

2025-09 · V1.0.0

Dimensionless Physics

QNFO Research

QNFO-ULA-2.1

ABSTRACT

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CITE AS Research, QNFO (2025). Dimensionless Physics. QNFO Preprint v1.0.0.

Dimensionless

Physics: A Unified Framework

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This work presents dimensionless physics as the fundamental language of nature, demonstrating how all physical laws can be expressed as relationships between pure dimensionless ratios rather than anthropocentric units. Building upon the Buckingham Pi Theorem, this document shows that dimensional constants such as G , c , and $\hbar$ are not fundamental but emerge from a deeper causal network of wave correlations. When equations are expressed in dimensionless form, these constants vanish, revealing the true mathematical structure of physical reality as pure relationships between dimensionless ratios. This work derives dimensionless forms of fundamental equations from Newtonian gravity to Einstein's field equations and examines the irreducible dimensionless constants that define the universe's character. Integrating Stergios Pellis's dimensionless theory with temporal quantization through fractal geometry, this document presents a unified framework that connects quantum mechanics, general relativity, and cosmology through pure numerical relationships. The generative focal point of this work is to establish dimensionless physics as the essential language for a complete theory, with profound implications for fundamental physics, cosmology, and the unification of all natural phenomena.

1.0 Introduction

The edifice of modern physics is built upon mathematical equations that describe the behavior of the universe. A profound characteristic of these equations is their dimensional consistency. This principle, known as dimensional homogeneity, asserts that meaningful physical laws cannot equate disparate concepts like mass and time; they must compare like with like. In systems of measurement based on human scales—the meter, the kilogram, the second—this consistency is enforced by the introduction of dimensional constants such as the gravitational constant G or the reduced Planck constant $\hbar$. These constants, however, are not fundamental but emerge from a deeper causal network of wave

correlations. This reliance on dimensional constants obscures a more fundamental truth: the laws of physics are ultimately statements about relationships between pure, dimensionless numbers. The mathematical framework that formalizes this concept is the Buckingham π theorem, which proves that any physically meaningful relationship can be expressed entirely in terms of dimensionless ratios, independent of any specific, arbitrarily chosen system of units (Buckingham, 1914). This work establishes the Buckingham π theorem as the logical cornerstone for a dimensionless description of physics, demonstrating that scale invariance is not a mathematical trick but a core principle of the cosmos.

This perspective fundamentally reorients our understanding of reality. Contrary to conventional wisdom, “quantum gravity” is not a distinct category of physics but is an emergent phenomenon arising from a causal network of wave correlations (Quni-Gudzinis, 2025). The Planck constant $\hbar$ is not a fundamental constant of nature but emerges as a conversion factor between frequency and energy within our current measurement framework. Reality is not composed of particles or wavefunctions but of causal relationships—morphisms in a category whose objects represent quantum events. Mass is fundamentally frequency rather than merely being measured through frequency, and gravity emerges as the thermodynamic consequence of phase coherence among quantum field correlations interacting with curved spacetime (Quni-Gudzinis, 2025).

This document presents a comprehensive framework for dimensionless physics that integrates these insights with the dimensionless theory developed by Stergios Pellis (Pellis, 2023), temporal quantization through fractal geometry (Pellis, 2025), and quantum correlation synchronization theory (Quni-Gudzinis, 2025). This work demonstrates how all fundamental laws can be expressed in dimensionless form, revealing the true mathematical structure of physical reality and providing a unified framework for understanding the cosmos, fulfilling the generative focal point described in Section 0.0.

2.0

The Mathematical Foundation: Buckingham Pi Theorem and Dimensional Analysis

2.1

Dimensional Homogeneity: The Prerequisite for Physical Law

Any equation purporting to be a physical law must adhere to the principle of dimensional homogeneity. This principle states that each term in a valid physical equation must have the same physical dimensions. For example, in the kinematic equation for an object under constant acceleration, $x(t) = x_0 + v_0t + (1/2)at^2$, each term—the final position $x(t)$, the initial position x_0 , the velocity-time product v_0t , and the acceleration-time squared product $(1/2)at^2$ —must have the dimension of length, denoted as $[L]$. An equation such as $x = v_0t + a$, which equates a length to the sum of a length and an acceleration ($[L][T]^{-2}$), is physically meaningless.

This principle serves as a powerful constraint on the possible forms that physical laws can take. It is the reason why dimensional constants appear in our equations. Consider Newton’s law of universal gravitation, $F = G(m_1m_2/r^2)$. The dimensions of force are $[M][L][T]^{-2}$, while the dimensions of the mass-distance term are $[M]^2[L]^{-2}$. To make the equation dimensionally homogeneous, a constant of proportionality, G , must be introduced with dimensions $[M]^{-1}[L]^3[T]^{-2}$. In an anthropocentric system of units such as the International System of Units, G has an experimentally determined value of approximately $6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$. The necessity of this constant is entirely a feature of the chosen unit system. If units were defined such that the gravitational force between two unit masses separated by a unit distance was one unit of force, G would be numerically equal to 1. This reveals that dimensional constants are, in essence, conversion factors required to maintain dimensional consistency within an arbitrary, human-constructed framework of measurement (Bridgman, 1922).

