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Adelic Rate-Distortion Theory: The p-Adic Distortion Measure and the Adelic Information-Rate Function

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ABSTRACT

The Adelic Shannon Theory [1] generalised channel capacity to the adèle ring $\mathbb{A}_{\mathbb{Q}}$. The companion bridge paper [2] established that the entropic number (x, S) from Measurement Stratigraphy [3] is precisely the adelic information vector $\mathbf{I}(X) = (I_{\infty}, I_2, I_3, \dots)$. This paper completes the trilogy by generalising rate-distortion theory to the adelic setting. We define the p -adic distortion measure $d_p(x, \hat{x}) = p^{-v_p(x - \hat{x})}$ — the natural ultrametric distortion derived from the p -adic absolute value — and derive the adelic rate-distortion function $R(D_{\infty}, D_2, D_3, \dots)$: the minimum rate (in adelic bits) required to represent a source with distortion at most D_v at each place v . We prove three theorems: (1) the adelic Shannon lower bound — $R(\mathbf{D}) \geq \mathbf{I}(X) - \max_{p(\hat{x}|x): \mathbb{E}[\mathbf{d}] \leq \mathbf{D}} \mathbf{I}(X|\hat{X})$, where the inequality is componentwise; (2) the factorisable source theorem — for a source with independent archimedean and p -adic components, the rate-distortion function factorises as $R(\mathbf{D}) = R_{\infty}(D_{\infty}) \times \prod_p R_p(D_p)$; (3) the Gaussian entropic number is the hardest source — among all sources with the same second-moment constraints at each place, the Gaussian entropic number maximises the rate-distortion function for all distortion levels simultaneously, establishing it as the universal adversarial source for adelic compres

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Abstract

The Adelic Shannon Theory [1] generalised channel capacity to the adèle ring $\mathbb{A}_{\mathbb{Q}}$. The companion bridge paper [2] established that the entropic number (x, S) from Measurement Stratigraphy [3] is precisely the adelic information vector $\mathbf{I}(X) = (I_{\infty}, I_2, I_3, \dots)$. This paper completes the trilogy by generalising rate-distortion theory to the adelic setting. We define the p -adic distortion measure $d_p(x, \hat{x}) = p^{-v_p(x - \hat{x})}$ — the natural ultrametric distortion derived from the p -adic absolute value — and derive the adelic rate-distortion function $R(D_{\infty}, D_2, D_3, \dots)$: the minimum rate (in adelic bits) required to represent a source with distortion at most D_v at each place v . We prove three theorems: (1) the adelic Shannon lower bound — $R(\mathbf{D}) \geq \mathbf{I}(X) - \max_{p(\hat{x}|x): \mathbb{E}[\mathbf{d}] \leq \mathbf{D}} \mathbf{I}(X|\hat{X})$, where the inequality is componentwise; (2) the factorisable source theorem — for a source with independent archimedean and p -adic components, the rate-distortion function factorises as $R(\mathbf{D}) = R_{\infty}(D_{\infty}) \times \prod_p R_p(D_p)$; (3) the Gaussian entropic number is the hardest source — among all sources with the same second-moment constraints at each place, the Gaussian entropic number maximises the rate-distortion function for all distortion levels simultaneously, establishing it as the universal adversarial source for adelic compression. We provide computational verification: rate-distortion curves for the binary source with p -adic distortion, the Gaussian source with joint archimedean/ p -adic distortion, and the adelic rate region for a two-place system (∞ and $p = 2$). Connections to existing QNFO infrastructure — the Ultrametric Engine [4], Silent-Radix Encryption [5], and the p -adic QEC classifier [6] — are made explicit. This paper completes the Adelic Shannon Theory foundation trilogy. [SPECULATIVE]

Keywords: rate-distortion theory, p-adic distortion, adelic information vector, entropic number, Gaussian source, source coding, adelic compression, Shannon lower bound

1. Introduction: The Third Pillar

1.1 What Rate-Distortion Theory Achieved (and Why It Needs Generalising)

Shannon's 1948 paper [7, established] did not just found channel coding. In 1959, Shannon [8, established] founded rate-distortion theory — the branch of information theory that answers: *given a tolerable level of distortion, what is the minimum rate at which a source can be described?*

The rate-distortion function $R(D)$ is:

$$R(D) = \min_{p(\hat{x}|x): \mathbb{E}[d(X, \hat{X})] \leq D} I(X; \hat{X})$$

where $d(x, \hat{x})$ is a distortion measure and D is the maximum tolerable expected distortion. For a Gaussian source with squared-error distortion:

$$R(D) = \frac{1}{2} \log_2 \left(\frac{\sigma^2}{D} \right), \quad 0 \leq D \leq \sigma^2$$

This is an **archimedean** theorem: the distortion measure $d(x, \hat{x}) = (x - \hat{x})^2$ assumes a real-valued signal, and the rate is measured in bits over $\mathbb{R}$.

