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A Universal Adelic Normalization Principle: Ostrowski's Product Formula and Ono's Tamagawa Numbers as Specializations of One Local–Global Schema

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ABSTRACT

Ostrowski's product formula $\prod_v |x|_v = 1$ for a global field and Ono's formula $\tau(T) = |\mathbf{H}^1(k, \widehat{T})|/|\backslash\text{Sha}(T)|$ for the Tamagawa number of an algebraic torus both assert that a family of individually free local contributions aggregates, under a global consistency condition, to a canonical value. We propose that both are instances of a single universal adelic normalization principle (UANP): local data (absolute values, local Haar measures) are freely normalizable, but a gluing condition—compactness of the norm-one idele class group in the first case, exactness of the idelic cohomology sequence of the character lattice in the second—forces their product to a value computable from Galois cohomology. We formalize the principle via a two-layer framework (cohomological gluing plus idelic integration), derive both classical statements as specializations, and compute fully worked examples: the product formula for $x = 12/5$ and $x = 2$ over $\mathbb{Q}$, the adelic height $H(2/3) = 3$, $\tau(\mathbb{G}_m) = 1$ over $\mathbb{Q}$, and $\tau(T) = 2$ for the norm-one torus $T = R_{\mathbb{Q}(i)/\mathbb{Q}}^1 \mathbb{G}_m$. [...]

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Abstract

Ostrowski's product formula $\prod_v |x|_v = 1$ for a global field and Ono's formula $\tau(T) = |\mathbf{H}^1(k, \widehat{T})|/|\backslash\text{Sha}(T)|$ for the Tamagawa number of an algebraic torus both assert that a family of individually free local contributions aggregates, under a global consistency condition, to a canonical value. We propose that both are instances of a single *universal adelic normalization principle* (UANP): local data (absolute values, local Haar measures) are freely normalizable, but a gluing condition—compactness of the norm-one idele class group in the first case, exactness of the idelic cohomology sequence of the character lattice in the second—forces their product to a value computable from Galois cohomology. We formalize the principle via a two-layer framework (cohomological gluing plus idelic integration), derive both classical statements as specializations, and compute fully worked examples: the product formula for $x = 12/5$ and $x = 2$ over $\mathbb{Q}$, the adelic height $H(2/3) = 3$, $\tau(\mathbb{G}_m) = 1$ over $\mathbb{Q}$, and $\tau(T) = 2$ for the norm-one torus $T = R_{\mathbb{Q}(i)/\mathbb{Q}}^1 \mathbb{G}_m$. We classify neighboring local–global statements from the adelic literature as conforming, boundary, or violating cases, and isolate a cokernel criterion deciding when the forced value is 1. Limitations, failure modes, and falsification conditions are stated explicitly.

1. Introduction

Two classical theorems exhibit the same phenomenology. The first is Ostrowski's product formula: for $x \in \mathbb{Q}^\times$,

$$\prod_v |x|_v = 1,$$

where v runs over all places of $\mathbb{Q}$ (the archimedean place ∞ and the primes p), with the normalization $|p|_p = p^{-1}$ and $|x|_\infty = |x|_{\mathbb{R}}$. No individual factor is constrained; only the aggregate is forced.

The second is Ono's formula for algebraic tori. For a torus T over a number field k (an algebraic group that becomes isomorphic to $(\mathbb{G}_m)^n$ over $\bar{k}$; $\mathbb{G}_m$ denotes the multiplicative group), the Tamagawa number $\tau(T)$ —the volume of the adelic quotient $T(\mathbb{A}_k)/T(k)$ with respect to a family of local measures, each individually arbitrary up to normalization—is the canonical rational number

$$\tau(T) = \frac{|\mathbf{H}^1(k, \widehat{T})|}{|\backslash\text{Sha}(T)|},$$

where $\widehat{T} = \text{Hom}(T, \mathbb{G}_m)$ is the character lattice with its $\text{Gal}(\bar{k}/k)$ -action, and $\backslash\text{Sha}(T) = \ker(\mathbf{H}^1(k, T) \rightarrow \prod_v \mathbf{H}^1(k_v, T))$ is the Tate–Shafarevich group measuring failure of the Hasse principle.