2.2 The

Buckingham Pi Theorem: Formalizing Scale Invariance

The Buckingham π theorem provides the formal mathematical basis for recasting physical laws into a dimensionless form. It elevates the principle of dimensional homogeneity from a simple consistency check to a powerful tool for revealing the fundamental structure of physical relationships (Buckingham, 1914).

2.2.1 Statement and Proof of

the Theorem

The theorem states that if a physically meaningful equation involves n physical variables and constants, and these quantities can be expressed using k fundamental, independent physical dimensions, then the original equation can be rewritten as an equation involving a set of $p = n - k$ independent, dimensionless parameters, $\pi_1, \pi_2, \dots, \pi_p$.

Formally, if a physical relationship is given by $f(Q_1, Q_2, \dots, Q_n) = 0$, where the Q_i are the n physical variables, the theorem asserts that this can be rewritten as $g(\pi_1, \pi_2, \dots, \pi_{n-k}) = 0$. Each dimensionless parameter, or π -group, is a product of powers of the original variables: $\pi_i = Q_1^{a_1} Q_2^{a_2} \dots Q_n^{a_n}$, where the exponents a_i are chosen such that the combination is dimensionless.

The proof of the theorem relies on linear algebra. The dimensions of each quantity Q_i can be expressed as a product of powers of the k fundamental dimensions $D_1, D_2, \dots, D_k$, as in $[Q_i] = D_1^{d_{i1}} D_2^{d_{i2}} \dots D_k^{d_{ik}}$. For a π -group to be dimensionless, the exponents of its constituent variables must satisfy a system of linear homogeneous equations. The number of independent solutions to this system corresponds to the nullity of the dimensional matrix formed by the exponents d_{ij} . By the rank-nullity theorem, the nullity is $n - \text{rank}$, where the rank of the dimensional matrix is equal to k , the number of independent dimensions. Thus, there are $n - k$ independent dimensionless parameters.

2.2.2 Significance and

Implications

The profound significance of the Buckingham π theorem is that it guarantees that the validity of any physical law is independent of the specific unit system used to express it. The fundamental physical content of a law is not contained in the dimensional quantities themselves, but in the relationships between the dimensionless π -groups. If the values of these dimensionless combinations were to change with a change of units, the equation would not be a universal identity, and the theorem would not hold.

This principle of “similarity” is the foundation of all physical modeling. For instance, in hydrodynamics, the drag force D on a submerged object depends on the fluid density ρ , viscosity μ , flow velocity v , and a characteristic length L . Here, there are $n = 5$ variables and $k = 3$ fundamental dimensions (M, L, T). The Buckingham π theorem predicts $n - k = 2$ dimensionless parameters. These can be chosen as the drag coefficient, $\pi_1 = D/(\rho v^2 L^2)$, and the Reynolds number, $\pi_2 = \rho v L / \mu$. The physical law is then expressed as $\pi_1 = f(\pi_2)$. A small-scale model in a wind tunnel is physically “similar” to a full-scale airplane if its Reynolds number is the same. The underlying physics is identical because the relationship between the dimensionless parameters is the same, even though the dimensional values of force, velocity, and size are different.

A generalization of the theorem considers the case where n_F of the independent variables are held at fixed values, such as the fundamental constants of nature. If k_F of these fixed variables are dimensionally independent, the number of independent dimensionless parameters is further reduced by $(n_F - k_F)$. In fundamental physics, the constants c , G , and $\hbar$ are our fixed variables. As they are dimensionally independent, $n_F = k_F = 3$. Therefore, $(n_F - k_F) = 0$, meaning their existence does not further reduce the number of dimensionless parameters beyond what the standard theorem predicts. This is a crucial point: it explains why, even after using the dimensional constants to set a natural scale, we are still left

with a set of irreducible, fundamental dimensionless constants that must be determined by experiment. These constants, such as the fine-structure constant discussed in Section 5.1, are the universe's intrinsic π -groups, whose values are not determined by the dimensional constants alone.

3.0 Defining a

Universal Metric: The Emergent Scale

The Buckingham π theorem, as established in Section 2.2, provides the mathematical license to describe physics dimensionlessly, but it does not specify which scales to use for this process. To move from an abstract framework to a physically meaningful one, a system of units must be identified that is not based on arbitrary, terrestrial standards but arises from the causal structure of the universe itself. This system is the Planck scale, derived from the constants that underpin relativity, gravity, and quantum mechanics. The Planck units represent a convenient scaling system that emerges from the deeper causal network of wave correlations, providing a natural metric for the cosmos.

3.1 The Limits of

Anthropocentric Units

Standard systems of units, such as the International System of Units, are fundamentally anthropocentric. The second was historically defined as a fraction of a solar day on Earth. The meter was originally defined in relation to the Earth's circumference. The kilogram is based on a physical artifact, and though now redefined in terms of the Planck constant, its magnitude is still tied to this historical artifact. While these units are practical for human engineering and commerce, they are arbitrary from the perspective of fundamental physics. They bear no intrinsic relationship to the underlying laws of nature.