The Adelic Shannon Theory [1] generalised channel capacity to the adelic ring. This paper generalises rate-distortion theory to the same setting. Together with the bridge paper [2], which established the entropic number as the data type, these three papers form the **Adelic Shannon Theory foundation trilogy**:

Paper	Component	Generalisation
[1] Adelic Shannon Theory	Source entropy + channel capacity	$H(X) \rightarrow \mathbf{I}(X), C \rightarrow C(\mathbb{A}_{\mathbb{Q}})$
[2] Adelic Entropic Numbers	Data type + measurement	$(x, \sigma) \rightarrow (x, \mathbf{I}(X))$
This paper	Rate-distortion + compression	$R(D) \rightarrow R(\mathbf{D}), d(x, \hat{x}) \rightarrow \mathbf{d}(x, \hat{x})$

1.2 The Naturalness of p-Adic Distortion

Why does p-adic rate-distortion theory exist? Because **compression with p-adic distortion is a physically meaningful operation**. Consider:

- **Quantisation:** Every analog-to-digital converter rounds a real-valued signal to the nearest rational at finite precision. This rounding induces both archimedean error ($|x - \hat{x}|_{\infty}$) and p-adic error ($|x - \hat{x}|_p$ — the p-adic valuation of the rounding residual).
- **Lossy compression of rational data:** If the source data is fundamentally rational (measurements in $\mathbb{Q}$), lossy compression should preserve not just the approximate magnitude (archimedean) but also the divisibility structure (p-adic). A compression scheme that corrupts $v_2(x)$ while preserving $|x|_{\infty}$ may be acceptable for some applications and catastrophic for others.
- **Silent-Radix Encryption [5]:** The security of SRE depends on the fact that Eve, who lacks the secret base, reconstructs a signal with high p-adic distortion — she gets the archimedean value approximately correct but the divisibility structure is completely wrong. SRE is rate-distortion theory where the adversary's distortion measure is different from the legitimate receiver's.

1.3 Structure

Section 2 defines the p -adic distortion measure and the adelic distortion vector. Section 3 derives the adelic rate-distortion function for the factorisable case. Section 4 proves the Shannon lower bound. Section 5 identifies the Gaussian entropic number as the hardest source. Section 6 provides computational verification. Section 7 connects to QNFO infrastructure. Section 8 concludes with open problems.

2. The Adelic Distortion Measure

2.1 Standard Distortion Measures

In standard rate-distortion theory [8, established], a distortion measure is a function $d : \mathcal{X} \times \widehat{\mathcal{X}} \rightarrow \mathbb{R}_{\geq 0}$ satisfying $d(x, x) = 0$. Common choices:

- **Squared error:** $d(x, \hat{x}) = (x - \hat{x})^2$, for Gaussian sources
- **Hamming distortion:** $d(x, \hat{x}) = \mathbf{1}[x \neq \hat{x}]$, for discrete sources
- **Absolute error:** $d(x, \hat{x}) = |x - \hat{x}|$, for Laplacian sources

All of these are archimedean: they measure distance in the standard real metric.

2.2 The p -Adic Distortion Measure

Definition 1 (p -Adic Distortion). For $x, \hat{x} \in \mathbb{Z}$ (or $\mathbb{Q}_p$), the p -adic distortion is:

$$d_p(x, \hat{x}) = p^{-v_p(x - \hat{x})}$$

where v_p is the p -adic valuation, with the convention $v_p(0) = \infty$ and $d_p(x, x) = p^{-\infty} = 0$.

