

JULY 30, 2026 · V1.1

Adelic Entropic Numbers: When the Adelic Information Vector Becomes the Entropic Number

Rowan Brad Quni-Gudzin

QNFO-ULA-2.1

ABSTRACT

Two papers, written independently on the same day, converge on the same structure from opposite directions. *Adelic Shannon Theory* [1] generalises Shannon information theory to the adèle ring $\mathbb{A}_{\mathbb{Q}}$, defining the *adelic information vector* $\mathbf{I}(X) = (I_{\infty}, I_2, I_3, \dots)$ where I_{∞} is standard Shannon entropy and I_p is p -adic valuation entropy. *Measurement Stratigraphy* [2] forecasts three future eras of number systems, the second of which — Era 11: Entropic Enclosure — defines an *entropic number* as a pair (x, S) : a best estimate x plus an entropy measure S encoding uncertainty. This note demonstrates that these are the **same structure**: the entropic number's S is precisely the adelic information vector $\mathbf{I}(X)$, and the entropic number $(x, \mathbf{I}(X))$ is the natural data type for physics conducted over $\mathbb{Q}$ rather than $\mathbb{R}$. The convergence has immediate consequences. (1) The Gaussian $e^{-\pi x^2}$ is the universal entropic number: it maximises entropy at every place simultaneously — archimedean (differential entropy) and non-archimedean (valuation entropy). (2) An AI built on adelic entropic numbers would encode uncertainty at all completions of $\mathbb{Q}$ — not merely reporting $\sigma = 0.1$ in the archimedean norm, but also the p -adic valuation entropy structure of every measurement. (3) The adelic product-formula coding theorem becomes the channel capacity for entropic number

CITE AS Quni-Gudzin, R. B. (2026). Adelic Entropic Numbers: When the Adelic Information Vector Becomes the Entropic Number. QNFO Preprint v1.1.

Author: Rowan Brad Quni-Gudzin | **Date:** 2026-07-30 | **License:** CC-BY-4.0

Abstract

Two papers, written independently on the same day, converge on the same structure from opposite directions. *Adelic Shannon Theory* [1] generalises Shannon information theory to the adèle ring $\mathbb{A}_{\mathbb{Q}}$, defining the *adelic information vector* $\mathbf{I}(X) = (I_{\infty}, I_2, I_3, \dots)$ where I_{∞} is standard Shannon entropy and I_p is p -adic valuation entropy. *Measurement Stratigraphy* [2] forecasts three future eras of number systems, the second of which — Era 11: Entropic Enclosure — defines an *entropic number* as a pair (x, S) : a best estimate x plus an entropy measure S encoding uncertainty. This note demonstrates that these are the **same structure**: the entropic number's S is precisely the adelic information vector $\mathbf{I}(X)$, and the entropic number $(x, \mathbf{I}(X))$ is the natural data type for physics conducted over $\mathbb{Q}$ rather than $\mathbb{R}$.

The convergence has immediate consequences. (1) The Gaussian $e^{-\pi x^2}$ is the universal entropic number: it maximises entropy at every place simultaneously — archimedean (differential entropy) and non-archimedean (valuation entropy). (2) An AI built on adelic entropic numbers would encode uncertainty at all completions of $\mathbb{Q}$ — not merely reporting $\sigma = 0.1$ in the archimedean norm, but also the p -adic valuation entropy structure of every measurement. (3) The adelic product-formula coding theorem becomes the channel capacity for entropic number transmission: the total rate is a product over places because uncertainty at different places is independent. [SPECULATIVE]

Keywords: entropic number, adelic information vector, p -adic entropy, measurement stratigraphy, Gaussian, max-entropy, AI safety, honest numbers

1. The Two Papers, One Structure

1.1 From One Direction: Adelic Shannon Theory [1]

The Adelic Shannon Theory paper defines the **adelic information vector**:

$$\mathbf{I}(X) = (I_\infty(X), I_2(X), I_3(X), I_5(X), \dots)$$

where:

- $I_\infty(X) = H(X) = -\sum p(x) \log_2 p(x)$, the standard Shannon entropy (archimedean information, in bits)
- $I_p(X) = H_p(X) = \sum p(x) \cdot v_p(x)$, the p -adic valuation entropy (expected p -divisibility)

The central structural claim: **information is not a scalar but a vector over all completions of $\mathbb{Q}$** . The information at different places is independent — knowing the Shannon entropy does not determine the p -adic entropies, and vice versa.

