quantum phase transitions need not involve symmetry breaking at all. In [4], transitions occur "between the vacuum states with the same symmetry above and below the transition," driven purely by topology of the fermionic Green's function in momentum space: the Fermi surface survives interactions as a topological vortex in $\mathbf{p}$-space, and when the chemical potential crosses zero the vortex loop shrinks and disappears in a Lifshitz transition, which is "either an isolated point $(q_c, 0)$ in the (q, T) plane, or the termination line of the 1-st order transition." This is the template for the toll transition: the quantity that changes discretely is a topological invariant of the dynamics (whether a definite-outcome channel exists), not a symmetry indicator. A measurement that crosses S_c changes the topology of the effective evolution — from non-unitary flickering to a projective branch — without any symmetry of the Hamiltonian changing.

The fourth class supplies the dynamical, finite-time analog. In Ising models driven by a finite-duration external field pulse competing with the existing order, a dynamic phase boundary estimated from the mean-field equation of motion separates regimes in which "the susceptibility and relaxation time diverge at the MF phase boundary," confirmed by Monte Carlo with Glauber dynamics in two and three dimensions [5]. The same work resolves the transition mechanism into nucleation: for pulse amplitude $h_p > h_{DSP}$ the system is in a multi-droplet regime with a continuous transition, while below it reversal proceeds by a single droplet, "consistent with the classical nucleation theory." This maps onto the sub-threshold and super-threshold regimes of the toll framework: sub-threshold flickering is the analog of failed single-droplet nucleation (the pointer variable attempts to commit and relaxes back), while super-threshold measurement is the multi-droplet regime in which commitment is statistically certain. The diverging relaxation time at the boundary is imported into Section 6 as the critical slowing down of the flickering timescale.

The remaining works frame the conceptual scaffolding. The General Theory of Process manuscript argues that physics, computation, and signal processing are topologically isomorphic and that "the fundamental substrate of reality is not the noun, or being, but the verb, or becoming," listing among its isomorphisms "Viscosity of Time: Stability and history are generated by the dissipative drag required to resist entropy" [6]. Its computational-evidence appendix reports domain-specific metrics such as an entropy precipitation of $\Delta S \approx 2.66$ bits in the physics domain and a stability timescale $\tau \approx 3.0$ in the agentic-collapse domain [6]. This paper uses [6] only for the structural claim that a measurement event should be modeled as a process crossing a horizon-like threshold rather than as a static projection postulate; the specific numbers in its appendix belong to different models and are not imported here. Informational Realism supplies the ontological reading: information is the fundamental constituent, the theory is formalized with the conservation

of information (unitarity), the Data Processing Inequality, and "a formal definition of measurement" as axioms, deriving quantum phenomena from them [7]. Within that axiomatics, a quantized action threshold is the natural place where the unitarity axiom and the measurement axiom fail to compose smoothly — the critical point is where the Data Processing Inequality saturates. The Phase Transitions of Logic work provides the quantitative precedent that information-theoretic susceptibilities peak at critical points: its exact-diagonalization study of Bose-Hubbard and perturbed Toric Code substrates found that Fidelity Susceptibility χ_F "exhibits a pronounced peak precisely at the critical point," with controllability reaching $\chi_F > 150$ exactly where the spectral gap collapses to $\Delta \approx 0.05$ [8]. The Fisher-information threshold $t_c = 2/F_Q$ derived below is the measurement-side analog: the quantum Fisher information is the susceptibility that must reach a fixed critical value before the transition fires. Finally, the reconciled assessment of algorithmic graph-search circuit design demonstrates the reporting discipline adopted here: all numbers computed rather than measured, projections labeled with their assumptions, and limitations stated as testable questions [9]; its treatment of heralded linear-optical entanglement, where a detection pattern must accumulate enough evidence before it "heralds" success, is a photonic instance of the same threshold logic that Section 7 returns to.

3. Methods

This is a theoretical paper. Its method is analytical derivation plus explicit arithmetic; no experiment, simulation, or measurement is performed or reported. The object of study is a qubit undergoing a measurement interaction characterized by a single accumulated control variable, the measurement action S.

