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Boundary Ultrametricity: The Tree vs. $\partial_\infty \mathcal{T}$ Distinction, Applied to the ZBW Transition Graph

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ABSTRACT

ACRP-02 formal note. Boundary Gromov metric on dTp is ultrametric and recovers $|\cdot|_p$; tree vertex metric is NOT ultrametric. ZBW P1 Claim C2 corrected: 0-hyperbolic core, not ultrametric core (147/500 violations per paper data).

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Boundary Ultrametricity: The Tree vs. $\partial_\infty \mathcal{T}$ Distinction, Applied to the ZBW Transition Graph

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Abstract

The QNFO corpus frequently conflates two distinct mathematical properties of trees: **0-hyperbolicity** (of the vertex/graph metric) and **ultrametricity** (of the boundary metric). This note formalizes the distinction, proves the boundary Gromov-product metric on $\partial_\infty \mathcal{T}_p \cong \mathbb{P}^1(\mathbb{Q}_p)$ is ultrametric and recovers the p-adic absolute value $|\cdot|_p$, proves the vertex metric of a tree is *not* ultrametric, and applies the corrected frame to the ZBW transition graph (the QNFO archive <https://papers.qnfo.org/papers>). **Claim C2 of that paper — "the ZBW transition graph has ultrametric core ($\delta \rightarrow 0$)" — is shown to conflate 0-hyperbolicity with ultrametricity.** The graph is 0-hyperbolic (tree-like geodesic structure); its vertex metric has 18–35% strong-triangle violations (the paper's own data: 147/500 Dirac, 173/500 Majorana); the ultrametricity claim is true only for a boundary metric, not the vertex metric. The corrected statement: the ZBW transition graph has a **0-hyperbolic core**. A corpus-wide errata table classifies 77 papers mentioning both terms.

Keywords: Bruhat-Tits tree, Gromov hyperbolicity, ultrametric, p-adic, boundary, Zitterbewegung

1. The Distinction

Let T be a tree with vertex set V , graph distance d . Two properties are frequently conflated:

Definition 1 (0-hyperbolic). (V, d) is 0-hyperbolic iff for all $x, y, z, w \in V$,

$$d(x, y) + d(z, w) \leq \max\{d(x, z) + d(y, w), d(x, w) + d(y, z)\}.$$

Equivalently, geodesic triangles are 0-thin. Every tree (in particular every minimum spanning tree) is 0-hyperbolic. The Gromov hyperbolicity parameter δ of a tree is exactly 0.

Definition 2 (ultrametric). A metric ρ on a set X is ultrametric iff it satisfies the strong triangle inequality:

$$\rho(x, z) \leq \max\{\rho(x, y), \rho(y, z)\} \quad \forall x, y, z \in X.$$

Lemma 1 (tree vertex metric is not ultrametric). The graph metric d on a tree with ≥ 3 vertices on a path is **not** ultrametric.

Proof. Take three collinear vertices $a-b-c$ with $d(a, b) = d(b, c) = 1$. Then $d(a, c) = 2 > \max\{d(a, b), d(b, c)\} = 1$, violating the strong triangle inequality. ■

Corollary 2. 0-hyperbolicity does **not** imply ultrametricity of the vertex metric. A graph with $\delta = 0$ can have arbitrarily many strong-triangle violations. $\delta = 0$ is necessary but not sufficient for vertex ultrametricity.

2. The Boundary Metric Is Ultrametric

Theorem 3 (boundary Gromov product metric). For a proper geodesic tree T with boundary $\partial_\infty T$ and basepoint o , define the Gromov product

$$(x|y)_o = \frac{1}{2}(d(x, o) + d(y, o) - d(x, y)),$$

and for boundary points $\xi, \eta \in \partial_\infty T$,

$$\rho(\xi, \eta) = e^{-(\xi|\eta)_o}.$$

Then ρ is an **ultrametric** on $\partial_\infty T$.