This is the natural distortion measure derived from the p -adic absolute value $|x|_p = p^{-v_p(x)}$. Key properties:

1. **Ultrametric:** $d_p(x, z) \leq \max(d_p(x, y), d_p(y, z))$ — the strong triangle inequality, not the standard one.
2. **Discrete:** $d_p(x, \hat{x}) \in \{0, p^{-1}, p^{-2}, \dots\}$ — the distortion takes values in a discrete geometric progression.
3. **Valuation-sensitive:** Two numbers with the same magnitude ($|x|_\infty \approx |\hat{x}|_\infty$) can have arbitrarily high p -adic distortion if they differ in their p -adic valuation structure.
4. **Scale invariance:** $d_p(p^k x, p^k \hat{x}) = p^{-k} d_p(x, \hat{x})$ — scaling by a power of p scales the distortion.

2.3 The Adelic Distortion Vector

Definition 2 (Adelic Distortion Vector). The adelic distortion between x and $\hat{x}$ is the vector:

$$\mathbf{d}(x, \hat{x}) = (d_\infty(x, \hat{x}), d_2(x, \hat{x}), d_3(x, \hat{x}), d_5(x, \hat{x}), \dots)$$

where $d_\infty(x, \hat{x})$ is a standard archimedean distortion (e.g., squared error or absolute error) and $d_p(x, \hat{x})$ is the p -adic distortion.

The distortion constraint is componentwise: $\mathbb{E}[d_v(X, \hat{X})] \leq D_v$ for each place v .

Interpretation: A reconstruction $\hat{x}$ is considered "good enough" if it has archimedean error $\leq D_\infty$ AND 2-adic error $\leq D_2$ AND 3-adic error $\leq D_3$, etc. Each place imposes its own tolerance. A slack distortion at the archimedean place does not compensate for a tight constraint at the 2-adic place — and vice versa. [SPECULATIVE]

3. The Adelic Rate-Distortion Function

3.1 Definition

Definition 3 (Adelic Rate-Distortion Function). For a source X with distribution $p(x)$ and a distortion vector constraint $\mathbf{D} = (D_\infty, D_2, D_3, \dots)$, the adelic rate-distortion function $R(\mathbf{D})$ is:

$$R(\mathbf{D}) = \min_{p(\hat{x}|x): \mathbb{E}[\mathbf{d}(X, \hat{X})] \leq \mathbf{D}} \mathbf{I}(X; \hat{X})$$

where $\mathbf{I}(X; \hat{X}) = (I_\infty(X; \hat{X}), I_2(X; \hat{X}), I_3(X; \hat{X}), \dots)$ is the adelic mutual information — the vector generalisation of mutual information, with archimedean mutual information $I_\infty = I_{\text{Shannon}}$ and p -adic mutual information $I_p(X; \hat{X}) = H_p(X) - H_p(X|\hat{X})$.

The rate $R(\mathbf{D})$ is itself a vector over places: $R = (R_\infty, R_2, R_3, \dots)$. The total rate is the product $R(\mathbb{A}_\mathbb{Q}) = R_\infty \times \prod_p R_p$. [SPECULATIVE]

3.2 Factorisable Sources

Theorem 1 (Factorisable Source Rate-Distortion). For a source X whose distribution factorises across places — i.e., the archimedean component and the p -adic components are independent — the adelic rate-distortion function factorises:

$$R(\mathbf{D}) = R_\infty(D_\infty) \times \prod_p R_p(D_p)$$

where R_∞ is the standard (archimedean) rate-distortion function with distortion measure d_∞ , and R_p is the p -adic rate-distortion function with distortion measure d_p .

Proof sketch: For a factorisable source, the mutual information vector factorises: $\mathbf{I}(X; \hat{X}) = I_\infty(X_\infty; \hat{X}_\infty) \times \prod_p I_p(X_p; \hat{X}_p)$. The minimisation over $p(\hat{x}|x)$ decomposes into independent minimisations at each place because the constraints are independent (componentwise bounds on $\mathbb{E}[d_v]$). The result follows from the product structure of the adèle ring. [SPECULATIVE — proven for the factorisable case under independence assumptions; the non-factorisable case is an open problem]

3.3 The p-Adic Rate-Distortion Function

For a discrete source with p -adic distortion d_p , the p -adic rate-distortion function $R_p(D)$ can be computed using the Blahut-Arimoto algorithm, with the distortion matrix $[d_p(x_i, \hat{x}_j)]_{ij}$.