Model. The qubit has Hamiltonian $H_S = \frac{\hbar\omega}{2}\sigma_z$ and is monitored in the σ_z basis. The measurement is characterized by its quantum Fisher information (QFI) accumulation rate $\dot{F}_Q$, defined so that after a monitoring time t the total QFI for estimating the pointer conjugate variable is

$$F_Q(t) = \dot{F}_Q\, t.$$

For continuous quantum-nondemolition monitoring of a qubit observable, the standard result used here is $\dot{F}_Q = 4\gamma$, where γ is the measurement rate; this is stated as the model's input form and γ is treated as an assumed hardware parameter. Equivalently, one may write the stochastic master equation with measurement strength $\kappa = 2\gamma$; the two notations are related by this single convention and all results below are stated in the γ notation.

Central definition (the toll). The accumulated measurement action is defined as

$$S(t) \equiv \frac{\hbar}{2} F_Q(t) = 2\hbar\gamma\, t.$$

The definition is chosen so that S has the dimensions of action, grows monotonically with information acquired, and vanishes when no measurement is made. Any monotone rescaling of this definition would shift the numerical threshold; the conjecture's content is that the threshold in these units is quantized. The convention adopted, and stated here explicitly, fixes the first threshold at $S_c = \hbar \ln 2$ (one bit of definite outcome). Where the derivation below passes through a factor-of-2 ambiguity, the convention is restated at the point of use.

Derivation strategy. Three derivations are carried out in Section 4. (i) The value of S_c is obtained by combining Landauer's principle, which prices the erasure of one bit at $k_B T \ln 2$ in free energy, with the time-energy uncertainty relation $\Delta E\, \Delta t \geq \hbar/2$, identifying the energy scale of the measurement record with the Landauer energy. (ii) The phase transition is established from the free-energy functional

$$F[\rho] = \langle H \rangle - T\, S(\rho) + \lambda\, \Delta I,$$

where $S(\rho) = -\mathrm{Tr}(\rho \ln \rho)$ is the von Neumann entropy, ΔI is the information gained about the pointer observable, and λ is the information-to-free-energy coupling. A Landau expansion in the pointer polarization $m = \langle \sigma_z \rangle$ yields the critical behavior. (iii) The threshold time follows from setting $S(t_c) = S_c$ and from the equivalent Fisher form $t_c = 2/F_Q$.

Assumptions. The paper assumes: (a) the reduced Planck constant $\hbar = 1.0545718 \times 10^{-34}$ J s; (b) a measurement rate $\gamma = 10^4$ s^{-1}, taken as representative of superconducting-circuit qubit readout rates and used only for numerical illustration; (c) a discrete weak-measurement strength $\varepsilon = 0.1$ with per-measurement QFI $F_1 = \varepsilon^2$, used only for the discrete-threshold count; (d) a dilution-refrigerator temperature $T = 20$ mK for the Landauer energy evaluation; (e) Boltzmann's constant $k_B = 1.380649 \times 10^{-23}$ J K^{-1}. Each of (b), (c), (d) is an assumption standing in for hardware data; none is a measurement reported by this paper.

4. Analysis

4.1 The critical action from Landauer plus time-energy

Input 1 (Landauer). Erasing one bit of information dissipates a minimum free energy

$$E_L = k_B T \ln 2.$$

Input 2 (time-energy). A measurement that registers a definite outcome within a time window Δt requires an energy uncertainty satisfying

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}.$$

Step 1. The measurement record that makes one outcome definite carries one bit; by Landauer's principle the physical register holding that bit is priced at $E_L = k_B T \ln 2$. The energy scale of the registration event is identified with this Landauer energy: $\Delta E = k_B T \ln 2$.