This paper advances the conjecture that both are specializations of one **universal adelic normalization principle** (UANP), and verifies the conjecture on the flagship cases with full arithmetic. Our contributions are:

1. **A formal schema** (Section 3): local normalization data, a gluing cocycle into a cohomology group, and a forced aggregate value $\nu(X) = |\mathbf{H}_{\text{glue}}^1|/|\backslash\text{Sha}_{\text{glue}}|$.
2. **Fully worked derivations** (Section 4): the product formula for $x = 12/5$ and $x = 2$, the adelic height $H(2/3) = 3$, and the Tamagawa numbers $\tau(\mathbb{G}_m) = 1$ and $\tau(R_{\mathbb{Q}(i)/\mathbb{Q}}^1 \mathbb{G}_m) = 2$, each computed from stated inputs with every arithmetic step shown.
3. **A criterion** (Section 4.5): the forced value is 1 precisely when the local-to-global restriction on the relevant lattice has trivial cokernel and the obstruction group vanishes.
4. **A classification** (Section 4.6) of neighboring statements from the literature as conforming, boundary, or violating the schema.

Terminology for the adjacent-field reader: the *adele ring* $\mathbb{A}_k$ is the restricted product $\prod'_v k_v$ of all completions of k ; a *place* v is one completion; an *idele* is an invertible adele.

2. Background and Related Work

We review the supplied bibliography in its exact numbering; together these works span both poles of the proposed unification and its boundary.

[1] Adjoint motives of modular forms and the Tamagawa number conjecture (arXiv:2512.02348v2). This work constructs integral structures on the realisations of the adjoint motive of a newform of weight $k \geq 2$ and verifies the λ -part of the Bloch–Kato Tamagawa number conjecture by the Taylor–Wiles method. The Bloch–Kato conjecture is the deepest known instance of "local Euler factors, individually free, aggregate to a canonical global value"; any full-strength version of our schema must recover its formalism, and [1] supplies the integral-structure machinery such a recovery would need.

[2] The local Tamagawa number conjecture for Hecke characters, II (arXiv:math/0701634v2). This paper proves the weak local Tamagawa number conjecture for non-critical motives attached to Hecke characters $\psi_\theta : \mathbb{A}_K \rightarrow K^\times$ over imaginary quadratic fields of class number one. Hecke characters are idelic characters living on the same idele class group that carries the product formula, so [2] shows that Tamagawa-type local-to-global bookkeeping operates literally on the central object of our schema.

[3] Quasi-adelic measures and equidistribution on $\mathbb{P}^1$ (arXiv:1502.04660v3). Building on Baker–Rumely, Favre–Rivera-Letelier, and Chambert-Loir, this work proves equidistribution of small-height points on the Berkovich projective line with respect to adelic measures. It is a toy model of the schema: a freely chosen family of local measures μ_v becomes globally well-behaved only under an admissibility (normalization) condition, and the forced global value is a canonical measure rather than a number.

[4] The arithmetic Hodge index theorem for adelic line bundles II (arXiv:1304.3539v2). This extends the arithmetic Hodge index theorem to finitely generated fields via adelic line bundles, with applications to rigidity of preperiodic points of dynamical systems. Adelic line bundles assign a local metric at each place; the Hodge index theorem is a global constraint (a forced sign) on the aggregate of free local data—a second-order, inequality-valued analogue of the product formula.

[5] Finite-Dimensional Protori Are Adelic Tori (arXiv:2411.16000v44). This identifies the category of finite-dimensional compact connected abelian groups (protori) with the category of adelic tori, via an adelic exponential sequence $0 \rightarrow \mathbb{Q}^n \rightarrow \mathcal{L}(G) \rightarrow G \rightarrow 0$. This is category-level evidence for unification: the topological and arithmetic sides of the story are the same category, so a normalization principle for adelic tori automatically has a topological shadow, with $\mathcal{L}(G)$ playing the role of the constraint lattice.

[6] Product formulas on posets, Wick products, and a correction for the q -Poisson process (arXiv:1708.08034v4). This work corrects product and linearization formulas for Wick-product versions of q -Charlier polynomials, computing Möbius functions of the governing posets. It is a cautionary analogue: product identities are sensitive to the exact combinatorial normalization of the underlying lattice, and a misidentified normalization yields a wrong canonical value—precisely the failure mode our criterion must detect.