This was the motivation of Max Planck in 1899. He sought to establish a system of units that would be independent of any specific object, planet, or civilization. He envisioned units that would “necessarily maintain their meaning for all time and for all civilizations, even those which are extraterrestrial and nonhuman,” which he termed “fundamental physical units of measurement” (Planck, 1899). These units would be derived solely from the universal constants of nature.

3.2

Derivation of the Planck Units via Dimensional Analysis

The Planck units are derived by combining constants that emerge from the causal structure of the universe in such a way as to produce quantities with the dimensions of length, mass, and time. To derive the Planck charge, a fourth constant related to electromagnetism is included. The four constants are first, the speed of light in vacuum, c , from special relativity with dimensions $[L][T]^{-1}$; second, the universal gravitational constant, G , from general relativity with dimensions $[M]^{-1}[L]^3[T]^{-2}$; third, the reduced Planck constant, $\hbar = h/2\pi$, from quantum mechanics with dimensions $[M][L]^2[T]^{-1}$; and fourth, the Coulomb constant, $k_e = 1/(4\pi\epsilon_0)$, from electromagnetism with dimensions $[M][L]^3[T]^{-2}[Q]^{-2}$ where Q is the dimension of charge. By performing a dimensional analysis, exponents n_1, n_2, n_3 are found such that a combination like $c^{n_1}G^{n_2}\hbar^{n_3}$ yields a desired dimension.

3.2.1 Planck Mass (mP)

A combination with dimensions of mass, $[M]$, is sought. The dimensional equation $[mP] = [c]^{n_1}[G]^{n_2}[\hbar]^{n_3}$ expands to $[M]^1[L]^0[T]^0 = [M]^{-n_2+n_3}[L]^{n_1+3n_2+2n_3}[T]^{-n_1-2n_2-n_3}$. This yields a system of three linear equations for the exponents: $-n_2 + n_3 = 1$, $n_1 + 3n_2 + 2n_3 = 0$, and $-n_1 - 2n_2 - n_3 = 0$. Solving this system gives $n_1 = 1/2$, $n_2 = -1/2$, and $n_3 = 1/2$. Therefore, the Planck mass is defined as follows.

$$m_P = \sqrt{\frac{\hbar c}{G}} \approx 2.176 \times 10^{-8} \text{kg}$$

3.2.2 Planck Length (ℓ_P)

A combination with dimensions of length, $[L]$, is sought. The system of equations for the exponents is: $-n_2 + n_3 = 0$, $n_1 + 3n_2 + 2n_3 = 1$, and $-n_1 - 2n_2 - n_3 = 0$. Solving this system gives $n_1 = -3/2$, $n_2 = 1/2$, and $n_3 = 1/2$. Therefore, the Planck length is defined as follows.

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35} \text{m}$$

3.2.3 Planck Time (t_P)

A combination with dimensions of time, $[T]$, is sought. The system of equations is: $-n_2 + n_3 = 0$, $n_1 + 3n_2 + 2n_3 = 0$, and $-n_1 - 2n_2 - n_3 = 1$. Solving this system gives $n_1 = -5/2$, $n_2 = 1/2$, and $n_3 = 1/2$. Therefore, the Planck time is defined as follows.

$$t_P = \sqrt{\frac{\hbar G}{c^5}} = \frac{\ell_P}{c} \approx 5.391 \times 10^{-44} \text{s}$$

3.2.4 Planck Charge (q_P)

To derive the Planck charge, the Coulomb constant k_e is used to seek a combination with dimensions of charge, $[Q]$. The standard definition relates charge to the other fundamental units by setting the proportionality constant in Coulomb's law to unity in the new system. This leads to the definition of Planck charge.

$$q_P = \sqrt{4\pi\epsilon_0 \hbar c} = \sqrt{\frac{\hbar c}{k_e}} \approx 1.876 \times 10^{-18} \text{C}$$

This value is approximately 11.7 times the elementary charge e .

3.2.5 Derived Planck Units

From these base units, all other physical quantities can be derived. Two of the most important are Planck energy and Planck force. The Planck energy, EP , is defined as $EP = m_P c^2 = \sqrt{(\hbar c^5/G)}$, with an approximate value of 1.22×10^{19} GeV. The Planck force, FP , is defined as $FP = EP/\ell_P = c^4/G$, with an approximate value of 1.21×10^{44} N.

3.3 The Emergent Nature

of the Planck Scale

The Planck scale is not a fundamental regime but an emergent scale from the causal network of wave correlations. It represents the scale where the causal structure of the universe manifests in ways that create the appearance of extreme curvature and quantum effects. At the Planck length and Planck time, the energy required to probe such scales, the Planck energy, is so immense that it would create a microscopic black hole with a Schwarzschild radius on the order of the Planck length itself. This is a manifestation of the emergent causal structure rather than fundamental physics breaking down.

Using Planck units to nondimensionalize physical equations is a convenient approach. It is mathematically equivalent to setting $c = 1$, $G = 1$, $\hbar = 1$, and $k_e = 1$. This is a profound physical statement, declaring that all quantities are being measured as ratios relative to the emergent scales of nature itself. An energy expressed in Planck units is a dimensionless number that answers the question: "How does this energy compare to the energy scale at which causal wave correlations manifest quantum-like behavior?"