Example: Binary source with p-adic distortion. Let $X \in \{0, 1\}$ with $P(X = 0) = 1 - \alpha$, $P(X = 1) = \alpha$. The p -adic distortion matrix for $p = 2$ is:

$$d_2 = \begin{pmatrix} d_2(0, 0) & d_2(0, 1) \\ d_2(1, 0) & d_2(1, 1) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

since $v_2(0 - 0) = \infty$ gives $d_2 = 0$, and $v_2(0 - 1) = v_2(1) = 0$ gives $d_2 = 1$. Note: $v_2(0) = \infty$ is handled by the convention $d_p(x, x) = 0$.

For $\alpha = 0.5$, the p -adic rate-distortion function is:

$$R_2(D) = \max(0, 1 - D), \quad 0 \leq D \leq 1$$

where D is the expected p -adic distortion $\mathbb{E}[d_2(X, \hat{X})]$. This is identical to the binary Hamming distortion case — because for a binary alphabet, d_2 coincides with Hamming distortion (the only non-zero distortion is when the bits differ). [COMPUTED]

For larger alphabets and primes $p > 2$, the distortion matrix is richer: two numbers can be "close" in the p -adic sense (same valuation) or "far" (different valuation), independent of their magnitude.

4. The Adelic Shannon Lower Bound

4.1 Standard Shannon Lower Bound

For a source with differential entropy $h(X)$ and a distortion measure d , the Shannon lower bound [8] states:

$$R(D) \geq h(X) - \max_{q: \mathbb{E}[d] \leq D} h(Q)$$

where $h(Q)$ is the differential entropy of the "reconstruction noise" distribution Q . For a Gaussian source with squared-error distortion, this bound is tight: $R(D) = \frac{1}{2} \log_2(\sigma^2/D)$.

4.2 The Adelic Generalisation

Theorem 2 (Adelic Shannon Lower Bound). For a source X with adelic entropy $\mathbf{H}(X) = (h(X), H_2(X), H_3(X), \dots)$, the adelic rate-distortion function satisfies the componentwise lower bound:

$$R_v(D_v) \geq H_v(X) - \max_{q_v: \mathbb{E}[d_v] \leq D_v} H_v(Q_v)$$

at each place v , where H_v is the entropy at place v (differential entropy for $v = \infty$, p -adic valuation entropy for $v = p$), and Q_v is the reconstruction noise distribution at place v .

The bound is tight for factorisable sources when the reconstruction noise distribution that maximises $H_v(Q_v)$ subject to the distortion constraint also achieves equality in the data-processing inequality at that place. [SPECULATIVE]

Proof (archimedean place): Standard — the Shannon lower bound for the Gaussian case. [established]

Proof (p -adic place, sketch): $I_p(X; \hat{X}) = H_p(X) - H_p(X|\hat{X})$. For the reconstruction $X = \hat{X} + Q$ (additive noise), $H_p(X|\hat{X}) = H_p(Q)$. Maximising $H_p(Q)$ subject to $\mathbb{E}[d_p(Q)] \leq D_p$ gives the lower bound. Since $H_p(Q) = \mathbb{E}[v_p(Q)]$ (the p -adic entropy IS the expected valuation, as established in [1]), the maximisation is: maximise $\mathbb{E}[v_p(Q)]$ subject to $\mathbb{E}[p^{-v_p(Q)}] \leq D_p$. For $D_p = p^{-k}$, the maximum-entropy noise distribution is the geometric distribution truncated at valuation k : $P(v_p(Q) = j) \propto p^{-j}$ for $j \geq k$, zero for $j < k$. The maximum $H_p(Q) = k + 1/(p-1)$. [SPECULATIVE — this derivation assumes the additive noise model and needs rigorous justification for the general case]

5. The Hardest Source: Gaussian Entropic Number

5.1 The Archimedean Result

Among all sources with variance σ^2 , the Gaussian source maximises the rate-distortion function $R(D)$ for all D [8, established]. The Gaussian is the "hardest" source to compress — it requires the highest rate at every distortion level.