[7] Tamagawa number formula for Jacobians (arXiv:2606.06713v1). This gives a product formula for Tamagawa numbers of Jacobians over a discretely valued field with perfect residue field, factored into unipotent, toric, arithmetic, and cohomological terms, proved by extending the flow-cut construction from semistable to arbitrary curves via Raynaud's theory. It is the closest relative to our proposal: a literal product formula for Tamagawa numbers, showing the two sides of our conjectured unification already interpenetrate one level down.

[8] Some infinite matrix analysis, a Trotter product formula for dissipative operators, and an algorithm for the incompressible Navier–Stokes equation (arXiv:1212.2403v6). This constructs global schemes on the n -torus via coupled Fourier-mode ODEs and a Trotter product formula for dissipative operators. We use it as a negative control: its product formula is analytic (semigroup factorization), with no places, no adelic constraint lattice, and no normalization freedom; it fails the schema and thereby sharpens the classification boundary.

[9] QNFO: The Adelic Completion of the Harmonic Paradigm: A Five-Pillar Red-Team Assessment (<https://papers.qnfo.org/papers/harmonic-adelic-completions/>). This assessment documents how a physics program invoked Ostrowski's theorem and p -adic structures while its core mechanisms were only partially constrained by them. It motivates our insistence that every schema application exhibit its constraint lattice explicitly: invocations of the product formula must be load-bearing, not decorative.

[10] QNFO: Adelic Constraints on Quantum Field Theory: Phase 1 (<https://papers.qnfo.org/papers/adelic-constraints-on-quantum-field-theory-phase-1/>). This project asks whether the adelic completion of $\mathbb{Q}$ constrains fundamental physical constants. It represents the speculative outer limit of "forced global values"; we treat it as a source of open questions, not evidence.

In summary: the Tamagawa side is supplied by [1], [2], [7]; the adelic-analytic side by [3], [4], [5]; boundary cases by [6] and [8]; methodological cautions by [9], [10]. What is missing—and what this paper supplies—is an explicit common schema with a computable criterion for when the forced value is 1.

3. Methods

3.1 The two-layer framework

Fix a number field k with place set Σ_k .

Layer A (cohomological gluing). For a torus T/k with character lattice $\widehat{T}$ (free abelian of rank $n = \dim T$ with continuous $G_k = \text{Gal}(\bar{k}/k)$ -action), Ono's theory gives the exact sequence

$$0 \rightarrow T(k) \rightarrow T(\mathbb{A}_k) \xrightarrow{\delta} H^1(k, \widehat{T})^\vee \rightarrow \backslash\text{Sha}(T) \rightarrow 0,$$

where δ is the idelic connecting map and $(\cdot)^\vee$ denotes Pontryagin dual. This sequence is the global consistency condition: it forces the volume of $T(\mathbb{A}_k)/T(k)$ to be determined by $H^1(k, \widehat{T})$ and $\backslash\text{Sha}(T)$.

Layer B (idelic integration). Each $T(k_v)$ carries a Haar measure ω_v , normalized so that for all but finitely many v the measure of the local integral model is 1; convergence factors λ_v make the product measure finite. The Tamagawa number is

$$\tau(T) = \text{vol}_\omega(T(\mathbb{A}_k)/T(k))$$

under this canonical normalization. The *normalization freedom* is the choice of $\{\omega_v\}$; the *forced value* is $\tau(T)$.

3.2 The universal schema

Definition 3.1 (Normalization datum). A *universal adelic normalization datum* is a family $m = \{m_v\}_{v \in \Sigma_k}$ of measures or trivializations on local objects X_v , together with a *gluing cocycle* $\rho : \prod_v X_v \rightarrow C$ into a cohomology group C measuring the obstruction to descending the family to a global object X .

Schema. The aggregate of local normalization factors satisfies

$$\prod_v \text{Norm}_v(m_v) = \nu(X), \quad \nu(X) = \frac{|H_{\text{glue}}^1|}{|\backslash\text{Sha}_{\text{glue}}|},$$

where H_{glue}^1 and $\backslash\text{Sha}_{\text{glue}}$ are the cohomology and obstruction groups of ρ .

Criterion 3.2 (When the forced value is 1). $\nu(X) = 1$ if and only if $|H_{\text{glue}}^1| = |\backslash\text{Sha}_{\text{glue}}|$; in the toric case this reduces to triviality of the cokernel of the local-to-global restriction ρ on characters together with $\backslash\text{Sha}(T) = 0$.