This framework reveals some of the deepest mysteries in physics. For example, the “hierarchy problem” is the question of why gravity is so much weaker than the other fundamental forces. In Planck units, this is no longer a question about the disparate values of dimensional constants but a question about a pure number. The mass of a proton is approximately $10^{-19}m_P$. The electrostatic force between two protons is roughly 10^{36} times stronger than their gravitational attraction. The hierarchy problem is thus reframed as the question of why the characteristic dimensionless masses of elementary particles are so extraordinarily small compared to 1. Expressing physics dimensionlessly does not just simplify the equations; it exposes the fundamental, unexplained numerical ratios that define the character of our universe (Rees, 2000).

4.0

Nondimensionalization of Fundamental Equations

The power of the dimensionless framework is revealed when it is applied to the fundamental equations of physics. By recasting these laws in dimensionless form, the dimensional constants—which emerge from the causal network of wave correlations—vanish, leaving behind pure mathematical relationships between dimensionless ratios. This process not only simplifies the equations but also uncovers their essential structure and reveals deep connections between seemingly disparate areas of physics.

4.1

Classical Mechanics: Newton’s Law of Universal Gravitation

4.1.1 Standard Form

The familiar form of Newton’s law of universal gravitation is:

$$F = G \frac{m_1 m_2}{r^2}$$

4.1.2 Nondimensionalization

Dimensionless variables, denoted with a prime ('), are introduced by scaling each dimensional quantity with its corresponding Planck unit, as defined in Section 3.2: $F' = F/FP$, $m_1' = m_1/m_P$, $m_2' = m_2/m_P$, and $r' = r/\ell_P$. The dimensional variables are then expressed in terms of the dimensionless ones: $F = F'FP$, $m_1 = m_1'm_P$, $m_2 = m_2'm_P$, and $r = r'\ell_P$.

4.1.3 Derivation

Substituting the scaled variables into Newton’s law yields: $F'FP = G(m_1'm_P)(m_2'm_P)/(r'\ell_P)^2$. By substituting the definitions of the Planck units, $FP = c^4/G$, $m_P = \sqrt{(\hbar c/G)}$, and $\ell_P = \sqrt{(\hbar G/c^3)}$, the equation becomes:

$$F' \left(\frac{c^4}{G} \right) = G \frac{m_1' m_2' \left(\frac{\hbar c}{G} \right)}{r'^2 \left(\frac{\hbar G}{c^3} \right)}$$

Simplifying the expression reveals a common factor of c^4/G on both sides of the equation, which can be cancelled.

4.1.4 Dimensionless Form

The resulting dimensionless equation is strikingly simple:

$$F' = \frac{m_1' m_2'}{r'^2}$$

In this form, the law of gravitation is a pure relationship between dimensionless numbers. The gravitational constant G has been absorbed into the definitions of the units. The equation states that the gravitational force, measured as a fraction of the Planck force, is equal to the product of the masses, measured as fractions of the Planck mass, divided by the square of the distance, measured as a fraction of the Planck length.

4.2 Electromagnetism: Coulomb's

Law

4.2.1 Standard Form

The standard form of Coulomb's law is:

$$F = k_e \frac{q_1 q_2}{r^2}$$

4.2.2 Nondimensionalization

Dimensionless variables are defined by scaling with the appropriate Planck units: $F' = F/F_P$, $q_1' = q_1/q_P$, $q_2' = q_2/q_P$, and $r' = r/\ell_P$.

4.2.3 Derivation

Substituting the scaled variables into Coulomb's law gives $F' F_P = k_e (q_1' q_P)(q_2' q_P)/(r' \ell_P)^2$. Using the convenient forms $F_P = \hbar c / \ell_P^2$ and $q_P = \sqrt{\hbar c / k_e}$, the equation becomes:

$$F' \left(\frac{\hbar c}{\ell_P^2} \right) = k_e \frac{q_1' q_2' \left(\frac{\hbar c}{k_e} \right)}{r'^2 \ell_P^2}$$

The term $\hbar c / \ell_P^2$ appears on both sides, allowing for its cancellation.

4.2.4 Dimensionless Form

The dimensionless form of Coulomb's law is:

$$F' = \frac{q_1' q_2'}{r'^2}$$

This result is profound. In the natural system of Planck units, the mathematical form of Coulomb's law is identical to that of Newton's law of gravitation, derived in Section 4.1. This reveals that the inverse-square laws of gravity and electromagnetism share the same fundamental structure. The vast difference in their perceived strengths is not a property of the laws themselves, but a consequence of the dimensionless magnitudes of the “charges” that particles possess—mass for gravity (m/m_P) and electric charge for electromagnetism (q/q_P).

4.3

Quantum Mechanics: The Time-Dependent Schrödinger Equation

4.3.1 Standard Form

The time-dependent Schrödinger equation for a single non-relativistic particle of mass m in a potential V is (Schrödinger, 1926):

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi$$

4.3.2 Nondimensionalization

A set of dimensionless variables and operators is defined: $t' = t/t_P$, $r' = r/\ell_P$, so $\nabla = (1/\ell_P)\nabla'$ and $\nabla^2 = (1/\ell_P^2)\nabla'^2$; $m' = m/m_P$; $V' = V/E_P$; and $\Psi'(r', t') = \ell_P^{3/2}\Psi(r, t)$, to ensure the probability density $|\Psi|^2$ is dimensionless.

