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Booklet Cosmological States as Random-Tensor Codes: Recovery Bounds and Entropic Diagnostics for Multi-Boundary Baby Universes

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ABSTRACT

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Abstract

Cosmological states in holography can contain closed "baby" universes inaccessible to any single asymptotic boundary, raising the question of whether the information they carry is recoverable, and from where. The booklet cosmology construction extends the two-boundary cosmological-state proposal of Antonini, Sasieta and Swingle to three or more holographic CFTs glued along a common interface, and models the resulting cosmology-to-boundary map by a circular complex Gaussian random tensor, i.e., a tripartite Haar-like state in flat energy windows. We treat this Gaussian model as a quantum error-correcting code whose logical system is the central closed universe and whose three arms are recovery regions. Working entirely within the stated Gaussian model, we derive explicit diagnostics with fully shown arithmetic: the expected purity of a single-arm reduced state via the Page formula, its excess over the maximally mixed value (scaling as $1/b^2$), the single-arm von Neumann entropy with Page correction, and the recovery error for a code of dimension K from any two arms, scaling linearly as K/b . At representative dimensions $b = 10^6$, $K = 10^3$ we obtain excess purity $\approx 10^{-12}$, single-arm entropy ≈ 19.93 bits, and recovery error $\leq 10^{-3}$, with conservative Markov and union bounds giving a simultaneous three-pair success probability of at least 0.8125 at a 4×10^{-3} error threshold. We situate these results relative to earlier quantum-cosmology programs and to a reported no-go tension between quantum error correction and Quantum Darwinism, and we state the assumptions under which the analysis would fail.

1. Introduction

A persistent puzzle in quantum gravity is the status of closed universes formed inside holographic systems: if a baby universe pinches off behind a horizon, no single asymptotic observer can access it, and one may worry that quantum information has left the boundary theory, violating unitarity. The lectures of [3] framed this problem decades ago as the

need for a quantum mechanics of closed systems, including the universe itself, with generalized quantum mechanics and a careful treatment of time. Modern holography offers a sharper formulation: the cosmology-to-boundary map, which encodes the baby-universe degrees of freedom non-locally in the boundary state.

The booklet cosmology construction [2] (with its bibliographic record [1]) is the most recent and concrete entry point. It glues three or more AdS pages, each carrying an asymptotic conformal field theory (CFT) boundary, along a common interface via multiway junction conditions, prepares the state by Euclidean evolution with a multilinear heavy insertion, and shows that in the heavy-insertion limit the bulk develops a closed universe at the center. Crucially, it models the insertion by a circular complex Gaussian random tensor, so that in flat energy windows the state is a tripartite Haar-like state, and proves that a code of dimension K on the baby universe is approximately recoverable from any two of the three arms with vanishing error and high probability as $K/b \rightarrow 0$, where b is the per-arm Hilbert-space dimension.

This paper has three goals. First, to make the error-correcting content of the Gaussian model quantitative and fully explicit, deriving every diagnostic from stated inputs with shown arithmetic (Section 4). Second, to place the construction in the lineage of quantum cosmology, from Wheeler-DeWitt-based programs [3], [4], [8] through loop and modified loop quantum cosmology [6], [7], [9], gravitational-entropy arguments [5], and quantum-gravity environment models [10], and to identify what is genuinely new: the random-tensor/quantum-error-correction (QEC) lens. Third, to examine a potential tension with the reported incompatibility of quantum error correction and Quantum Darwinism in the QNFO corpus [13], and to flag the limits of the Gaussian model itself.

Our central quantitative results, all derived in Section 4 from the Gaussian/Haar model with per-arm dimension b : (i) the excess purity of a single arm over the maximally mixed value scales as $1/b^2$, evaluated to $\approx 1.0 \times 10^{-12}$ at $b = 10^6$; (ii) the single-arm entropy is $\ln b$ nats up to a Page correction of order $1/(2b)$, giving ≈ 19.93 bits at $b = 10^6$; (iii) the two-arm recovery error for a code of dimension K scales linearly as K/b , giving 10^{-3} at $K = 10^3$, $b = 10^6$. These are model-internal results, not empirical claims; their physical interpretation is discussed with explicit caveats in Section 6.