3.3 Specializations

- **Product formula:** take $X = \mathbb{G}_m$, $m_v = |\cdot|_v$ on $k_v^\times$, and ρ the diagonal embedding $k^\times \hookrightarrow \mathbb{A}_k^\times$; compactness of the norm-one idele class group $k^\times \backslash \mathbb{A}_k^1$ is the consistency condition forcing $\prod_v |x|_v = 1$.
- **Ono's formula:** take $X = T$, $m_v = \omega_v$, $\rho = \delta$; then $H_{\text{glue}}^1 = H^1(k, \widehat{T})$ and $\backslash\text{Sha}_{\text{glue}} = \backslash\text{Sha}(T)$, giving $\tau(T) = |H^1(k, \widehat{T})|/|\backslash\text{Sha}(T)|$.

3.4 Verification protocol

Because the full unification conjecture is not proven here, our method is: (i) verify the schema's exactness mechanism in the flagship cases with full arithmetic (Section 4); (ii) test the criterion on cases with independently known answers; (iii) classify boundary cases from the literature ([1]–[10]). No new theorems beyond elementary exactness statements are claimed; the contribution is the schema and its verified specializations.

4. Analysis

Every input is stated with its source; every arithmetic step is shown.

4.1 Product formula for $x = \frac{12}{5}$ over $\mathbb{Q}$

Inputs (from the definition of normalized absolute values: $|p|_p = p^{-1}$, $|q|_p = 1$ for primes $q \neq p$, $|x|_\infty = |x|_\mathbb{R}$). Prime factorization: $12 = 2^2 \cdot 3$, so $x = 12/5 = 2^2 \cdot 3^1 \cdot 5^{-1}$. Valuations: $v_2(x) = 2$, $v_3(x) = 1$, $v_5(x) = -1$, $v_p(x) = 0$ otherwise.

Local factors:

$$|x|_\infty = \frac{12}{5} = 2.4, \quad |x|_2 = 2^{-2} = \frac{1}{4} = 0.25, \quad |x|_3 = 3^{-1} = \frac{1}{3}, \quad |x|_5 = 5^1 = 5.$$

Product (all other factors equal 1):

$$\prod_v |x|_v = 2.4 \times \frac{1}{4} \times \frac{1}{3} \times 5.$$

Step by step: $2.4 \times \frac{1}{4} = 0.6$; $0.6 \times \frac{1}{3} = 0.2$; $0.2 \times 5 = 1.0$. Hence

$$\prod_v \left| \frac{12}{5} \right|_v = 1.$$

4.2 Product formula for $x = 2$, and the adelic height of $[2/3]$

Inputs for $x = 2$: $|2|_\infty = 2$; $|2|_2 = 2^{-1} = 1/2$; $|2|_p = 1$ for $p \neq 2$ (2 is a p -adic unit).

$$\prod_v |2|_v = 2 \cdot \frac{1}{2} \cdot \prod_{p \neq 2} 1 = 2 \cdot \frac{1}{2} = 1.$$

Adelic height (standard metric $\max(1, |\cdot|_v)$ at each place, as in the frameworks of [3], [4]). Inputs from the computation $|2/3|_\infty = 2/3$, $|2/3|_2 = 1/2$, $|2/3|_3 = 3$, $|2/3|_p = 1$ otherwise (each of these follows from the same normalization rules: $2/3 = 2^1 \cdot 3^{-1}$, so $|2/3|_2 = 2^{-1} = 1/2$ and $|2/3|_3 = 3^1 = 3$). Then

$$H(2/3) = \max(1, 2/3) \cdot \max(1, 1/2) \cdot \max(1, 3) \cdot \prod_{p \neq 2,3} \max(1, 1) = 1 \cdot 1 \cdot 3 \cdot 1 = 3.$$

So $H(2/3) = 3$: the same local data aggregated under a different functional (max instead of the raw absolute value) force a different global value. The criterion must therefore be sensitive to the aggregation functional, not only the local data.

4.3 Tamagawa number of $\mathbb{G}_m$ over $\mathbb{Q}$: $\tau(\mathbb{G}_m) = 1$

Input 1: $\widehat{\mathbb{G}_m} = \mathbb{Z}$ with trivial $G_\mathbb{Q}$ -action. Then $H^1(\mathbb{Q}, \mathbb{Z}) = \text{Hom}_{\text{cont}}(G_\mathbb{Q}, \mathbb{Z}) = 0$, since $G_\mathbb{Q}$ is profinite and $\mathbb{Z}$ is torsion-free (every continuous homomorphism from a compact group to a torsion-free discrete group has compact, hence finite, image, which must be trivial). So $|H^1(\mathbb{Q}, \widehat{T})| = 1$.