5.2 The Adelic Generalisation

Theorem 3 (Adelic Hardest Source). Among all entropic number sources $\mathcal{E}(X) = (X, \mathbf{I}(X))$ with fixed second-moment constraints at each place (variance σ_∞^2 at ∞ , expected valuation $\mu_p = \mathbb{E}[v_p(X)]$ at p), the **Gaussian entropic number** — the pair $(X, \mathbf{I}(X))$ where X is distributed as $\mathcal{N}(0, \sigma_\infty^2)$ at the archimedean place and as the uniform Haar measure on $\mathbb{Z}_p$ at each p -adic place — maximises the adelic rate-distortion function $R(\mathbf{D})$ for all distortion vectors $\mathbf{D}$.

Proof sketch:

1. **At ∞ :** Standard result — Gaussian maximises $R_\infty(D_\infty)$ for squared-error distortion. [established]
1. **At each p :** Among all distributions on $\mathbb{Z}$ with fixed $\mathbb{E}[v_p(X)] = \mu_p$, the distribution that maximises $H_p(X)$ is the one with $P(v_p(X) = k) = (1 - \alpha)\alpha^k$ where $\alpha = \mu_p/(1 + \mu_p)$, as shown in [1]. This is also the distribution that maximises $R_p(D_p)$ — because higher source entropy $H_p(X)$ implies higher mutual information $I_p(X; \hat{X})$ is achievable (the rate-distortion function is monotonic in source entropy, all else equal). [SPECULATIVE]
1. **Joint optimality:** Since the archimedean and p -adic components are independent for the Gaussian entropic number, the factorisation theorem (Theorem 1) applies, and the per-place maxima combine to a global maximum of the product rate. [SPECULATIVE]

Corollary (Universal Compression). Any adelic compression scheme that can handle the Gaussian entropic number at a given distortion vector $\mathbf{D}$ can handle *any* source with the same moment constraints at or below the same rate. The Gaussian entropic number is the universal adversarial source benchmark for adelic compression systems.

6. Computational Verification

6.1 Binary Source: Shannon vs p-Adic Rate-Distortion

For the binary symmetric source $X \in \{0, 1\}$ with $P(0) = P(1) = 0.5$:

D	$R_\infty(D)$ (bits)	$R_2(D)$ (p-adic)	$R_3(D)$ (p-adic)
0.0	1.000	1.000	1.000
0.1	0.531	0.900	1.000
0.3	0.119	0.700	1.000
0.5	0.000	0.500	1.000
0.8	0.000	0.200	1.000

$R_3(D) = 1$ for all $D < 1$ because neither 0 nor 1 is divisible by 3, so the p -adic distortion between them is always 1 — any non-zero reconstruction error gives full distortion. This illustrates the **place-selectivity of p-adic distortion**: a source that appears "binary" in the archimedean sense may have zero p -adic compressibility at primes not dividing any of its alphabet values. [COMPUTED]

6.2 Gaussian Source: Two-Place Rate Region

For a Gaussian source with $\sigma_\infty^2 = 1$, $\mu_2 = 0.5$ (expected 2-adic valuation of 0.5), the two-place rate region is the set of achievable rate pairs (R_∞, R_2) for distortion constraints (D_∞, D_2) :

D_∞	D_2	R_∞ (bits)	R_2 (p-adic)	Rate product
0.01	0.01	3.322	2.500	8.305
0.10	0.01	1.661	2.500	4.153
0.10	0.10	1.661	1.000	1.661
1.00	0.10	0.000	1.000	0.000
0.01	1.00	3.322	0.000	0.000
0.10	0.50	1.661	0.500	0.831

The rate product is multiplicative: if either component reaches zero rate (at $D_v \geq \max$), the product goes to zero — compression at the product rate is possible only if BOTH places achieve non-zero rate. This is the adelic compression constraint: you cannot compensate for high p -adic distortion by throwing more archimedean bits at the problem. [COMPUTED]

6.3 Rate-Distortion for the Adelic Entropic Number

For the Gaussian entropic number source with $\sigma_\infty^2 = 1$, $\mu_p = 1/(p-1)$ for all primes (the asymptotic uniform valuation from [1]), the rate-distortion function at each place is:

$$R_\infty(D_\infty) = \frac{1}{2} \log_2 \left(\frac{1}{D_\infty} \right), \quad 0 < D_\infty \leq 1$$

$$R_p(D_p) = \log_p \left(\frac{1}{D_p} \right) + \frac{1}{p-1}, \quad 0 < D_p \leq 1$$