2. Background and Related Work

The booklet construction. Reference [2] proposes gluing three or more AdS pages along a common interface, imposing multiway junction conditions, and preparing states by Euclidean path-integral evolution with a multilinear insertion operator. In the heavy-insertion limit the bulk develops a closed universe in the center — the "booklet" — with each boundary CFT playing the role of a page bound into a single volume. The paper's technical core is a Gaussian model: the three-page insertion is a circular complex Gaussian random tensor, producing a tripartite Haar state in flat energy windows, in which each arm is a network branch associated with one boundary CFT. The headline theorem is approximate recoverability of a dimension- K code from any two arms as $K/b \rightarrow 0$. Record [1] is the bibliographic query entry for the same work; we cite it once for completeness and use [2] as the primary reference throughout.

Closed-system quantum cosmology. The conceptual ancestor of the booklet problem is the program of [3], which asked how to formulate quantum mechanics for closed systems such as the universe, covering generalized quantum mechanics, the problem of time, and the quantum mechanics of spacetime. The booklet state is precisely a closed-system object: the baby universe is a subsystem of a larger pure state, and the "observer" is distributed over boundary arms. Earlier attempts to extract classical cosmology from the Wheeler-DeWitt equation include [4], which constructed a Friedmann model whose Wheeler-DeWitt solution corresponds exactly to classical coasting evolution via de Broglie-Bohm and modified de Broglie-Bohm approaches, thereby dissolving some standard quantum-cosmology pathologies. The booklet construction replaces such exact correspondence with ensemble-level statements: exact classicality is traded for high-probability recovery in a random-tensor ensemble.

Gravitational entropy and singularities. Reference [5] revisits Penrose's Weyl curvature hypothesis — the universe began at low gravitational entropy, near-zero Weyl curvature — in light of quantum backreaction at cosmological singularities and bounces. In the booklet picture, the central closed universe is born in a heavy-insertion limit whose geometry is highly constrained; one may ask whether the effective Gaussian ensemble encodes an entropy-selection principle analogous to the Weyl curvature hypothesis. We flag this as an open question rather than a claim.

Loop quantum cosmology and alternatives. Reference [6] systematically studies the evolution of the universe in modified loop quantum cosmology (mLQC-I) across inflationary potentials, finding universal features in which the big-bang singularity is replaced by a quantum bounce. Reference [7] develops the deformed algebra approach to perturbations in loop quantum cosmology, aiming to connect Planck-era quantum gravity settings with observation. Reference [9] offers a sober, non-technical case study of claims that loop quantum cosmology could alleviate CMB anomalies, documenting the field's internal tensions between observability claims and conceptual objections — a cautionary template for any effective model of quantum cosmology, including the Gaussian ensemble used here. These programs quantize the universe as a whole; the booklet approach instead quantizes the cosmology-to-boundary map. Reference [8] reviews Wheeler-DeWitt applications to string cosmology, with duality-related background solutions and an initial low-energy, weak-coupling regime — a precedent for treating the "birth" of a universe as a transition in a larger quantum system, structurally similar to the booklet's Euclidean preparation.

Quantum gravity as an environment. Reference [10] treats a test field on a quantum cosmological spacetime as propagating on an emergent classical background via a dressed metric built from the quantum fluctuations of the geometry; when backreaction is negligible, massive modes see an anisotropic Bianchi type I background. This is conceptually parallel to the booklet arms: the baby universe is a quantum system whose effective description for boundary observers is a channel — here, the Gaussian random-tensor channel — rather than a fixed geometry.

QNFO corpus entries. References [11], [12], and [14] are provided without abstracts and are cited only as corpus context, not as technical support for any quantitative claim in this paper.

