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Discrete Scale Invariance Without a Universal Factor: Why the Efimov Data Refute Factor-2 Log-Periodicity Across Fundamental Physics

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ABSTRACT

Discrete scale invariance (DSI) — invariance under rescaling by a fixed factor rather than by any factor — is established in few-body quantum physics, Dirac semimetals, and renormalization-group limit cycles. A recurring conjecture holds that the scaling factor itself might be universal, with the dyadic value 2 appearing across unrelated domains. This paper tests that conjecture against the published few-body and condensed-matter evidence. We model each confirmed DSI system as a log-periodic spectrum $E_n = E_0 \exp(-2\pi n/s_0)$ and extract the scaling factor from published values. The three-boson Efimov spectrum has scaling factor 22.694, which we show corresponds to an RG index $s_0 = 2\pi/\ln(22.694) = 2.0124$ and a log-periodic period $\Delta(\ln E) = 3.1222$, i.e. 4.505 octaves — a factor of 4.505 larger than the dyadic period $\ln 2 = 0.69315$. Scaling traps realize continuously tunable factors controlled by the slope of a spatially varying scattering length, and spin-orbit-coupled three-boson systems obey a generalized scaling law in a five-dimensional parameter space, so no single factor is fixed. We conclude that DSI is real and broadly shared, but the specific factor-2 universality hypothesis is excluded by existing data at the level of a factor 11.347 per spectral step. The practical consequence is a sharp falsification criterion: any claimed universal dyadic periodicity in a physical spectrum must survive comparison against the measured Efimov period of 3.1222 in $\ln E$, which it cannot if the period is 0.69315.

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Abstract

Discrete scale invariance (DSI) — invariance under rescaling by a fixed factor rather than by any factor — is established in few-body quantum physics, Dirac semimetals, and renormalization-group limit cycles. A recurring conjecture holds that the scaling factor itself might be universal, with the dyadic value 2 appearing across unrelated domains. This paper tests that conjecture against the published few-body and condensed-matter evidence. We model each confirmed DSI system as a log-periodic spectrum $E_n = E_0 \exp(-2\pi n/s_0)$ and extract the scaling factor from published values. The three-boson Efimov spectrum has scaling factor 22.694, which we show corresponds to an RG index $s_0 = 2\pi/\ln(22.694) = 2.0124$ and a log-periodic period $\Delta(\ln E) = 3.1222$, i.e. 4.505 octaves — a factor of 4.505 larger than the dyadic period $\ln 2 = 0.69315$. Scaling traps realize continuously tunable factors controlled by the slope of a spatially varying scattering length, and spin-orbit-coupled three-boson systems obey a generalized scaling law in a five-dimensional parameter space, so no single factor is fixed. We conclude that DSI is real and broadly shared, but the specific factor-2 universality hypothesis is excluded by existing data at the level of a factor 11.347 per spectral step. The practical consequence is a sharp falsification criterion: any claimed universal dyadic periodicity in a physical spectrum must survive comparison against the measured Efimov period of 3.1222 in $\ln E$, which it cannot if the period is 0.69315.

1. Introduction

Experimental groups measuring few-body bound states in ultracold atoms, transport in Dirac materials, and spectra near charged impurities all face the same analysis problem: when a spectrum repeats itself under rescaling, what is the scaling factor, and does it carry fundamental meaning? The answer determines whether an observed oscillation is a fingerprint of a specific microscopic boundary condition, a tunable property of an engineered trap, or evidence for a new symmetry of nature. Misreading the factor leads to misassigned mechanisms: a trap-engineered period can be mistaken for a universal constant, or a genuine Efimov tower can be fitted with the wrong period and yield wrong scattering parameters.

The stakes are concrete. In the cold-atom case, the atom-loss resonances that signal Efimov states occur at critical scattering lengths whose ratios encode the scaling factor; extracting that factor is the primary quantitative output of the experiment, and it feeds into determinations of three-body parameters used across the ultracold-atom community [4]. In Dirac semimetals, quasibound states around charged impurities affect transport, and whether their spectrum shows a discrete scaling symmetry under magnetic field determines what a magnetotransport measurement can reveal about the underlying Dirac Hamiltonian [1]. If a universal factor-2 discrete scaling symmetry governed fundamental physics, a single measured period in one domain would calibrate expectations in every other domain, and deviations would signal new physics. If instead each system carries its own factor — set by boundary conditions, trap geometry, or spin-orbit coupling — then every domain must be calibrated independently, and factor-2 claims in new data require their own dynamical explanation.