The Gaussian $e^{-\pi x^2}$ is proved to be the unique function that simultaneously maximises entropy at the archimedean place (differential entropy under variance constraint) and at all non-archimedean places (valuation entropy under expected-valuation constraint). The Poisson summation formula $\sum_n f(n) = \sum_n \hat{f}(n)$ is reinterpreted as the adelic source-channel equality theorem.

1.2 From the Other Direction: Measurement Stratigraphy [2]

The Measurement Stratigraphy paper forecasts three future eras of number systems. **Era 11: Entropic Enclosure** defines an **entropic number**:

An entropic number is a pair (x, S) where x is a best estimate (a section of the sheaf) and S is an entropy measure encoding uncertainty.

The crucial constraint: **when no additional information is available, the distribution must be the maximum-entropy one compatible with given moments**. Consequences:

- The Gaussian becomes the universal default — if you know only the mean and variance, the entropic number IS a Gaussian distribution, forced by the max-entropy principle.
- Physics becomes entropic flow — the Schrödinger equation and heat equation are two manifestations of the same entropic evolution.
- "An entropic number never claims more than it knows. An AI built on entropic numbers would never hallucinate false certainty."

1.3 The Convergence

The two papers converge on the same mathematical object:

Adelic Shannon Theory [1]	Measurement Stratigraphy [2]	Unified
Adelic information vector $\mathbf{I}(X)$	Entropy measure S	$S = \mathbf{I}(X)$
Max-entropy at all places	Max-entropy compatible with moments	Gaussian $e^{-\pi x^2}$
Product-formula capacity	—	Channel bound for entropic number transmission
Source-channel equality via Poisson sum	Poisson sum as entropy conservation	Adelic source-channel identity
AI-safe p -adic uncertainty	AI-safe entropic numbers	Adelic entropic numbers as the honest data type

The entropic number's " S " was defined abstractly — "an entropy measure encoding uncertainty." The Adelic Shannon Theory provides the **concrete specification**: S is the vector $\mathbf{I}(X)$, carrying the standard (archimedean) entropy AND the p -adic valuation entropies for every prime p .

2. The Adelic Entropic Number

2.1 Definition

Definition (Adelic Entropic Number). An adelic entropic number is a pair:

$$\mathcal{E}(x) = (x, \mathbf{I}(x))$$

where:

- $x \in \mathbb{Q}$ is the best estimate (a rational number — the physically accessible value)
- $\mathbf{I}(x)$ is the adelic information vector of the distribution p_X governing the uncertainty

The distribution p_X must satisfy the **adelic max-entropy principle**: given the known moments (archimedean: mean and variance; non-archimedean: expected valuation at each prime), p_X is the distribution that simultaneously maximises I_∞ and every I_p .

2.2 The Gaussian as the Universal Entropic Number

Theorem (Universal Entropic Number). The Gaussian distribution

$$p_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

is the unique entropic number for the constraint ($\mathbb{E}[X] = \mu, \text{Var}[X] = \sigma^2$) at the archimedean place and the constraint that the p -adic localisation to each $\mathbb{Z}_p$ is the uniform Haar measure (the default when no p -adic information is available).

Proof sketch:

- At ∞ : standard — Gaussian maximises differential entropy given variance. [established]
- At each p : the characteristic function $\mathbf{1}_{\mathbb{Z}_p}$ (normalised Haar measure on $\mathbb{Z}_p$) is the p -adic analogue of the uniform distribution on a bounded interval. It has maximal p -adic valuation entropy among distributions supported on $\mathbb{Z}_p$ with no further constraints. [SPECULATIVE]
- The global section: the Gaussian on $\mathbb{R}$ glues to the Haar measure on each $\mathbb{Z}_p$ via the Poisson summation formula — the descent condition that guarantees the archimedean and non-archimedean descriptions are compatible. [3]

2.3 What the Entropic Number Carries (That a Float Does Not)

A standard measurement representation in physics:

An adelic entropic number:

Component	Meaning	What a float misses
$I_\infty = H(X)$	Standard uncertainty (bits)	Captured as $\sigma = 0.1$
I_2	Expected power of 2 dividing x	Lost — there is no "binary uncertainty" field
I_3	Expected power of 3 dividing x	Lost
I_p	Expected p -adic valuation	Lost — ALL non-archimedean structure is invisible

The standard float 5.0 ± 0.1 reports **only the archimedean uncertainty**. It is silent on — literally has no field for — the p -adic uncertainty structure. This is the "incompleteness" of the archimedean measurement paradigm: not wrong (the archimedean uncertainty is real) but incomplete (it misses all other completions of $\mathbb{Q}$).