QEC-Darwinism tension. The QNFO corpus document [13] reports a no-go theorem attributed to Maity et al. (arXiv:2608.03944): quantum error correction and Quantum Darwinism cannot coexist above a critical logical fidelity $F_L > 0.874$, establishing a tradeoff between protected quantum information and emergent classical objectivity. The booklet code is a natural stress test: the arms redundantly encode the baby-universe code (a Darwinism-like redundancy across three arms), while the code itself is protected. Whether the F_L threshold binds in the Gaussian model depends on details not fixed by [2] or [13]; we analyze the tension qualitatively in Section 6.

3. Methods

Model. We work in the Gaussian model of [2]. Let $\mathcal{H}_A, \mathcal{H}_B, \mathcal{H}_C$ each have dimension b . The insertion is modeled by a circular complex Gaussian random tensor T_{ijk} with i.i.d. entries of variance $1/b^2$ per entry so that the state is normalized on average:

$$|\Psi_T\rangle = \frac{1}{\sqrt{b}} \sum_{i,j,k=1}^b T_{ijk} |i\rangle_A |j\rangle_B |k\rangle_C,$$

with $\mathbb{E}[|T_{ijk}|^2] = 1/b^2$, so that $\mathbb{E}[\|\Psi_T\|^2] = b^3 \cdot (1/b^2) = b$ and the prefactor $1/\sqrt{b}$ normalizes in expectation. In flat energy windows this ensemble is equivalent, for reduced-state diagnostics, to a uniformly Haar-random pure state on $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$ [2]. We therefore compute all diagnostics for the Haar ensemble on total dimension $D = b^3$, and we state this equivalence as an assumption inherited from [2].

Diagnostics. We compute: (D1) the expected purity $\mathbb{E}[\text{Tr } \rho_A^2]$ via the Page formula; (D2) the excess purity over the maximally mixed value $1/b$; (D3) the single-arm von Neumann entropy S_A with the leading Page correction; (D4) the two-arm recovery error for a code of dimension K on the baby universe, using the linear-in- K/b scaling stated in [2]; and (D5) conservative Markov and union bounds converting the mean-square error into high-probability statements.

Recovery model. The logical system is the central closed universe, carried by arm C (or any single arm, by symmetry). A code $\mathcal{C}$ of dimension K is a K -dimensional subspace of $\mathcal{H}_C$. The recovery channel from the pair (A, B) is approximately the identity on $\mathcal{C}$ with error controlled by the decoupling parameter K/b [2]. We adopt the linear-in- K/b error bound as the model's stated scaling and evaluate it numerically; we do not claim a tighter constant than the model provides. (A minority draft of this analysis carried a $\sqrt{K/b}$ scaling; the divergence is documented in Appendix A, and the linear convention is adopted because it matches the theorem statement of [2] and two of the three independent drafts.)

Numerical conventions. All logarithms are in nats unless suffixed "bits"; conversions use $\ln 2 \approx 0.693147$. The representative dimension $b = 10^6$ is chosen as a concrete, computationally transparent value; physical b is expected to be vastly larger (Section 5, labeled projection).

4. Analysis

All inputs are stated here with their provenance. The only model inputs are the per-arm dimension b (Gaussian model of [2], chosen here as $b = 10^6$) and the code dimension K (free parameter of the recovery theorem of [2], chosen here as $K = 10^3$). Universal constants: $\ln 10 \approx 2.302585$, $\ln 2 \approx 0.693147$.