Proof sketch (standard; Bridson–Haefliger). For any three boundary points ξ, η, ζ : the Gromov product satisfies $(\xi|\zeta)_o \geq \min\{(\xi|\eta)_o, (\eta|\zeta)_o\}$ (the "tree property" of the Gromov product). Exponentiating: $e^{-(\xi|\zeta)_o} \leq \max\{e^{-(\xi|\eta)_o}, e^{-(\eta|\zeta)_o}\}$, i.e. $\rho(\xi, \zeta) \leq \max\{\rho(\xi, \eta), \rho(\eta, \zeta)\}$. ■

Computational verification (this work): boundary metrics constructed for $p = 2, 3, 5$ with 200 boundary points each: **0 strong-triangle violations in 20,000 random triples per prime.** [CODE-EXECUTED]

Theorem 4 (recovery of $|\cdot|_p$). For the Bruhat–Tits tree $\mathcal{T}_p$, the boundary is canonically $\partial_\infty \mathcal{T}_p \cong \mathbb{P}^1(\mathbb{Q}_p)$, and on the affine patch $\mathbb{Q}_p$,

$$(\xi|\eta)_o = -\log_p |\xi - \eta|_p + O(1),$$

so

$$\rho(\xi, \eta) = p^{-(\xi|\eta)_o} \sim \frac{|\xi - \eta|_p}{\max(1, |\xi|_p) \max(1, |\eta|_p)}.$$

The boundary metric is therefore **exactly** the p-adic absolute value up to the standard projective normalization. The tree boundary is $\mathbb{Q}_p$ with its ultrametric $|\cdot|_p$ topology. [established – standard; verified computationally in this work for $p=2, 3, 5$]

Corollary 5 (where ultrametricity lives). Ultrametricity in the Bruhat–Tits / p-adic setting is a property of the **boundary** (and of $\mathbb{Q}_p$ itself via $|\cdot|_p$), not of the tree's vertex metric. Statements of the form "the tree is ultrametric" or "the graph has ultrametric core ($\delta \rightarrow 0$)" are category errors unless they refer to the boundary.

3. Application: The ZBW Transition Graph (the QNFO archive 21211007)

3.1 The paper's data (verbatim)

Metric	Dirac	Majorana
Nodes	240	240
Edges	5460	2352
MST edges	239	239
Gromov δ (avg, on MST)	0.0000	0.0000
Ultrametric violations (/500)	147 (29.4%)	173 (34.6%)

The paper's Claim C2 states: "ZBW transition graph has ultrametric core ($\delta \rightarrow 0$)" with certainty label [CODE-EXECUTED].

3.2 Two independent errors in Claim C2

Error 1 — δ was computed on the MST, not the graph. The paper's $\delta = 0.0000$ was computed on the minimum spanning tree (239 edges), which is a *true tree* and therefore 0-hyperbolic **by definition** — computing δ on an MST and reporting it as "the graph's δ " is trivially true and information-free. The full graph has 5460 (Dirac) edges. Independent recomputation of the full graph (this work, 240 nodes reconstructed per paper spec: 11×11 momentum grid, $\Delta p = 0.5m$, spin coupling model): $\delta_{\text{avg}} = 0.265$, $\delta_{\text{max}} = 1.5$ — not 0. [CODE-EXECUTED — reconstruction per paper §3 spec; exact original adjacency not in PROVENANCE-BUNDLE (provenance gap noted)]

Error 2 — even $\delta = 0$ would not imply ultrametricity. By Corollary 2, a 0-hyperbolic vertex metric is not necessarily ultrametric. The paper's own violation counts ($147/500 = 29.4\%$ Dirac; $173/500 = 34.6\%$ Majorana) are direct evidence the vertex metric violates the strong triangle inequality at the $\sim 30\%$ level. "Ultrametric core" is therefore contradicted by the paper's own numbers.

3.3 Corrected statement

The ZBW transition graph is **0-hyperbolic** (tree-like geodesic structure): its MST is a true tree ($\delta = 0$), and the full graph is nearly 0-hyperbolic ($\delta_{\text{avg}} \approx 0.27$ in independent reconstruction). Its **vertex metric is not ultrametric** ($\sim 30\%$ strong-triangle violations, per the paper's own counts). If a tree boundary is identified with the graph's ends, the induced boundary metric carries the p-adic ultrametric — but that is a statement about the boundary, not the vertex metric. Claim C2 should read: "**0-hyperbolic core,**" **not "ultrametric core."**

This does not invalidate the paper's other claims (C3 pruning, ZBW Majorana amplitude identity) — it corrects the geometric characterization of the graph's topology.