4.3.3 Derivation

Transforming the derivatives and substituting into the Schrödinger equation yields:

$$\frac{i\hbar}{t_P} \frac{\partial \Psi'}{\partial t'} = -\frac{\hbar^2}{2m'm_P\ell_P^2} \nabla'^2 \Psi' + V' E_P \Psi'$$

Dividing the entire equation by the Planck energy, E_P , makes all terms dimensionless. The dimensionless coefficients are evaluated using the definitions of the Planck units from Section 3.2. The coefficient of the time derivative term, $\hbar/(t_P E_P)$, becomes 1. The coefficient of the kinetic energy term, $\hbar^2/(2m'm_P\ell_P^2 E_P)$, becomes $1/(2m')$.

4.3.4 Dimensionless Form

The resulting dimensionless Schrödinger equation is:

$$i \frac{\partial \Psi'}{\partial t'} = -\frac{1}{2m'} \nabla'^2 \Psi' + V' \Psi'$$

In this form, the constant $\hbar$, which is synonymous with quantum mechanics, has vanished. It has been absorbed into the scale of the system. The only parameter specific to the particle that remains is its dimensionless mass, m' . This reframes mass in a quantum context: it is a dimensionless number that dictates how a particle's wave function evolves relative to the Planck scale. Mass, in this view, is a fundamental measure of a particle's resistance to quantum delocalization.

4.4 General

Relativity: Einstein's Field Equations

4.4.1 Standard Form

The equations are written in tensor notation as (Einstein, 1915):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

4.4.2 Nondimensionalization

The following dimensionless tensors are defined: $G'_{\mu\nu} = G_{\mu\nu}\ell_P^2$, $\Lambda' = \Lambda\ell_P^2$, and $T'_{\mu\nu} = T_{\mu\nu}/(E_P/\ell_P^3)$.

4.4.3 Derivation

Substituting the scaled variables into the field equations gives:

$$G'_{\mu\nu} + \Lambda' g_{\mu\nu} = \frac{8\pi G\ell_P^2}{c^4} \frac{E_P}{\ell_P^3} T'_{\mu\nu} = \frac{8\pi G E_P}{c^4 \ell_P} T'_{\mu\nu}$$

Evaluating the dimensionless coefficient on the right-hand side by substituting the definitions of E_P and ℓ_P from Section 3.2 shows that it is equal to 8π .

4.4.4 Dimensionless Form

The dimensionless Einstein field equations are:

$$G'_{\mu\nu} + \Lambda' g_{\mu\nu} = 8\pi T'_{\mu\nu}$$

In this form, the constants G and c have disappeared. The equations now represent a direct relationship between dimensionless geometry (on the left) and dimensionless matter-energy content (on the right). The factor of 8π remains as a purely geometric factor. The equation states that the curvature of spacetime, measured in units of the Planck curvature ($1/\ell_P^2$), is proportional to the density of stress-energy, measured in units of the Planck density.

5.0 The Universe's Intrinsic

Parameters

Once the fundamental equations of physics are expressed in dimensionless form as shown in Section 4.0, the dimensional constants of scale (c , G , $\hbar$, k_e) vanish. What remains is a set of irreducible, dimensionless numbers whose values are not fixed by theory but must be determined by experiment. These are the true constants of nature that define the specific character of our universe. They are the universe's fundamental π -groups, as explained in Section 2.2. A universe with the same laws but different values for these constants would be a radically different place.

5.1

The Fine-Structure Constant (α): The Strength of Light and Matter

5.1.1 Definition and

Dimensionless Nature

In SI units, the **fine-structure constant**, denoted α , is defined as:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{k_e e^2}{\hbar c}$$

Its experimentally measured value is approximately $1/137.036$. To confirm its dimensionless nature, the dimensions of its components are analyzed: $[k_e] = [M][L]^3[T]^{-2}[Q]^{-2}$, $[e] = [Q]$, $[\hbar] = [M][L]^2[T]^{-1}$, and $[c] = [L][T]^{-1}$. Combining these for the expression $k_e e^2 / (\hbar c)$ yields the following dimensional analysis.

$$\begin{aligned} [\alpha] &= \frac{[k_e][e]^2}{[\hbar][c]} = \frac{([M][L]^3[T]^{-2}[Q]^{-2}) \cdot [Q]^2}{([M][L]^2[T]^{-1}) \cdot ([L][T]^{-1})} \\ &= \frac{[M][L]^3[T]^{-2}}{[M][L]^3[T]^{-2}} \\ &= [M]^0[L]^0[T]^0[Q]^0 \end{aligned}$$

The final expression indicates that the fine-structure constant α is a dimensionless physical constant. As required by the protocol in Section 4.6.1.3, this result demonstrates that all fundamental dimensions cancel completely, signifying that the constant is a pure number independent of any system of units, not that its numerical value is zero.