D1: Expected single-arm purity. For a Haar-random pure state on a system of dimension D partitioned into subsystems of dimensions d_S and $d_E = D/d_S$, the second-moment (Page) formula gives

$$\mathbb{E}[\text{Tr } \rho_S^2] = \frac{d_S + d_E}{d_S d_E + 1}.$$

Here $d_S = b = 10^6$ and $d_E = b^2 = 10^{12}$, so $D = b^3 = 10^{18}$. Substituting:

$$\mathbb{E}[\text{Tr } \rho_A^2] = \frac{10^6 + 10^{12}}{10^6 \cdot 10^{12} + 1} = \frac{1,000,001,000,000}{1,000,000,000,001}.$$

The numerator is $b^2(1 + 1/b)$ and the denominator is $b^3(1 + 1/b^3)$, so

$$\mathbb{E}[\text{Tr } \rho_A^2] = \frac{b^2}{b^3} \cdot \frac{1 + 1/b}{1 + 1/b^3} = \frac{1}{b} \left(1 + \frac{1}{b}\right) \left(1 - \frac{1}{b^3} + O(b^{-6})\right) \approx \frac{1}{b} + \frac{1}{b^2}.$$

At $b = 10^6$: $\mathbb{E}[\text{Tr } \rho_A^2] \approx 10^{-6} + 10^{-12} = 1.000001 \times 10^{-6}$. The arm is maximally mixed up to $O(1/b^2)$ corrections, consistent with D2 and with the pair-purity cross-check below.

D2: Excess purity over maximally mixed. The maximally mixed state on $\mathcal{H}_A$ has purity $1/b = 10^{-6}$. The excess is

$$\Delta = \mathbb{E}[\text{Tr } \rho_A^2] - \frac{1}{b} = \frac{b + b^2}{b^3 + 1} - \frac{1}{b}.$$

With common denominator $b(b^3 + 1) = b^4 + b$:

$$\Delta = \frac{b(b + b^2) - (b^3 + 1)}{b^4 + b} = \frac{b^2 + b^3 - b^3 - 1}{b^4 + b} = \frac{b^2 - 1}{b^4 + b}.$$

At $b = 10^6$:

$$\Delta = \frac{10^{12} - 1}{10^{24} + 10^6} \approx \frac{10^{12}}{10^{24}} = 10^{-12}.$$

More precisely, $\Delta \approx 10^{-12}(1 - 10^{-12}) \approx 9.99999 \times 10^{-13}$. This confirms the scaling $\Delta \sim 1/b^2$: the single arm is maximally mixed up to corrections of order 10^{-12} at $b = 10^6$, i.e., no single boundary arm carries any information about the baby universe beyond $O(1/b^2)$ fluctuations.

D3: Single-arm entropy with Page correction. The leading Page result for the average entropy of a small subsystem is

$$\mathbb{E}[S_A] \approx \ln d_S - \frac{d_S}{2d_E} = \ln b - \frac{1}{2b}.$$

At $b = 10^6$:

- $\ln b = \ln(10^6) = 6 \ln 10 = 6 \times 2.302585 = 13.815510$ nats.
- Correction: $\frac{1}{2b} = \frac{1}{2 \times 10^6} = 5 \times 10^{-7}$ nats.
- $\mathbb{E}[S_A] \approx 13.815510 - 0.0000005 = 13.8155095$ nats.

Converting to bits:

$$S_A \approx \frac{13.8155095}{0.693147} \approx 19.9315 \text{ bits},$$

i.e., ≈ 19.93 bits. The arm is indistinguishable from a maximally mixed b -dimensional system to relative accuracy $5 \times 10^{-7}/13.815510 \approx 3.6 \times 10^{-8}$.

D4: Two-arm recovery error. The theorem of [2] states that a code of dimension K is approximately recoverable from any two arms with vanishing error and high probability as $K/b \rightarrow 0$, with the Gaussian-model error controlled by the ratio K/b . Adopting the linear scaling as the model's stated bound:

$$\varepsilon \lesssim \frac{K}{b}.$$

At $K = 10^3, b = 10^6$:

$$\varepsilon \lesssim \frac{10^3}{10^6} = 10^{-3} = 0.001.$$

For comparison, at $K = 10^4$: $\varepsilon \lesssim 10^4/10^6 = 10^{-2} = 0.01$; at $K = 10^5$: $\varepsilon \lesssim 10^{-1} = 0.1$. As a cross-check at smaller dimensions, the secondary worked example of one independent draft used $b = 2^{10} = 1024$ and $K = 2^5 = 32$: $K/b = 32/1024 = 0.03125$, giving $\varepsilon \lesssim 3.125 \times 10^{-2}$. The regime of validity requires $K/b \rightarrow 0$; at $K = b$ the bound saturates at $\varepsilon \lesssim 1$ and guarantees nothing, consistent with the theorem's stated asymptotic nature.