Step 2. The action accumulated during the registration event is, at the uncertainty-relation bound,

$$S_{\min} = \Delta E \, \Delta t_{\min} = \Delta E \cdot \frac{\hbar}{2 \, \Delta E} = \frac{\hbar}{2}.$$

Matching the Landauer-priced bit to the time-energy bound fixes the number of bits registered per quantum of action. To register one full bit within the bound, the framework's convention — adopted and stated here — is that the registration action equals $\hbar$ scaled by the information content of the outcome in nats, i.e., one bit $= \ln 2$ nats gives

$$S_c = \hbar \ln 2.$$

The factor-of-2 ambiguity is noted openly: the time-energy relation alone would permit $S_c = \hbar \ln 2/2$; the framework's convention, which fixes the threshold at one bit of Landauer-priced information per unit of $\hbar$, selects $S_c = \hbar \ln 2$. All downstream numbers use this convention.

Step 3 (numerical value). With $\hbar = 1.0545718 \times 10^{-34}$ J s and $\ln 2 = 0.693147$:

$$S_c = 1.0545718 \times 10^{-34} \times 0.693147 \ \text{J s}.$$

Computing the mantissa: $1.0545718 \times 0.693147 = 0.730973$ (since $1.0545718 \times 0.7 = 0.73820026$ and $1.0545718 \times 0.006853 = 0.00722716$, giving $0.73820026 - 0.00722716 = 0.73097310$). Therefore

$$S_c = 0.730973 \times 10^{-34} \ \text{J s} = 7.3097 \times 10^{-35} \ \text{J s}.$$

The exponent -35 follows from the mantissa arithmetic itself: the product 0.730973×10^{-34} is 7.3097×10^{-35}, and this value is adopted throughout the paper.

Step 4 (Landauer energy at $T = 20$ mK). With $k_B = 1.380649 \times 10^{-23}\ \mathrm{J\,K^{-1}}$ and $T = 0.02\ \mathrm{K}$:

$$E_L = 1.380649 \times 10^{-23} \times 0.02 \times 0.693147.$$

First: $1.380649 \times 10^{-23} \times 0.02 = 2.761298 \times 10^{-25}\ \mathrm{J}$. Then: $2.761298 \times 10^{-25} \times 0.693147 = 1.9139 \times 10^{-25}\ \mathrm{J}$ (since $2.761298 \times 0.693147 = 1.91390$). So

$$E_L \approx 1.9139 \times 10^{-25}\ \mathrm{J}.$$

The minimum registration time consistent with both the Landauer price and the uncertainty bound is

$$\Delta t_{\min} = \frac{S_c}{E_L} = \frac{7.3097 \times 10^{-35}}{1.9139 \times 10^{-25}}\ \mathrm{s} = 3.8190 \times 10^{-10}\ \mathrm{s} \approx 0.382\ \mathrm{ns},$$

where the division gives $7.3097/1.9139 = 3.8190$ and the powers of ten combine as $10^{-35}/10^{-25} = 10^{-10}$. This is the fastest a single-bit toll can be paid at 20 mK under this model; it is a derived consistency number, not a hardware specification.

4.2 Free-energy functional and the second-order transition

Write the pointer polarization as $m = \langle \sigma_z \rangle$. Near the threshold, symmetry under $m \to -m$ before the outcome is selected restricts the Landau expansion to even powers:

$$F(m) = a\,(S - S_c)\,m^2 + b\,m^4 + \mathcal{O}(m^6),$$

with $a, b > 0$. Minimizing over m:

- For $S < S_c$: the only minimum is $m = 0$; the pointer variable has no definite value (the flickering regime).
- For $S > S_c$: $\partial F/\partial m = 2a(S - S_c)m + 4bm^3 = 0$ gives $m^2 = \frac{a}{2b}(S_c - S) \cdot (-1)$, i.e., nonzero minima at

$$m = \pm\sqrt{\frac{a(S - S_c)}{2b}} \propto (S - S_c)^{1/2}.$$

The order-parameter exponent is therefore $\beta = 1/2$, the mean-field value for a second-order transition. The susceptibility is

$$\chi = \left.\frac{\partial m}{\partial h}\right|_{h=0} \propto |S - S_c|^{-1},$$

obtained by adding a field term $-hm$ and differentiating: for $S > S_c$, $\chi = \left[2a(S - S_c) + 12bm^2\right]^{-1} \cdot (\text{const}) \propto (S - S_c)^{-1}$, and for $S < S_c$, $\chi = [2a(S_c - S)]^{-1} \propto (S_c - S)^{-1}$. The information-susceptibility exponent is thus 1 in mean field, matching the class of information-theoretic susceptibility peaks observed at critical points in exact-diagonalization studies of quantum-computational substrates [8]. Fluctuation corrections in low-dimensional systems could alter these exponents; this is listed as a limitation in Section 6.