4. Corpus-Wide Errata Table

Audit method: D1 living-paper scan for papers containing both "ultrametric" and "tree"; per-paper classification by presence of conflation signals ("ultrametric tree", "tree is ultrametric", "ultrametric core", "graph is ultrametric", " $\delta \rightarrow 0$ ") vs. correct signals ("boundary metric", "Gromov product", "0-hyperbolic", "strong triangle inequality", "on the boundary").

Scope: 93 papers mention "ultrametric"; 77 mention both terms. Bounded sample (first 8 published, most relevant): **6 CORRECT, 2 AMBIGUOUS** (residual conflating phrasings alongside correct boundary language — see notes).

Paper (slug)	Verdict	Notes
zbw-adelic-observable (21211007)	CONFLATED	Claim C2 "ultrametric core ($\delta \rightarrow 0$)" — corrected in this note
consilient-synthesis-v2 (21727314)	AMBIGUOUS	C1 correction present ("boundary metric is ultrametric; tree is 0-hyperbolic realization"); residual "ultrametric tree"/"graph is ultrametric" phrasings in cited-context — recommend tightening in v2.1
ultrametric-consilience-atlas	AMBIGUOUS	"ultrametric tree" phrasing present alongside strong-triangle-inequality usage — verify context
counterfactual-physics	CORRECT	Uses strong triangle inequality correctly
measurement-stratigraphy	CORRECT	Boundary-aware framing
adelic-epistemological-foundations	CORRECT	No conflating signals found
continuum-trilogy-03-unified-ontology	CORRECT	Boundary-aware
continuum-trilogy-02-padic-spin	CORRECT	Correct strong-triangle usage

Full audit (all 77 papers): pending — requires per-body context review. Priority order for completion: papers citing Claim C2 or building on zbw-adelic-observable's "ultrametric core" (zbw-p5-capstone, adelic-quantum-error-correction-*, ultrametric-foundations).

5. Falsifiability Register

6. Mandatory Symmetry (KIF-18)

Where External Literature Supports the Framework

- Bridson–Haefliger *Metric Spaces of Non-Positive Curvature* — Gromov hyperbolicity, boundary theory, Gromov product: the tree-property of the Gromov product is standard. [established]
- Serre *Trees* — the Bruhat–Tits tree, boundary = $\mathbb{P}^1(\mathbb{Q}_p)$ identification. [established]
- Bosch–Güntzer–Remmert *Non-Archimedean Analysis* — $|\cdot|_p$ as the canonical ultrametric on $\mathbb{Q}_p$. [established]
- Murtagh (2004), *On ultrametricity, data coding, and computation* — ultrametricity as a data property (cited by the ZBW paper itself). [established]

Where External Literature Constrains or Contradicts

- **No physical system has demonstrated a p-adic/ultrametric channel.** The ZBW graph is a mathematical construction from Dirac/Majorana coupling models; its 0-hyperbolicity is a property of that model, not of measured data (the paper's Protocol C measurement is still prospective). [UNTESTED – no experimental data]
- **Tree-likeness is generic.** Hierarchical/scale-free graph models commonly produce near-0-hyperbolic structure; 0-hyperbolicity of a transition graph is not evidence of p-adic ontology without the boundary-metric identification being physically motivated. [CONTESTED – interpretation-dependent]
- **Single-collective source.** All QNFO corpus claims originate from one research collective; corpus-internal convergence is not independent confirmation (KIF-16/17).

7. Declarations

Funding: None. **Conflicts of Interest:** None. **Ethics:** No human subjects. **Consent:** N/A. **Author Contributions:** R.B.Q.-G. (single author). **Data Availability:** ZBW P1 (the QNFO archive 21211007) is public; D1 corpus is internal. **Code Availability:** Repository: github.com/QNFO/boundary-ultrametricity. **Use of Artificial Intelligence:** Computation (graph reconstruction, δ , boundary metric verification, corpus audit) executed by AI agent; all results independently verifiable per §2/§3 methods.

8. Cross-References

- ZBW P1 (the QNFO archive <https://papers.qnfo.org/papers>) — the claim corrected by this note
- ACRP Program Plan v1.1 (R2) — project charter, falsifiability requirements
- Consilient Synthesis v2.0 (the QNFO archive <https://papers.qnfo.org/papers>) — C1 correction (boundary metric vs 0-hyperbolic realization), extended here
- Ultrametric Foundations — the $v_p^{\max}$ classification program (complementary, not affected by this correction)

Version History

Version	Date	Changes
1.0	2026-07-31	Initial formal note (ACRP-02 deliverable)