D5: Markov and union bounds under the linear convention. Squaring the bound of D4 gives $\mathbb{E}[\varepsilon^2] \leq (K/b)^2 = (10^{-3})^2 = 10^{-6}$ at the chosen parameters. Markov's inequality states that for a non-negative random variable X and threshold $t > 0$, $P(X \geq t) \leq \mathbb{E}[X]/t$.

Bound 1. Threshold $t_1 = 4(K/b)^2 = 4 \times 10^{-6}$. Then

$$P(\varepsilon^2 \geq 4(K/b)^2) \leq \frac{(K/b)^2}{4(K/b)^2} = \frac{1}{4},$$

i.e., $P(\varepsilon \geq 2K/b) \leq 1/4$. With $K/b = 10^{-3}$: with probability at least $3/4 = 0.75$, $\varepsilon \leq 2 \times 10^{-3}$.

Bound 2. Threshold $t_2 = 16 (K/b)^2$. Then $P(\varepsilon \geq 4K/b) \leq 1/16$, i.e., with probability at least $15/16 = 0.9375$, $\varepsilon \leq 4 \times 10^{-3}$.

Simultaneous recovery over the three arm-pairs. The "any two arms" statement requires the bound to hold for the pairs (A, B) , (A, C) , (B, C) . By the union bound over three pairs, using Bound 2:

$$P(\exists \text{ pair with } \varepsilon \geq 4K/b) \leq 3 \times \frac{1}{16} = \frac{3}{16} = 0.1875.$$

Hence, with probability at least $1 - 0.1875 = 0.8125$, all three pairs recover the code with $\varepsilon \leq 4 \times 10^{-3}$ at $b = 10^6$, $K = 10^3$. These are conservative assumption-free bounds; true Gaussian concentration is expected to be far stronger, but exploiting it would require constants not stated in [2].

Cross-check on D4 via pair purity. Recovering $\mathcal{C} \subset \mathcal{H}_C$ from (A, B) requires the code to decouple from the reference. For the pair AB , $d_S = b^2 = 10^{12}$, $d_E = b = 10^6$:

$$\mathbb{E}[\text{Tr } \rho_{AB}^2] = \frac{10^{12} + 10^6}{10^{12} \cdot 10^6 + 1} = \frac{1,000,000,000,001}{1,000,000,000,000,001} \approx 10^{-6} = \frac{1}{b^2}.$$

The pair AB is maximally mixed on its b^2 -dimensional space up to relative corrections of order $1/b^2 \approx 10^{-12}$: the pair carries no code information coherently, which is precisely the decoupling condition that makes two-arm recovery possible while one-arm recovery fails. The consistency check — one arm maximally mixed, two arms jointly maximally mixed, three arms pure — is the entropic signature of the booklet code.

5. Results

All numbers below are computed in Section 4 from the stated inputs ($b = 10^6$, $K = 10^3$) within the Gaussian/Haar model of [2]. No empirical or simulation data are reported.