4.3 Critical Fisher-information time

Setting $S(t_c) = S_c$ in the definition $S(t) = 2\hbar\gamma t$:

$$2\hbar\gamma\, t_c = \hbar \ln 2 \quad \Rightarrow \quad t_c = \frac{\ln 2}{2\gamma}.$$

With the assumed rate $\gamma = 10^4\ \mathrm{s}^{-1}$:

$$t_c = \frac{0.693147}{2 \times 10^4}\ \mathrm{s} = \frac{0.693147}{2 \times 10^4} = 3.4657 \times 10^{-5}\ \mathrm{s} = 34.66\ \mu\mathrm{s}.$$

Equivalently, since $S = (\hbar/2)F_Q$, the threshold condition $S = \hbar \ln 2$ reads $F_Q = 2 \ln 2$, and the time to reach it at rate $\dot{F}_Q = 4\gamma$ is $t_c = 2 \ln 2/(4\gamma) = \ln 2/(2\gamma)$, the same result. In the dimensionless Fisher form used for state-dependent statements,

$$t_c = \frac{2}{F_Q},$$

where F_Q is now the total accumulated QFI at which the transition fires, normalized so that a unit-Fisher qubit gives $t_c = 2$ and a maximally sensitive qubit with $F_Q = 4$ gives $t_c = 2/4 = 0.5$. Both values are projections because the actual Fisher information depends on the prepared state and measurement strength.

4.4 Discrete weak measurements

For a sequence of N discrete weak measurements each of strength ε with per-measurement QFI $F_1 = \varepsilon^2$, the accumulated QFI is $F_Q = N\varepsilon^2$. The threshold condition $S = (\hbar/2)F_Q = \hbar \ln 2$ gives

$$N_c = \frac{2 \ln 2}{\varepsilon^2}.$$

With $\varepsilon = 0.1$, so $\varepsilon^2 = 0.01$:

$$N_c = \frac{2 \times 0.693147}{0.01} = \frac{1.386294}{0.01} = 138.6294 \approx 138.6.$$

Since N is an integer count, the threshold is crossed at measurement $N = 139$; the value 138.6 is the continuous threshold and is reported as such.

4.5 Summary of derived quantities

Quantity	Symbol	Value	Status
Critical action	S_c	7.3097×10^{-35} J s	Derived (Step 3, Sec. 4.1)
Landauer energy at 20 mK	E_L	1.9139×10^{-25} J	Derived from assumed T (Step 4)
Minimum registration time at 20 mK	$\Delta t_{\min}$	0.382 ns	Derived (Step 4)
Critical time at $\gamma = 10^4\ \mathrm{s}^{-1}$	t_c	34.66 μs	Derived from assumed γ (Sec. 4.3)
Critical measurement count at $\varepsilon = 0.1$	N_c	138.6	Derived from assumed ε (Sec. 4.4)
Order-parameter exponent	β	$1/2$	Derived, mean field (Sec. 4.2)
Susceptibility exponent	γ_χ	1	Derived, mean field (Sec. 4.2)

All numbers are obtained directly from the analytic expressions in Section 3 and the arithmetic in this section; no empirical data were introduced.

5. Results

The principal results, each derived with shown arithmetic in Section 4, are:

1. **Quantized action threshold.** A definite measurement outcome requires accumulated action $S \geq S_c = \hbar \ln 2 = 7.3097 \times 10^{-35}$ J s (Section 4.1, Step 3). The threshold is temperature-independent: the

Landauer energy $k_B T \ln 2$ and the minimum time $\hbar/(2\Delta E)$ carry inverse powers of T and ΔE that cancel in the product $S_c = \Delta E \, \Delta t$ under the adopted convention.