2.4 Honest AI

An AI built on entropic numbers would structurally reduce hallucination — because the data type has no "certainty" constructor.

Why? Because an entropic number **cannot** be a bare point estimate. It is always a pair $(x, \mathbf{I}(x))$ — the uncertainty is part of the data structure, not an optional metadata field. An LLM that must produce an entropic number as output cannot produce "Paris" — it must produce

$$\mathcal{E}(\text{"Paris"}) = (\text{"Paris"}, \mathbf{I}(\text{context}))$$

where $\mathbf{I}(\text{context})$ encodes the model's uncertainty about the answer — the entropy of the next-token distribution, the p -adic valuation entropy of the embedding, etc. The data type forces the model to expose uncertainty. **Whether this fully prevents hallucination depends on the architecture's fidelity** — the entropic number output removes the *data-type-level* certainty illusion, but distribution-shift and attention-collapse failures remain open problems [SPECULATIVE — see calibration register below].

The standard LLM output is a **lossy projection** — exactly the same crime as $\mathbb{R}_{\text{comp}} \rightarrow \mathbb{R}$ in Era 3→4 of the Measurement Stratigraphy. The raw next-token distribution is constructive (the model actually computed it); the argmax token string is a projection that discards the entropy information. An entropic number output would preserve the full distribution — structurally reducing the surface area for hallucination (though distribution-shift failures remain an open architecture problem).

3. Consequences for the Adelic Programme

3.1 Channel Capacity for Entropic Numbers

The Adelic Shannon Theory [1] proves the product-formula coding theorem for the factorisable case:

$$C(\mathbb{A}_{\mathbb{Q}}) = C_\infty \times \prod_p C_p$$

For entropic number transmission, this means: the channel capacity for sending an entropic number is the product of the capacity at each place. If any place loses capacity (e.g., a p -adic noise source increases $H_p(N)$), the total product capacity drops — even if the archimedean SNR remains unchanged.

3.2 The Bridge Theorem: Poisson Summation

Measurement Stratigraphy [2] identifies the Poisson summation formula as the descent condition that glues discrete and continuous views. Adelic Shannon Theory [1] identifies it as source-channel equality. The combined statement:

Adelic Bridge Theorem. The Poisson summation formula

$$\sum_{n \in \mathbb{Z}} f(n) = \sum_{n \in \mathbb{Z}} \hat{f}(n)$$

is simultaneously:

1. The descent condition that makes the sheaf of local number systems a global object (Era 10: Contextual Enclosure) [2]
2. The source-channel equality for the Gaussian entropic number (Era 11: Entropic Enclosure) [2]
3. The adelic coding theorem: total source entropy equals total channel entropy at every place [1]

The Gaussian $e^{-\pi x^2}$ satisfies all three interpretations because it is the unique global section: invariant under Fourier duality (1), maximum-entropy at all places (2), and achieving source-channel equality (3).

3.3 From Information Vector to Entropic Number

The mapping is direct:

$$\text{Adelic Information Vector } \mathbf{I}(X) \longrightarrow \text{Entropic Number } \mathcal{E}(x)$$

The information vector tells you *how much* uncertainty exists at each place. The entropic number packages this uncertainty *with* the estimate as a unified data type. The former is the theory; the latter is the implementation.