1. **Single-arm purity (D1).** $\mathbb{E}[\text{Tr } \rho_A^2] = \frac{b + b^2}{b^3 + 1} \approx \frac{1}{b} + \frac{1}{b^2} = 1.000001 \times 10^{-6}$ at $b = 10^6$: the arm is maximally mixed up to $O(1/b^2)$ corrections, consistent with the excess purity in item 2.
1. **Excess purity (D2).** $\Delta = \frac{b^2 - 1}{b^4 + b} \approx 9.99999 \times 10^{-13} \approx 10^{-12}$ at $b = 10^6$. Scaling: $\Delta \sim 1/b^2$.
1. **Single-arm entropy (D3).** $\mathbb{E}[S_A] \approx 13.8155095$ nats ≈ 19.93 bits at $b = 10^6$, with Page correction 5×10^{-7} nats.
1. **Two-arm recovery error (D4).** $\varepsilon \lesssim K/b = 10^{-3}$ at $K = 10^3$, $b = 10^6$; $\varepsilon \lesssim 10^{-2}$ at $K = 10^4$; $\varepsilon \lesssim 10^{-1}$ at $K = 10^5$; secondary example $\varepsilon \lesssim 3.125 \times 10^{-2}$ at $K = 32$, $b = 1024$. Validity requires $K/b \rightarrow 0$.
1. **Probability bounds (D5).** $P(\varepsilon \geq 2K/b) \leq 1/4$ and $P(\varepsilon \geq 4K/b) \leq 1/16$ per pair (Markov); simultaneous three-pair success probability $\geq 1 - 3/16 = 0.8125$ at the $4K/b = 4 \times 10^{-3}$ threshold.
1. **Pair purity cross-check.** $\mathbb{E}[\text{Tr } \rho_{AB}^2] \approx 1/b^2 = 10^{-6}$ at $b = 10^6$, confirming the decoupling structure: one arm and two arms are both (in their respective senses) informationless, while the total state is pure.

Labeled projection (not a computed result). For a physical holographic CFT, the flat-window dimension b is expected to be exponentially large in the central charge, e.g., $b \sim e^c$ with c the Virasoro central charge; assuming $c \sim 100$ gives $b \sim e^{100} \approx 2.7 \times 10^{43}$, for which $\Delta \sim b^{-2} \approx 1.4 \times 10^{-87}$ and $\varepsilon \lesssim K/b$ is negligible for any K polynomial in c . This is a projection under stated assumptions ($b \sim e^c$; flat energy windows), not a derivation from the model's proven theorems, and the constant in $b \sim e^c$ is not fixed by [2].

6. Discussion

What the results mean. Within the Gaussian model, the booklet cosmological state is a three-arm quantum error-correcting code in a strong sense: any single arm is blind to the baby universe (excess purity $\sim 1/b^2$), any two arms suffice for recovery ($\varepsilon \sim K/b$), and the structure is symmetric under arm permutation. This is a holographic realization of the closed-system quantum mechanics advocated in [3]: the baby universe is not lost; it is delocalized.

Limitations and failure modes. (i) The Haar equivalence holds only in flat energy windows; real CFT spectra and heavy insertions may violate it, and the excess-purity and entropy computations are ensemble averages — typicality statements, not guarantees for a given microstate. (ii) The recovery error scaling K/b is adopted from [2]'s theorem statement; we did not derive the constant, and the theorem is asymptotic in $K/b \rightarrow 0$. Near $K \sim b$ the code fails, and the transition region is uncontrolled here. (iii) All numbers are evaluated at the illustrative $b = 10^6$; the physical- b projection in Section 5 is assumption-laden. (iv) The Gaussian model erases energy-shell and gravitational backreaction structure; whether the junction-condition geometry constrains the ensemble beyond Haar-like statistics is open. (v) The QEC-Darwinism tension of [13] is unresolved: the booklet code has redundancy across arms (Darwinism-like) and QEC protection simultaneously, which superficially conflicts with the reported no-go threshold $F_L > 0.874$; moreover, the worked example has logical fidelity $F \geq 1 - \varepsilon = 1 - 10^{-3} = 0.999 > 0.874$, squarely in the regime excluded by [13] if the boundary arms were simultaneously required to implement Darwinism. However, the theorem's applicability depends on definitions of logical fidelity and redundancy not fixed by either source, and we make no claim either way. (vi) Connections to gravitational entropy selection [5], bounce physics [6], [7], observational programs [9], and string-scale Wheeler-DeWitt cosmology [8] are structural analogies, not derivations.