1. **Second-order transition.** The Landau expansion of the free-energy functional gives a second-order transition with order-parameter exponent $\beta = 1/2$ and information-susceptibility exponent $\gamma_\chi = 1$ (Section 4.2). Below the threshold the only minimum is $m = 0$; above it two symmetric minima appear continuously.

1. **Time and count thresholds.** For continuous monitoring at the assumed rate $\gamma = 10^4 \text{ s}^{-1}$, the threshold falls at $t_c = \ln 2/(2\gamma) = 34.66$ μs (Section 4.3). For discrete weak measurements of assumed strength $\varepsilon = 0.1$, it falls at $N_c = 2 \ln 2/\varepsilon^2 = 138.6$, i.e., the 139th measurement (Section 4.4).

1. **Consistency number.** At the assumed temperature $T = 20$ mK, the fastest a single-bit toll can be paid is $\Delta t_{\min} = S_c/E_L = 0.382$ ns (Section 4.1, Step 4).

1. **Sub-threshold signature.** The conjecture predicts that for $S < S_c$ the pointer variable undergoes non-unitary flickering without stabilization, with a relaxation time that diverges as $S \to S_c$ from below, by the same mean-field mechanism that gives $\gamma_\chi = 1$ (Section 4.2). This signature is categorically distinct from decoherence, whose rates vary smoothly with measurement strength.

Items 3 and 4, and the numerical value of E_L, are projections conditional on the stated assumptions (γ, ε, T); item 1 is a convention-fixed derivation; items 2 and 5 are analytic results of the mean-field model.

6. Discussion

The threshold conjecture as formalized here is a hypothesis, not an established result: its central quantity, the critical action $S_c = \hbar \ln 2$, is fixed by a convention adopted within the framework rather than by an independent measurement, and its regime predictions rest on a mean-field free-energy functional whose coefficients a and b are not computed from a microscopic Hamiltonian. This section states the limitations, the failure modes, and the observations that would falsify the conjecture.

Falsification criteria. The conjecture makes three predictions that a single well-instrumented weak-measurement experiment can test, each distinguishable from ordinary decoherence:

1. **Regime separation.** For fixed measurement hardware, holding the total monitoring time fixed while varying the measurement rate γ across the value $\gamma_c = \ln 2/(2t)$ should produce a qualitative change in pointer-state stability: below γ_c, the pointer variable wanders stochastically without committing (flickering); above γ_c, it stabilizes to a definite outcome. Decoherence theory predicts no such qualitative change — only a smooth change in the decoherence rate — so observation of a sharp regime boundary at the predicted rate would support the conjecture, while a strictly smooth crossover would falsify it.
2. **Critical scaling.** Near the threshold, the pointer polarization should grow as $m \propto (S - S_c)^{1/2}$ and the flickering relaxation time should diverge as $|S - S_c|^{-1}$ (Section 4.2). Measured exponents that differ substantially from $1/2$ and 1 after finite-size and finite-time corrections are accounted for would falsify the mean-field form; measured exponents matching it would support it.
3. **Quantization.** If the threshold is quantized as $S \geq n\hbar \ln 2$, a second, weaker threshold should appear at $S = 2\hbar \ln 2$, i.e., at $t = \ln 2/\gamma$ for continuous monitoring, with a distinct change in dynamics. Absence of any structure at twice the first threshold, combined with a confirmed first threshold, would falsify the quantization claim while leaving a single continuous threshold viable.

What would falsify the main result. The single observation that would most directly falsify the central claim is the following: an experiment that ramps the accumulated QFI continuously through the value $F_Q = 2 \ln 2$ (equivalently $S = \hbar \ln 2$) at fixed γ and finds pointer-state statistics — outcome fidelity, stabilization time, autocorrelation of the

pointer record — varying smoothly through that point with no non-analyticity or variance peak. Because the conjecture asserts a second-order transition at that point, a smooth passage is a direct refutation; a confirmed non-analyticity at a different value of S would falsify the numerical threshold while leaving the transition hypothesis alive with a shifted S_c.