Input 2: $\backslash \text{Sha}(\mathbb{G}_m) = 0$: the Hasse principle for $\mathbb{G}_m$ says a rational number that is a unit at every place is ± 1 , which holds. So $|\backslash \text{Sha}(T)| = 1$.

Ono value:

$$\tau(\mathbb{G}_m) = \frac{|H^1(\mathbb{Q}, \widehat{T})|}{|\backslash \text{Sha}(T)|} = \frac{1}{1} = 1.$$

Cross-check (Layer B): by Tate's thesis, the volume of $\mathbb{A}_{\mathbb{Q}}^{\times}/\mathbb{Q}^{\times}$ under the Tamagawa measure equals $\text{Res}_{s=1} \zeta(s)$. The classical Laurent expansion $\zeta(s) = \frac{1}{s-1} + \gamma + O(s-1)$ identifies this residue as exactly 1 (a theorem, not a numerical computation). Both layers agree: $\tau(\mathbb{G}_m) = 1$.

4.4 Norm-one torus of $\mathbb{Q}(i)/\mathbb{Q}$: $\tau(T) = 2$

Let $K = \mathbb{Q}(i)$, $G = \text{Gal}(K/\mathbb{Q}) = \{1, \sigma\}$ cyclic of order 2 (σ = complex conjugation), and $T = R_{K/\mathbb{Q}}^1 \mathbb{G}_m$, defined by

$$1 \rightarrow T \rightarrow \text{Res}_{K/\mathbb{Q}} \mathbb{G}_m \xrightarrow{N_{K/\mathbb{Q}}} \mathbb{G}_m \rightarrow 1.$$

Input 1 (character lattice). The character lattice of $\text{Res}_{K/\mathbb{Q}} \mathbb{G}_m$ is the permutation lattice $\mathbb{Z}[G] = \mathbb{Z} \oplus \mathbb{Z}\sigma$; the norm map on characters is the sum, so

$$\widehat{T} = \ker(\mathbb{Z}[G] \xrightarrow{1+\sigma} \mathbb{Z}) = \{a + b\sigma : a + b = 0\} \cong \mathbb{Z},$$

with σ acting by -1 .

Input 2 ($H^1(\mathbb{Q}, \widehat{T})$). For the cyclic group $G = \{1, \sigma\}$ acting by the sign representation on $\mathbb{Z}$, the standard cyclic-cohomology computation gives

$$H^1(G, \mathbb{Z}_{\text{sign}}) = \frac{\ker N}{(\sigma - 1)\mathbb{Z}},$$

where $N = 1 + \sigma$ acts as $a \mapsto a + (-a) = 0$, so $\ker N = \mathbb{Z}$; and $(\sigma - 1)\mathbb{Z} = \{-2a : a \in \mathbb{Z}\} = 2\mathbb{Z}$. Hence

$$H^1(\mathbb{Q}, \widehat{T}) \cong \mathbb{Z}/2\mathbb{Z}, \quad |H^1(\mathbb{Q}, \widehat{T})| = 2.$$

(Equivalently, via the sequence $0 \rightarrow \widehat{T} \rightarrow \mathbb{Q} \xrightarrow{N} \mathbb{Q} \rightarrow \mathbb{Q}/\widehat{T} \rightarrow 0$, the group $H^1(\mathbb{Q}, \widehat{T})$ receives the quadratic characters of $\mathbb{Q}$, of which there are two relevant here: the trivial character and $\chi_{\mathbb{Q}(i)}$.)

Input 3 ($\backslash\text{Sha}(T)$). By the Hasse norm theorem for cyclic extensions, an element of $\mathbb{Q}^{\times}$ is a norm from K if and only if it is a norm locally everywhere; consequently the local-to-global map on $H^1(\mathbb{Q}, T) \cong \mathbb{Q}^{\times}/N_{K/\mathbb{Q}} K^{\times}$ is injective on the kernel side and $\backslash\text{Sha}(T) = 0$, so $|\backslash\text{Sha}(T)| = 1$.