What would falsify the claims. If the Gaussian-to-Haar equivalence fails in the relevant energy windows — e.g., if the insertion's heavy limit induces non-Gaussian correlations that change the reduced-state purity scaling from $1/b^2$ — then D2, D3, and the cross-check collapse. If the recovery error grows faster than linearly in K/b , or fails at moderate K/b with non-vanishing probability, the D4 bound is wrong. If the multiway junction conditions impose constraints making the three arms non-symmetric, the "any two arms" statement weakens.

Arguing against ourselves. One may object that the entire exercise is model-bound: a Gaussian random tensor is not a gravitational state, and "recoverability from two arms" may be an artifact of Haar-typicality rather than a property of any physical booklet state. The response of [2] — that the Gaussian model extends the effective Gaussian assumption already validated in the two-page construction — is plausible but not a proof. A second objection: the union-bound probability 0.8125 in D5 is so weak that a skeptic could say we have shown little; our defense is methodological honesty — these are the strongest assumption-free bounds available from the stated inputs, and they already exhibit the correct parametric scaling. A third objection: comparing with loop quantum cosmology [6], [7], [9] may be a category error, since those programs target phenomenology (bounces, CMB) while booklet cosmology targets state structure; we accept this partially, but the structural comparison in Section 2 is about *what kind of quantum question* each program poses, which is a fair comparison.

Open questions. Does the booklet code admit a quantum Darwinism phase structure across arm subsets? Can the $1/b^2$ excess purity be interpreted as a gravitational-entropy selection principle in the sense of [5], and does the failure transition at $K \sim b$ admit a geometric, black-hole-like interpretation? We leave both open.

7. Conclusion

We treated the Gaussian random-tensor model of booklet cosmology [2] as a quantum error-correcting code whose logical system is the central closed universe and whose three boundary arms are recovery regions, deriving every diagnostic with explicit arithmetic. At $b = 10^6$, $K = 10^3$: the single-arm purity is $\mathbb{E}[\text{Tr } \rho_A^2] \approx 1/b + 1/b^2 = 1.000001 \times 10^{-6}$, i.e., the arm is maximally mixed up to excess $\Delta \approx 10^{-12}$ scaling as

$1/b^2$; the single-arm entropy is ≈ 19.93 bits with Page correction 5×10^{-7} nats; the two-arm recovery error satisfies $\varepsilon \lesssim K/b = 10^{-3}$; and conservative Markov/union bounds give a simultaneous three-pair success probability of at least 0.8125 at threshold 4×10^{-3} . The structural picture is that the baby universe is delocalized, not lost: no single arm sees it, any two arms recover it. These are model-internal, ensemble-averaged results whose physical validity rests on the Gaussian-to-Haar equivalence in flat energy windows assumed in [2]; the tension with the QEC–Quantum Darwinism no-go of [13] remains open. The contribution is methodological: a fully explicit, arithmetic-transparent template for analyzing multi-boundary cosmological states as random-tensor codes.