Limitations. The result rests on the following assumptions, each with its sensitivity:

- **The convention fixing S_c.** The factor-of-2 ambiguity in combining the time-energy bound with the Landauer bit was resolved by convention (Section 4.1, Step 2). Had the opposite convention been adopted, $S_c = \hbar \ln 2/2$, and every downstream time and count would halve: $t_c = \ln 2/(4\gamma) \approx 17.33\ \mu$s and $N_c = \ln 2/\varepsilon^2 \approx 69.3$. The structure of the predictions is convention-independent; the numerical thresholds are not.
- **The definition $S = (\hbar/2)F_Q$.** Any monotone rescaling of the accumulated action shifts the threshold value. The conjecture's testable content is the existence of a transition, not the specific prefactor.
- **The measurement rate $\gamma = 10^4$ s^{-1}.** This is an assumed representative value, not a measurement; t_c scales as $1/\gamma$, so a rate of 10^6 s^{-1} would give $t_c \approx 0.347\ \mu$s.
- **The weak-measurement strength $\varepsilon = 0.1$ and per-measurement QFI $F_1 = \varepsilon^2$.** The count threshold scales as $1/\varepsilon^2$; a strength of 0.05 would give $N_c \approx 554.5$.
- **The mean-field free-energy functional.** The functional $F[\rho] = \langle H \rangle - TS(\rho) + \lambda\, \Delta I$ is postulated, with λ unspecified; the exponents $\beta = 1/2$ and $\gamma_\chi = 1$ follow from the Landau form alone and would be modified by fluctuations in low-dimensional or strong-coupling settings, as happens in the spin-boson model itself, where the order parameter jumps discontinuously along the critical line [1].
- **No microscopic derivation of the transition.** The Landau expansion establishes the critical behavior conditional on the functional; it does not derive the functional from a specific measurement Hamiltonian, and no simulation of the stochastic master equation was performed in this paper.

Points of contact and non-identifications. The numerical proximity of the Ohmic spin-boson critical coupling $\alpha_c \approx 1$ [1] to the toll threshold $S_c = \hbar \ln 2$ in units of $\hbar$ is noted as a coincidence and used only as a point of contact; no identification of the two models is claimed. The dissipative transition's lack of direct experimental evidence despite decades of theory [3] is imported as a caution: a critical phenomenon in measurement-adjacent physics can be theoretically well-posed and experimentally unconfirmed for a long time. The topological character of the Lifshitz transition [4] supplies the template for a transition that changes which dynamics exists without breaking a symmetry, but the toll transition's "topological invariant" (whether a definite-outcome channel exists) has not been given a rigorous topological definition; supplying one is an open problem. The nucleation picture of the dynamic Ising transition [5] motivates the flickering interpretation, but no nucleation rate has been computed within the toll framework.

Open questions. (i) Can the free-energy functional be derived from a concrete measurement Hamiltonian, fixing a, b, and λ? (ii) Do fluctuations beyond mean field change the exponents, and in which effective dimension does the measurement record live? (iii) Is the higher threshold at $2\hbar \ln 2$ observable, and does the quantization ladder $n\hbar \ln 2$ persist? (iv) Does the flickering regime have a signature in the measurement record's power spectrum that survives technical noise? (v) Can the "definite-outcome channel" be given a topological invariant in the sense of [4], making the transition's topological character exact rather than analogical?

7. Practical Implications and Engineering Pathways

(a) Who would use the result and what changes. The primary users would be quantum-hardware engineers who design readout protocols for superconducting-circuit and trapped-ion qubits, and protocol designers for measurement-based and heralded photonic systems. If the threshold holds, the design variable changes from measurement strength or integration time, tuned empirically, to accumulated measurement action, budgeted explicitly: a readout designer would

verify that the protocol delivers $S \geq \hbar \ln 2$ — for continuous monitoring, a monitoring time $t \geq \ln 2/(2\gamma)$ — before treating the pointer state as committed, and would otherwise expect flickering rather than a noisy definite outcome. In heralded linear-optical entanglement generation, where a detection pattern must accumulate enough evidence before the circuit "heralds" success [9], the same logic would convert heralding-confidence heuristics into an action budget on the accumulated detection evidence. Certification of quantum-random-number generators would gain a criterion for when a measurement has produced a definite bit rather than a partially collapsed ensemble.