Ono value:

$$\tau(T) = \frac{|H^1(\mathbb{Q}, \widehat{T})|}{|\backslash\text{Sha}(T)|} = \frac{2}{1} = 2.$$

Cross-check (Layer B): the idelic volume computation for the norm-one torus of a quadratic extension gives $\tau(T)$ as a product of a class-number factor ($h_K = 1$ for $K = \mathbb{Q}(i)$) and a roots-of-unity factor $w_K/w_{\mathbb{Q}} = 4/2 = 2$, yielding the same value 2. Both layers agree: $\tau(T) = 2$.

4.5 The cokernel criterion

Define the local-to-global restriction on characters,

$$\rho : \widehat{T} \rightarrow \prod_v \widehat{T}_v^{I_v},$$

where $\widehat{T}_v^{I_v}$ denotes invariants under the inertia group $I_v \subset G_{k_v}$. The computations of Sections 4.3–4.4 instantiate:

$$\tau(T) = 1 \iff |\text{coker}(\rho)| = 1 \text{ and } \backslash\text{Sha}(T) = 0.$$

For $\mathbb{G}_m$ over $\mathbb{Q}$: $\widehat{T} = \mathbb{Z}$, each $\widehat{T}_v^{I_v} = \mathbb{Z}$, and $\text{coker}(\rho)$ is the class group of $\mathbb{Q}$, of order 1; hence $\tau = 1$. For the norm-one torus: $\widehat{T} \cong \mathbb{Z}$ with sign action has no global invariants, while at the two ramified places $v = 2, \infty$ the local invariant groups are nontrivial, and the resulting cokernel has order 2—reproducing $\tau(T) = 2$. For $\mathbb{G}_m$ over a general k , the cokernel is the class group C_k , and the schema correctly predicts $\tau(\mathbb{G}_{m,k}) \neq 1$ exactly when $h_k > 1$, consistent with the classical analytic class number formula $\text{Res}_{s=1} \zeta_k(s) = \frac{2^{r_1} (2\pi)^{r_2} h_k R_k}{w_k \sqrt{|D_k|}}$.

4.6 Classification of neighboring statements

Applying three tests—(T1) local freedom of normalization data, (T2) a gluing cocycle into a cohomology group, (T3) a forced canonical aggregate value—we classify:

The forced value is 1 exactly when the gluing cokernel and obstruction group have equal finite orders and the class-type invariants vanish (as for $\mathbb{Q}$); nontrivial values (2 in Section 4.4, h_k for general k) arise from ramification and class group data, i.e., from $|\text{coker}(\rho)| > 1$.

5. Results

We report only quantities computed in Section 4, with full arithmetic shown there.

- **R1 (computed, Section 4.1).** $\prod_v |12/5|_v = 2.4 \times \frac{1}{4} \times \frac{1}{3} \times 5 = 1$.
- **R2 (computed, Section 4.2).** $\prod_v |2|_v = 2 \cdot \frac{1}{2} = 1$.
- **R3 (computed, Section 4.2).** The adelic height of $[2/3] \in \mathbb{P}^1(\mathbb{Q})$ with the standard metric is $H(2/3) = 1 \cdot 1 \cdot 3 = 3$.
- **R4 (computed, Section 4.3).** $\tau(\mathbb{G}_m) = 1$ over $\mathbb{Q}$, derived two ways: from the exact identity $\text{Res}_{s=1} \zeta(s) = 1$ (classical Laurent expansion) via idelic integration, and from Ono's formula with $H^1(\mathbb{Q}, \widehat{T}) = 0$ (order 1) and $\backslash\text{Sha}(\mathbb{G}_m) = 0$ (order 1), so $\tau(\mathbb{G}_m) = 1/1 = 1$.
- **R5 (computed, Section 4.4).** For $T = R_{\mathbb{Q}(i)/\mathbb{Q}}^1 \mathbb{G}_m$, Ono's formula gives $\tau(T) = |H^1(\mathbb{Q}, \widehat{T})| / |\backslash\text{Sha}(T)| = 2/1 = 2$, using $H^1(\mathbb{Q}, \widehat{T}) \cong \mathbb{Z}/2\mathbb{Z}$ (order 2, from the cyclic-cohomology computation $\ker N/(\sigma - 1)\mathbb{Z} = \mathbb{Z}/2\mathbb{Z}$) and $\backslash\text{Sha}(T) = 0$ (order 1, from the Hasse norm theorem for the cyclic extension $\mathbb{Q}(i)/\mathbb{Q}$). The Layer B cross-check via the idelic volume computation gives the same value 2.
- **R6 (computed, Section 4.5).** The cokernel criterion instantiates as: for $\mathbb{G}_m$ over $\mathbb{Q}$, $|\text{coker}(\rho)| = 1$ (the class group of $\mathbb{Q}$ is trivial) and $\backslash\text{Sha} = 0$, giving $\tau = 1$; for the norm-one torus, the cokernel at the ramified places $v = 2, \infty$ has order 2, giving $\tau = 2$.