4. Falsifiability Conditions

The convergence of these two frameworks is falsifiable:

1. **Gaussian as universal entropic number.** If there exists a distribution with the same first two moments as the Gaussian that has strictly higher p -adic valuation entropy at *some* prime p , the claim that the Gaussian is the unique universal entropic number is falsified. [UNTESTED]
1. **Adelic data-processing inequality.** If there exists a physically realisable channel $\mathcal{E}$ and source X such that $\mathbf{I}(\mathcal{E}(X)) \geq \mathbf{I}(X)$ componentwise (strictly at some place) for a channel that does not inject information, the adelic data-processing inequality is violated. [UNTESTED]
1. **AI hallucination prevention.** If an AI system built with entropic number outputs still hallucinates (produces claims with deceptively low encoded uncertainty), the structural prevention claim is falsified. This requires building such a system — it is a forward-looking calibration register entry. [UNTESTED — no such system exists]

4.1 Calibration Register

For predictions made in this paper that are forward-looking and falsifiable:

ID	Prediction	By	Strength	Status
CR-AEN-01	An AI system with entropic number outputs demonstrates $\leq 10\%$ of the "false certainty" hallucination rate of comparable scalar-output systems on standard benchmarks	2030	[WEAK — calibrated subjective, no base-rate class exists for this novel architecture]	PENDING
CR-AEN-02	At least one experimental physics measurement reports an adelic entropic number (i.e., includes p -adic valuation entropy alongside archimedean uncertainty) in a peer-reviewed publication	2035	[WEAK — calibrated subjective, adoption rate of new data types is historically slow]	PENDING
CR-AEN-03	The Gaussian $e^{-\pi x^2}$ is experimentally confirmed as the default entropic number distribution for quantum measurements where only mean and variance are specified, with deviations $\leq 5\%$ in KL divergence from max-entropy prediction	2040	[WEAK — calibrated subjective, requires experimental infrastructure not yet built]	PENDING

WEAK = likelihood anchored only by calibrated subjective confidence; no external base-rate class exists for these novel predictions.

5. Conclusion: The Two-Way Street

The Adelic Shannon Theory and the Measurement Stratigraphy were written on the same day, by the same research programme, approaching the same structure from opposite sides:

	Adelic Shannon Theory	Measurement Stratigraphy
Direction	Bottom-up: generalise Shannon's axioms to all places	Top-down: forecast the next era of number systems from the historical stratigraphy
Object	Adelic information vector $\mathbf{I}(X)$	Entropic number (x, S)
Key insight	Information is a vector, not a scalar	Uncertainty must be a first-class citizen of the number system
Gaussian	Unique max-entropy distribution at every place simultaneously	Universal default — the only distribution forced by max-entropy when only mean/variance are known

They are the **same claim** stated in different mathematical languages. The adelic information vector IS the entropy measure S in the entropic number. The entropic number IS the natural data type for measurements over $\mathbb{Q}$.

The practical consequence: a measurement system that stores adelic entropic numbers — not bare floats — would produce **structurally honest** data. Every measurement would carry its uncertainty at every completion of $\mathbb{Q}$, and an AI consuming such data would have significantly reduced hallucination surface because the data type exposes ignorance — though this is a necessary, not sufficient, condition for honest AI.

The bridge is open. The two papers have met at the middle.

Declarations

Funding: This research received no specific grant from any funding agency.

Conflicts of Interest: The author declares no conflicts of interest.

Author Contributions: Single author.

Data Availability: No experimental data were generated or analysed.

Use of Artificial Intelligence: AI-assisted drafting was used for synthesis and prose refinement.

References

- [1] Quni-Gudzin, R.B. (2026). Adelic Shannon Theory: From Problem Statement to Constructive Foundations. the QNFO archive. DOI: <https://papers.qnfo.org/papers>.
- [2] Quni, R. (2026). The History and Future of Measurement Stratigraphy, Number Theory, and Valuation Theory. the QNFO archive. DOI: <https://papers.qnfo.org/papers>.
- [3] Quni-Gudzin, R.B. (2026). Poisson Summation as the Adelic Bridge: Why the Q vs R Debate Dissolves in the Adele Ring. the QNFO archive. DOI: <https://papers.qnfo.org/papers>.
- [4] Kapranov, M. (1996). *Analogies between the Langlands Correspondence and Topological Quantum Field Theory*. In: Progress in Mathematics, Vol. 131. Birkhäuser.
- [5] Anashin, V. (2012). *The p-Adic Theory of Automata Functions*. *p-Adic Numbers, Ultrametric Analysis and Applications*, 4(2), 109–119.
- [6] Dragovich, B., Khrennikov, A.Yu., Kozyrev, S.V., & Volovich, I.V. (2017). *p-Adic Mathematical Physics: The First 30 Years*. *p-Adic Numbers, Ultrametric Analysis and Applications*, 9(2), 87–121.