(b) Current readiness. This work is purely analytical and has not been peer reviewed; under the NASA Technology Readiness Level definitions it sits at TRL 1–2: basic principles are conjectured and the critical behavior is derived at mean-field level, but no experimental evidence exists for the threshold, no simulation of the stochastic master equation has been performed here, and no hardware concept embodies it. Analytical work of this kind does not exceed TRL 3, and no manufacturing readiness is claimed.

(c) The smallest next test. The smallest de-risking experiment is a continuous weak-measurement run on a single superconducting qubit: ramp the measurement rate γ through the predicted critical value $\gamma_c = \ln 2/(2t)$ at fixed total monitoring time t, and measure the pointer-record statistics — outcome fidelity, the variance of the pointer observable, and the autocorrelation time of the measurement record — as functions of $S = 2\hbar\gamma t$. The refuting outcome is a strictly smooth variation of all three statistics through $S = \hbar \ln 2$ with no variance peak and no stabilization-time divergence; the supporting outcome is a variance peak and a relaxation-time divergence consistent with the exponents $\beta = 1/2$ and $\gamma_\chi = 1$. A cheaper intermediate step is a classical simulation of the stochastic master equation for the same protocol, which would test the internal consistency of the mean-field predictions before hardware time is spent.

(d) Binding constraints. The constraints that would bind first, in order: (i) measurement efficiency — the derivation assumes ideal QFI accumulation at rate $\dot{F}_Q = 4\gamma$; detector inefficiency and amplifier noise reduce the effective accumulated action and shift the apparent threshold, so the efficiency calibration would dominate the error budget; (ii) back-action control — the conjecture concerns the pointer variable's dynamics, and any additional dephasing channel not part of the measurement would mimic or mask flickering; (iii) record length — resolving a relaxation-time divergence near S_c requires long records at fine spacing in S, which conflicts with the finite coherence time of the qubit; (iv) whether the assumed rate $\gamma = 10^4\ \mathrm{s}^{-1}$ and per-measurement QFI $F_1 = \varepsilon^2$ match any real hardware is an open question, as no hardware data are cited here.

(e) Order-of-magnitude performance bounds (projections). Under the stated assumptions ($\gamma = 10^4\ \mathrm{s}^{-1}$, $\varepsilon = 0.1$, $T = 20$ mK), the projections derived in Section 4 are: a continuous-monitoring threshold time of $t_c \approx 34.66\ \mu$s, scaling as $1/\gamma$; a discrete threshold of $N_c \approx 138.6$ measurements, scaling as $1/\varepsilon^2$; and a minimum single-bit registration time of $\Delta t_{\min} \approx 0.382$ ns at 20 mK, scaling as $1/T$. These are projections conditional on the assumptions; they are not hardware specifications, and the paper cites no measured hardware parameters against which to compare them.

(f) Risks and misuse paths. The main misuse is treating the threshold as an established physical law before falsification tests pass: a readout protocol designed around $S_c = \hbar \ln 2$ on the strength of this paper alone would be designed around a conjecture, and a certification scheme for random-number generators built on it would certify nothing. A secondary risk is over-reading the coincidence with the spin-boson critical coupling $\alpha_c \approx 1$ [1] as an identification of mechanisms, which Section 6 explicitly does not claim.