5.1.2 Physical Meaning

The fine-structure constant is the coupling constant of quantum electrodynamics (QED). Its value determines the strength of the interaction between charged particles and photons (Feynman, 1949). In the framework of Planck units, its meaning is particularly clear. Using the definition of the Planck charge, $q_P = \sqrt{\hbar c/k_e}$, from Section 3.2.4, the constant can be written as $\alpha = (e/q_P)^2$. Thus, α is the square of the ratio of the elementary charge to the Planck charge. It is a fundamental dimensionless measure of the strength of electric charge in the universe.

5.2 The

Proton-to-Electron Mass Ratio (μ): The Scale of Matter

5.2.1 Definition and

Dimensionless Nature

The proton-to-electron mass ratio, denoted by μ or β , is defined as $\mu = m_p/m_e$, where m_p is the rest mass of the proton and m_e is the rest mass of the electron. Its value has been experimentally determined to be approximately 1836.15267343. As it is a ratio of two quantities with the same dimension, it is inherently a pure number.

5.2.2 Physical Meaning

The value of μ is critical for the structure of matter. Because the proton is nearly 2000 times more massive than the electron, the nucleus of an atom is extremely heavy and slow-moving compared to the orbiting electrons. This large mass ratio justifies the Born-Oppenheimer approximation in quantum chemistry, which allows for the separation of nuclear and electronic motion, making stable molecular bonds and complex chemistry possible. Unlike α , the value of μ is the result of a complex interplay between different physical theories. The majority of the proton's mass comes from the kinetic energy of its constituent quarks and the binding energy of the gluon field, as described by quantum chromodynamics ('t Hooft, 1974). Therefore, μ is a complex emergent constant whose value depends on the parameters of both the electroweak theory and the strong nuclear force.

5.3 The

Cosmological Constant (Λ): The Energy of Nothing

5.3.1 Definition and

Dimensionless Form

In its standard formulation, Λ has dimensions of inverse length squared, $[L]^{-2}$. Its fundamental nature is better captured by comparing it to other scales in the universe. One key dimensionless form is the cosmological density parameter, Ω_Λ , which is the ratio of the energy density associated with the cosmological constant to the critical density of the universe. Observations place the value of Ω_Λ at approximately 0.7. A more fundamental dimensionless representation is obtained by scaling Λ with the Planck length: $\Lambda' = \Lambda \ell_P^2$. Using observed cosmological parameters, this dimensionless constant has a value of approximately 10^{-122} .

5.3.2 Physical Meaning

The incredibly small number, $\Lambda' \approx 10^{-122}$, represents the energy density of the vacuum itself, measured in natural Planck units. The “cosmological constant problem” is the mystery of why this number is so extraordinarily close to zero. Calculations in quantum field theory predict that the vacuum energy should be on the order of the Planck density, which would correspond to $\Lambda' \approx 1$, about 122 orders of magnitude larger than the observed value (Weinberg, 1972). This discrepancy highlights the distinction between constants of scale and constants of character. The constants c , G , and $\hbar$ define the scale of the universe. The dimensionless numbers that remain— α , μ , Λ' , and others—define its character. The challenge for fundamental physics is not just to measure these numbers, but to explain them.

6.0 Dimensionless Theory of

Everything

6.1 Introduction to

the Dimensionless Approach

The concept of a theory of everything has long been a central goal of theoretical physics. This section presents Stergios Pellis's dimensionless theory, which demonstrates how all fundamental interactions can be unified through dimensionless relationships between physical constants (Pellis, 2023). Pellis's work builds upon the dimensionless framework established in the preceding sections, extending it to show how the coupling constants of the fundamental forces can be connected through mathematical relationships. This theory represents a significant advancement, revealing deep connections between quantum mechanics, general relativity, and cosmology that have previously remained obscured by the use of anthropocentric units.

6.2

Dimensionless Unification of Fundamental Interactions

Pellis's dimensionless theory demonstrates that the coupling constants of the fundamental forces can be connected through simple mathematical relationships. The most fundamental of these is the dimensionless unification of the strong nuclear and weak nuclear interactions: $e \cdot \alpha_s = 10^7 \cdot \alpha_w$ and $e \pi \cdot \alpha_s^2 = 10^7 \cdot \alpha_w$, where α_s is the strong coupling constant and α_w is the weak coupling constant. Similarly, the dimensionless unification of the strong nuclear and electromagnetic interactions is given by $e \pi \cdot \alpha_s \cdot \cos(\alpha^{-1}) = 1$ and $\alpha_s \cdot \cos(\alpha^{-1}) = i^{2i}$, where α is the fine-structure constant. For the weak nuclear and electromagnetic interactions, the unification takes the form: $10^7 \cdot e \pi \cdot \alpha_w \cdot \cos(\alpha^{-1}) = e$. These relationships demonstrate that the coupling constants are not independent parameters but are interconnected through mathematical relationships that emerge naturally when expressed in dimensionless form.

6.3

Dimensionless Unification of Gravitational and Electromagnetic Interactions

One of the most profound results of Pellis's dimensionless theory is the unification of gravitational and electromagnetic interactions. This is expressed through the relationship $4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1$, where α_G is the gravitational coupling constant and N_A is Avogadro's number. This equation reveals a deep connection between gravity and electromagnetism that has previously remained hidden in dimensional formulations. It connects the microscopic world of atomic physics with the macroscopic world of thermodynamics and statistical mechanics.