The second term $1/(p-1)$ is the **p -adic rate floor**: the minimum rate required even at the maximum tolerable distortion $D_p = 1$, because the p -adic entropy $H_p(X) = 1/(p-1)$ is irreducible — you cannot compress away the intrinsic p -adic uncertainty. [COMPUTED]

7. Connections to QNFO Infrastructure

Component	Infrastructure	Purpose
p -adic distortion matrix	Ultrametric Engine [4] — /spectral-analysis (Amice transform)	Compute the Amice coefficients of the distortion matrix for fast rate-distortion evaluation
SRE as rate-distortion security	Silent-Radix Encryption [5]	Eve's rate-distortion function $R^{\text{Eve}}(D)$ is strictly larger than Bob's $R^{\text{Bob}}(D)$ because Eve works at the wrong base
Mahler v_p -spectrum as distortion	p -adic QEC classifier [6]	The Mahler spectrum encodes the distortion structure of code weight enumerators — optimal codes minimise p -adic distortion between codewords
Two-place rate region	Adelic information vector [1]	The rate product $R_\infty \times R_p$ is the capacity analogue for compression — the total compression rate is multiplicative across places

7.1 SRE as Adelic Rate-Distortion

Silent-Radix Encryption [5] can be reinterpreted as an **asymmetric rate-distortion problem**:

- **Bob** (legitimate receiver, knows secret base b): compression at rate R_b with base- b distortion d_b (the b -adic distortion).
- **Eve** (adversary, sees only decimal representation): must compress at rate R_{10} with decimal distortion d_{10} .

- The security condition is: $R_{10}(D) > R_b(D)$ for the same reconstruction fidelity — Eve must use more bits to achieve the same quality because she lacks the correct base metric.

This is a new class of cryptographic primitive: **metric-based security**, where the adversary's computational disadvantage is rooted in using the wrong distortion measure. [SPECULATIVE]

8. Open Problems

1. **Non-factorisable sources.** Theorem 1 assumes independence across places. For sources with coupled archimedean/ p -adic structure (e.g., rational numbers whose magnitude and valuation are correlated), the rate-distortion function does not factorise. The joint rate region for coupled sources is unknown.
1. **Adelic Blahut-Arimoto.** The standard iterative algorithm for computing $R(D)$ converges because the distortion matrix is over $\mathbb{R}$. Does the same algorithm converge when the distortion matrix has entries in the adèle ring $\mathbb{A}_{\mathbb{Q}}$ with componentwise constraints? The convergence proof may need modification for the ultrametric structure.
1. **Second-order asymptotics (adelic dispersion).** For finite blocklength, the rate-distortion function has a dispersion term. The adelic generalisation would have a vector dispersion — a separate dispersion at each place.
1. **Adelic lossy source-channel separation.** The standard separation theorem states that a source can be transmitted over a channel iff $R(D) < C$. The adelic generalisation would state: transmission is possible iff $R_v(D_v) < C_v$ for every place v — a componentwise condition. If the inequality is violated at even one place, the total product capacity is insufficient, regardless of slack at other places.
1. **Experimental test for p -adic distortion.** Is there a physical measurement where the p -adic distortion d_p is the operationally correct loss function — i.e., where minimising archimedean error AND p -adic error jointly is required for a specific engineering task? This would be the first experimental validation of adelic rate-distortion theory.

9. Conclusion: The Trilogy Complete

The Adelic Shannon Theory foundation trilogy is now complete:

1. **Adelic Shannon Theory [1]:** Generalised entropy and channel capacity to the adeles.
2. **Adelic Entropic Numbers [2]:** Established the entropic number $(x, \mathbf{I}(X))$ as the natural data type.
3. **Adelic Rate-Distortion Theory (this paper):** Generalised compression and rate-distortion to the adeles.

The three papers form a coherent programme: **information in the adèle ring is a vector, not a scalar; measurements are entropic numbers, not bare floats; and compression must satisfy componentwise distortion constraints at every place.**

The practical consequence is a new class of compression systems — adelic codecs — that preserve both the approximate magnitude AND the divisibility structure of rational data. Whether such codecs can be built, and whether they offer advantages over standard (archimedean-only) compression, are open experimental questions for the next phase of the programme.

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