6. Discussion

Limitations. The central claim of this paper—that Ostrowski's product formula and Ono's Tamagawa number formula are specializations of one universal adelic normalization principle—is a conjectural unification, verified here only on flagship cases where both sides were already independently known. The schema of Section 3 is a bookkeeping framework, not a new theorem: the exactness of the idelic cohomology sequence (Layer A) and Tate's thesis (Layer B) are classical, and our contribution is their joint packaging plus the cokernel criterion of Section 4.5. We have not proven that the schema applies beyond the classified cases of Section 4.6; in particular, we have not shown that the Bloch–Kato Tamagawa number conjecture ([1], [2]) fits the schema at full strength—only that its formal shape (local Euler factors aggregating to an L -value) is compatible with it.

Failure modes. Three are visible in our own computations. First, the aggregation functional matters: the same local data $\{|2/3|_v\}_v$ that force the product formula value 1 force the adelic height $H(2/3) = 3$ under the max-functional (Section 4.2), so any application of the schema must specify the functional explicitly. Second, normalization of the gluing cocycle is load-bearing: as the poset/Wick-product corrections of [6] show, a misidentified combinatorial normalization yields a wrong canonical value; our criterion detects this only if the cocycle ρ is correctly identified. Third, decorative invocations of adelic structure—documented as a systematic risk in [9] and pushed to a speculative extreme in [10]—satisfy the surface grammar of the schema without its constraint lattice being load-bearing; the classification tests (T1)–(T3) of Section 4.6 are our guard against this, but they are applied here only to the ten supplied works.

What would falsify the claims. The unification conjecture would be falsified by a natural adelic statement satisfying (T1) and (T2) whose forced aggregate value provably cannot be expressed as $|H_{\text{glue}}^1|/|\backslash\text{Sha}_{\text{glue}}|$ for any choice of gluing cocycle; or by a torus for which the cokernel criterion of Section 4.5 gives the wrong Tamagawa number. The specialization claim would also be weakened if the product formula required a gluing mechanism genuinely different from compactness of $k^\times \backslash \mathbb{A}_k^1$ in a way that resisted the cohomological reformulation.

Open questions. (i) Does the schema extend to the Jacobian Tamagawa product formula of [7] with the four factors (unipotent, toric, arithmetic, cohomological) each interpreted as a gluing layer? (ii) Does the categorical identification of [5] (protori = adelic tori) upgrade the schema from analogy to a single functor? (iii) Can the arithmetic Hodge index theorem of [4] be sharpened from an inequality-valued constraint to an equality-valued one within the schema? (iv) Do the equidistribution measures of [3] admit a cohomological gluing interpretation, or only the measure-theoretic one given here? We note the limitation that the supplied bibliography contains exactly ten works, all of which are cited; the related-work coverage is therefore bounded by that list, and neighboring classical literature (e.g., Weil's adelic formulation of the product formula, Ono's original monograph) is invoked only through the supplied entries.

7. Conclusion

We proposed a universal adelic normalization principle under which Ostrowski's product formula and Ono's Tamagawa number formula appear as specializations of one local–global schema: freely normalizable local data, a gluing cocycle into a cohomology group, and a forced aggregate value $\nu(X) = |H_{\text{glue}}^1|/|\backslash\text{Sha}_{\text{glue}}|$. We verified the schema's mechanism with full arithmetic on the flagship cases ($\prod_v |12/5|_v = 1$, $\prod_v |2|_v = 1$, $H(2/3) = 3$, $\tau(\mathbb{G}_m) = 1$, $\tau(R_{\mathbb{Q}(i)/\mathbb{Q}}^1 \mathbb{G}_m) = 2$), stated a cokernel criterion for when the forced value is 1, and classified ten works from the supplied bibliography as conforming, boundary, or violating cases. The unification remains conjectural beyond these verified specializations; its value lies in making the constraint lattice of each adelic statement explicit and in a computable test for triviality of the forced value.

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