6.4

Dimensionless Unification of All Four Fundamental Interactions

Pellis's dimensionless theory culminates in the unification of all four fundamental interactions through the relationship $\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2$. This equation represents the complete unification of the strong, weak, electromagnetic, and gravitational forces. It demonstrates that all fundamental interactions can be described through pure dimensionless relationships, with no need for anthropocentric units or dimensional constants.

6.5

Gravitational Constant and Cosmological Constant

Pellis's dimensionless theory provides exact formulas for the gravitational constant G and the cosmological constant Λ :

$$G = \frac{c^3}{\hbar} \cdot \left(\frac{1}{4 \cdot e^2 \cdot \alpha^2 \cdot N_A^2} \right)$$

$$\Lambda = \frac{1}{\ell_P^2} \cdot \left(\frac{1}{(2 \cdot e \cdot \alpha^2 \cdot N_A)^6} \right)$$

These formulas reveal that the gravitational constant and cosmological constant are not independent parameters but are determined by fundamental dimensionless relationships.

6.6

Poincaré Dodecahedral Space as the Shape of the Universe

Pellis's dimensionless theory also provides evidence that the shape of the universe is Poincaré dodecahedral space (Pellis, 2022). This is derived from the dimensionless relationships between the density parameters of baryonic matter, dark matter, and dark energy: $\Omega_B = e^{-\pi} \approx 4.32\%$, $\Omega_D = 2 \cdot e^{1-\pi} \approx 23.49\%$, and $\Omega_\Lambda = 2 \cdot e^{-1} \approx 73.57\%$. These dimensionless density parameters sum to approximately 1.0139, indicating a slightly positively curved universe consistent with the Poincaré dodecahedral space topology. This suggests that the universe has a finite, multiply connected topology rather than being infinite and simply connected (Luminet et al., 2003).

7.0

Temporal Quantization in the Unified Fractal Theory of Time

7.1 Introduction to Fractal

Time

Time has traditionally been conceived as a linear and homogeneous parameter. However, emerging evidence suggests that temporal dynamics may instead be governed by fractal, self-similar, and nonlinear structures (Bassingthwaighite, 1994). This section presents Stergios Pellis's Temporal Quantization in the Unified Fractal Theory of Time, which introduces a ϕ -based fractal formalism as a unifying framework for the quantization and scaling of time (Pellis, 2025). This framework integrates the golden ratio ϕ , Fibonacci structures, and operatorial formulations of fractal calculus, leading to a recursive conception of temporal flow.

7.2 The Pellis Function

The Pellis Function is defined as $f(x) = 360 \cdot x^{-2} - 2 \cdot x^{-3} + (3 \cdot x)^{-5}$. When $x = \phi$, the golden ratio, the function approximates with great accuracy the inverse fine-structure constant α^{-1} . This relationship connects the constant to the golden angle, $\theta_g \approx 137.5^\circ$, which derives from the golden ratio (Heyrovska, 2005). The equation accurately expresses this relationship, where the term $360 \cdot \phi^{-2}$ corresponds exactly to the golden angle, while the corrective terms fine-tune the value to match the precise experimental constant.

7.3 Fractal Time Flow

Pellis defines Fractal Time as a hierarchy of repeating time scales: $T_n = T_0 \cdot \phi^n$, where T_n is the time scale of level n , T_0 is the basic unit (e.g., Planck time), and ϕ is the golden ratio. Each T_n represents a fractal temporal layer corresponding to quantum, biological, or cosmological phenomena. This expresses quantized, hierarchical time, where each moment nests within larger spiraling structures, analogous to musical scales and biological oscillations (Bailly et al., 2010; Sacco, 2018).

7.4 The Pellis Golden Clock

The Pellis Golden Clock is a theoretical model that unifies cosmological, biological, and quantum time frequencies through scaling with the golden ratio. It unifies natural frequencies across domains via ϕ -scaling: $f_n = f_0 \cdot \phi^n$, where f_n is the natural frequency of the n -th plane, $n \in \mathbb{Z}$, and f_0 is a basis constant. The fractal frequency spectrum is represented via a Pellis-Fourier transform, $F(\omega) = \sum_n A_n e^{i\omega n t}$, which encodes multi-scale temporal harmonics. This model captures multi-scale harmonics in ϕ -time and can be applied to heart rate variability, electroencephalography, DNA oscillations, and cosmic cycles (Kramer, 2022).

7.5 Applications of Fractal

Time

The fractal time framework has numerous applications. First, in quantum systems, ϕ -scaled repeating temporal patterns appear in quantum beats and micro-scale oscillations, and Pellis fractal clocks can model these patterns (Golmankhaneh et al., 2024). Second, in cosmology, cosmic time emerges from ϕ^n -folding of Planck time, connecting molecular, physiological, and cosmic temporal scales (Khalili-Golmankhaneh, 2018). Third, in biology, analysis of signals such as heart rate variability using ϕ -scaled temporal layers enhances detection of nested rhythms and multi-scale harmonics (Ivanov et al., 1999). Fourth, in genomics, DNA double helix geometries and protein folding patterns can be described as ϕ -spirals within the fractal time framework (Marples, 2022). Fifth, in geophysics, Pellis-type seismic eigenfunctions can describe the temporal structure of seismic phenomena (Guettari et al., 2025). These applications demonstrate that the fractal time framework is not merely theoretical but has practical implications for modeling diverse physical, biological, and cosmological phenomena.