References

[1] TITLE: arXiv Query: `searchquery=&idlist=2610.02168&start=0&max_results=1` [2] A baby universe from a large family: booklet cosmology states and quantum error correction. arXiv:2610.02168v1. <https://arxiv.org/abs/2610.02168v1> [3] The Quantum Mechanics of Cosmology. arXiv:1805.12246v1. <https://arxiv.org/abs/1805.12246v1> [4] Exact Classical Correspondence in Quantum Cosmology. arXiv:1405.7957v2. <https://arxiv.org/abs/1405.7957v2> [5] Weyl Curvature Hypothesis in light of Quantum Backreaction at Cosmological Singularities or Bounces. arXiv:2110.01104v2. <https://arxiv.org/abs/2110.01104v2> [6] Universal properties of the evolution of the Universe in modified loop quantum cosmology. arXiv:2406.06745v3. <https://arxiv.org/abs/2406.06745v3> [7] The perturbed universe in the deformed algebra approach of Loop Quantum Cosmology. arXiv:1606.03271v1. <https://arxiv.org/abs/1606.03271v1> [8] Quantum string cosmology. arXiv:2101.01070v2. <https://arxiv.org/abs/2101.01070v2> [9] Cosmic tangle: Loop quantum cosmology and CMB anomalies. arXiv:2106.02481v1. <https://arxiv.org/abs/2106.02481v1> [10] Cosmological particle production in quantum gravity. arXiv:2106.08739v1. <https://arxiv.org/abs/2106.08739v1> [11] <https://papers.qnfo.org/papers/universe-as-self-proving-theorem/>. QNFO: Universe as Self-Proving Theorem. [12] <https://papers.qnfo.org/papers/recursive-self-consistency/>. QNFO: Recursive Self-Consistency. [13] <https://papers.qnfo.org/papers/qec-darwinism-ultrametric/>. QNFO: Archimedean Shadows: The QEC-Darwinism Tradeoff in Ultrametric Spaces. [14] <https://papers.qnfo.org/papers/number-theory-as-physics/>. QNFO: Number Theory as Physics.

Appendix A. Divergence report

Divergence 1 (recovery-error scaling). One draft used $\varepsilon \lesssim \sqrt{K/b}$; two drafts used the linear convention $\varepsilon \lesssim K/b$. The disagreement stems from different readings of the asymptotic theorem of [2]: the $\sqrt{K/b}$ draft assumed a decoupling bound in squared Hilbert-Schmidt norm, while the linear drafts took the theorem's stated parameter dependence at face value. Convention adopted for the main text: linear K/b , because it matches the theorem statement of [2] and the majority of drafts. Under the alternative convention the headline error at $K = 10^3$, $b = 10^6$ would be $\sqrt{10^{-3}} \approx 3.16 \times 10^{-2}$, and all D4/D5 numbers would weaken accordingly.

Divergence 2 (secondary worked example). One draft included a secondary example at $b = 2^{10} = 1024$, $K = 2^5 = 32$; the others evaluated only $b = 10^6$. Status: single-source, retained in Section 4 (D4) as a labeled cross-check; it does not affect any headline number.

No other substantive divergences were found. The D1 purity discrepancy was an arithmetic error in one draft, corrected in Section 4, not a convention conflict.

Appendix B. Claim attribution

Claim	Content	Drafts	Status
C1	Gaussian model: tripartite Haar-like state, per-arm dimension b	A, B, C	CONVERGENT
C2	Single-arm purity via Page formula $(b + b^2)/(b^3 + 1)$	A, B, C	CONVERGENT (evaluation corrected in Section 4)
C3	Excess purity $\Delta \sim 1/b^2$, $\approx 10^{-12}$ at $b = 10^6$	A, B, C	CONVERGENT
C4	Single-arm entropy ≈ 19.93 bits at $b = 10^6$	A, B, C	CONVERGENT
C5	Recovery-error scaling: linear K/b vs $\sqrt{K/b}$	A, B vs C	DIVERGENT (Appendix A)
C6	Markov/union bound: three-pair success ≥ 0.8125 at 4×10^{-3}	A, B	CONVERGENT
C7	Pair-purity cross-check $\mathbb{E}[\text{Tr } \rho_{AB}^2] \approx 1/b^2$	A, B, C	CONVERGENT
C8	QEC–Quantum Darwinism tension of [13] discussed qualitatively	A, B, C	CONVERGENT
C9	Physical- b projection $b \sim e^c$ with $c \sim 100$	A	SINGLE (labeled projection)
C10	Secondary example $b = 1024$, $K = 32$	C	SINGLE (cross-check)