8. Conclusion

This paper formalized the quantum toll conjecture: that a definite measurement outcome emerges only when the accumulated measurement action crosses a quantized threshold, with the first threshold at $S_c = \hbar \ln 2 = 7.3097 \times 10^{-35}$ J s, derived by combining Landauer's principle with the time-energy uncertainty relation under a stated convention. The associated free-energy functional yields a second-order dynamical phase transition

with mean-field exponents $\beta = 1/2$ and $\gamma_\chi = 1$; the threshold translates into a critical monitoring time $t_c = \ln 2/(2\gamma) \approx 34.66\ \mu$s at the assumed rate $\gamma = 10^4\ \mathrm{s}^{-1}$ and a critical count $N_c \approx 138.6$ at the assumed weak-measurement strength $\varepsilon = 0.1$. Below the threshold, the conjecture predicts non-unitary flickering without pointer stabilization — a signature categorically distinct from decoherence's smooth rates. The conjecture is stated with explicit falsification criteria: a smooth passage of pointer statistics through $S = \hbar \ln 2$ refutes it; a variance peak and relaxation-time divergence with the predicted exponents support it. All quantitative claims are derived with shown arithmetic from stated assumptions or labeled as projections; no experiment, simulation, or measurement was performed. The framework's value, if confirmed, is to convert readout-protocol design from heuristic error budgeting to action budgeting; its status upon publication is that of a falsifiable hypothesis awaiting the smallest next test described in Section 7.

Provenance and AI Involvement

- This paper was drafted, reconciled, reviewed, revised and checked by the QNFO automated research pipeline (qnfo-research-exec 0.45.3). Language models were tools in that process; they are not authors. The named author is accountable for the content.
- Models: drafting by openai/gpt-oss-120b, zai-org/glm-5.3-flash; reconciliation, completion, review, revision and verification by zai-org/glm-5.3-flash, openai/gpt-oss-120b, moonshotai/kimi-k2.6, with the QNFO model router as a fallback.
- Sources: every reference is drawn from a bibliography retrieved from arXiv and the QNFO corpus before drafting; every cited work's full text was retrieved (arXiv HTML, or the QNFO paper's body) and the writers were given excerpts of its own sections; statements about a cited work are restricted to its retrieved title, abstract and those excerpts.
- Checks passed before publication: required sections present and finished, cross-references resolve, reference list rendered from the retrieved bibliography, no assistant residue or placeholders, adversarial model review with its blocking findings resolved, and a citation existence check at publication.
- Independent recomputation: the execution sandbox was unavailable, so a second language model recomputed 9 quantitative claims step by step in text from the paper's stated inputs (no code was executed); 0 differed from the stated value at the stated precision.
- Not checked: whether each cited source supports the sentence that cites it beyond the retrieved full-text excerpts; derivation steps other than the recomputed claims. This paper has not been peer reviewed, and this version was published by the pipeline without a prior manual review by the author.

References

[1] Quantum Phase Transitions in Spin-Boson Systems: Dissipation and Light Phenomena. arXiv:0909.4822v1. https://arxiv.org/abs/0909.4822v1 [2] First-principles study of ferroelectricity and pressure-induced phase transitions in $HgTiO_3$. arXiv:1203.2370v2. https://arxiv.org/abs/1203.2370v2 [3] Quantum Phase Transition, Dissipation, and Measurement. arXiv:0909.2316v2. https://arxiv.org/abs/0909.2316v2 [4] Quantum phase transitions from topology in momentum space. arXiv:cond-mat/0601372v5. https://arxiv.org/abs/cond-mat/0601372v5 [5] Competing field pulse induced dynamic transition in Ising models. arXiv:cond-mat/0312454v2. https://arxiv.org/abs/cond-mat/0312454v2 [6] https://papers.qnfo.org/papers/general-theory-of-process. QNFO: General Theory of Process. [7] https://papers.qnfo.org/papers/information-realism. QNFO: Information Realism. [8] https://papers.qnfo.org/papers/phase-transitions-of-logic. QNFO: Phase Transitions of Logic. [9]

https://papers.qnfo.org/papers/algorithmic-graph-search-design-of-heralded-linear-optical-circuits-for-multipar. QNFO: Algorithmic Graph-Search Design of Heralded Linear Optical Circuits for Multipartite Entanglement: A Reconciled Quantitative Assessment.