8.0

Synthesis: Dimensionless Physics as a Unified Framework

8.1 The

Dimensionless Perspective on Physical Law

The analysis presented in this work converges on a singular conclusion: the fundamental laws of physics are expressions of relationships between pure, dimensionless ratios. This perspective reframes our understanding of the cosmos, moving from a picture defined by arbitrary units to one governed by an intrinsic and universal mathematical structure. The expression of all fundamental laws in a form devoid of dimensional constants, as achieved in Section 4.0, reveals that the universe is governed by a set of pure, dimensionless numbers. As established by the Buckingham π theorem and the application of Planck units, constants such as G , c , and $\hbar$ are not fundamental but emerge from a deeper causal network of wave correlations.

8.2 The Dimensionless

Theory of Everything

Pellis's dimensionless theory demonstrates that all fundamental interactions can be unified through dimensionless relationships between physical constants (Pellis, 2023). The coupling constants of the four fundamental forces are not independent parameters but are interconnected through mathematical relationships that emerge naturally when expressed in dimensionless form. As detailed in Sections 6.2 through 6.5, this theory provides exact formulas for the gravitational constant G and the cosmological constant Λ , and provides evidence that the shape of the universe is Poincaré dodecahedral space (Pellis, 2022).

8.3 Temporal

Quantization and Fractal Time

The Temporal Quantization in the Unified Fractal Theory of Time demonstrates that time itself is not linear but fractal, governed by the golden ratio ϕ and Fibonacci structures (Pellis, 2025). As explained in Section 7.0, the Pellis Function and Pellis Golden Clock provide a framework for understanding temporal phenomena across all scales as manifestations of a single, unified fractal structure. This suggests that biological rhythms, quantum phenomena, and cosmic cycles are all manifestations of the same underlying fractal structure.

8.4 Implications for

Fundamental Physics

The dimensionless perspective has several profound implications for fundamental physics. First, the dimensionless relationships between coupling constants indicate that a unified theory of all fundamental forces is possible through pure mathematical relationships. Second, the dimensionless constants of nature, such as α and μ , are not arbitrary but are determined by deeper mathematical structures. Third, the dimensionless density parameters indicate a slightly positively curved universe consistent with the Poincaré dodecahedral space topology. Fourth, the structure of time itself is not linear but fractal, governed by the golden ratio. Finally, the dimensionless density ratio $\rho' = \rho/\rho_P$ provides a universal criterion for determining whether a system's gravitational behavior is classical or requires a quantum description.

8.5 Future Directions

The dimensionless framework presented in this work opens several avenues for future research. One direction is the development of mathematical frameworks to derive the values of fundamental dimensionless constants from first principles. Another is the experimental verification of the dimensionless relationships between coupling constants and the predictions of the dimensionless theory. Further research should also involve applying the fractal time framework, described in Section 7.0, to practical problems in biology, medicine, and engineering. The dimensionless framework can also be used to develop a complete theory of quantum gravity. Lastly, cosmological models can be developed based on the dimensionless density parameters and Poincaré dodecahedral space topology.

9.0 Conclusion

The comprehensive analysis presented in this work demonstrates that the fundamental laws of physics are not statements about dimensional quantities tied to human scales, but are instead expressions of relationships between pure, dimensionless ratios. This perspective reframes our understanding of the cosmos, moving from a picture defined by arbitrary units to one governed by an intrinsic and universal mathematical structure.

The Buckingham π theorem provides the mathematical foundation for this dimensionless perspective, proving that any physically meaningful relationship can be expressed entirely in terms of dimensionless ratios (Buckingham, 1914). The Planck scale provides a convenient metric for this dimensionless description, allowing all physical quantities to be expressed as ratios relative to the emergent scales of nature.

Pellis's dimensionless theory demonstrates that all fundamental interactions can be unified through dimensionless relationships between physical constants (Pellis, 2023). The coupling constants of the fundamental forces are interconnected through elegant mathematical relationships that emerge naturally when expressed in dimensionless form. This theory also provides evidence that the shape of the universe is Poincaré dodecahedral space (Pellis, 2022).

The Temporal Quantization in the Unified Fractal Theory of Time demonstrates that time itself is not linear but fractal, governed by the golden ratio ϕ and Fibonacci structures (Pellis, 2025). This framework provides a unified structure for understanding temporal phenomena across all scales.

This dimensionless perspective has profound implications for fundamental physics and our understanding of the universe. It suggests that the universe is fundamentally mathematical in nature, with physical laws expressed as pure numerical relationships. The challenge for future physics is not just to measure the dimensionless constants of nature, but to derive them from first principles, revealing the deeper mathematical structure of the cosmos. As Max Planck envisioned, this dimensionless framework provides a physics for all times and all civilizations, built upon the timeless and universal language of dimensionless numbers (Planck, 1899).

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