This paper resolves that question by extracting the scaling factors that the published literature actually reports and comparing them against the dyadic hypothesis. The comparison is decisive. The three-boson Efimov factor is 22.694 [4], not 2; the scaling-trap factor is continuously tunable through the slope of the scattering-length profile [2]; the Dirac-semimetal factors depend on magnetic-field regime and dimensionality [1]; and

the log-periodic modulations proposed in quantum-gravity contexts carry an arbitrary period parameter ω [3]. The broader intuition behind the factor-2 conjecture — that discrete scaling loops can replace continuous scale invariance — has solid empirical support; the specific universal value does not.

The contributions of this paper are:

- A derivation, from the published Efimov scaling factor 22.694 [4], of the renormalization-group index $s_0 = 2.0127$, the log-periodic period $\Delta(\ln E) = 3.1222$, and the octave count 4.505, establishing quantitatively how far the confirmed few-body factor lies from the dyadic value (Section 4).
- A classification of confirmed discrete-scale-invariant systems by what fixes their scaling factor — boundary condition, trap slope, magnetic regime, or dynamical exponent — showing that in every cited system the factor is either non-dyadic or continuously adjustable (Sections 2 and 4).
- A falsification criterion for universal dyadic periodicity, stated as a measurable period in $\ln E$, together with the observation that factor 2 does appear in analysis conventions (dyadic partitions of frequency space [11]) and must therefore be distinguished from factor 2 as physics (Section 5).
- An assessment of what a viable research program on discrete scaling loops should claim — the existence and structure of log-periodicity, not the universality of its factor — and the smallest experimental test that would settle the matter for a new system (Sections 6 and 7).

2. Background and Related Work

The Efimov effect is the canonical example of discrete scale invariance in quantum mechanics. Three identical bosons near a two-body s -wave resonance can exhibit an infinite tower of bound states with a geometric energy sequence, reflecting a broken continuous scaling symmetry. The scaling factor is not arbitrary: for equal-mass bosons the three-boson spectrum has the discrete scaling factor 22.694, a number large enough that measuring signatures of two consecutive trimer levels is experimentally challenging, since the scattering length must simultaneously exceed the van der Waals length r_{vdW} and remain below the de Broglie wavelength λ_{dB} [4]. Despite this window constraint, the discrete scaling symmetry of the standard Efimov scenario has been confirmed experimentally by monitoring atom losses in an ultracold thermal gas of Cs atoms as a function of the s -wave scattering length, with loss enhancement at the critical scattering lengths $a_-^{(n)}$ and $a_*^{(n)}$ where trimer levels cross the three-atom and atom-dimer thresholds. This measured factor of 22.694 is the single most direct datum against a universal factor 2, and we use it as the anchor of the quantitative analysis in Section 4.

The factor is also not protected against modification. Adding one-dimensional spin-orbit coupling to the three-boson Hamiltonian introduces a parametric dependence of the relative Hamiltonian on the center-of-mass momentum vector, and the resulting spectrum obeys a generalized radial scaling law in a five-dimensional parameter space associated with a discrete scaling symmetry [4]. A scaling law whose factor lives in a five-dimensional parameter space cannot carry a single universal value; the same physical interaction class supports a continuum of factors. This is the strongest structural argument in the literature against factor universality within few-body physics itself.

A second line of evidence comes from engineered systems in which the factor is deliberately made tunable. When the scattering length is made proportional to the distance from the center of the system — realizable with ultracold atoms through a magnetic- or optical-field-induced Feshbach resonance with spatially varying field intensity, or through a confinement-induced resonance with transverse confinement lengths varying longitudinally — two particles are trapped about the center and their spectrum exhibits discrete scale invariance whose scale factor is controlled by the slope of the scattering length [2]. The proposal explicitly notes the resemblance to the Efimov effect while emphasizing practical advantages of the two-body trapped system. Because the slope is an experimental dial, the scaling factor in these systems is continuously adjustable by construction; a universal factor 2 is not merely absent here, it is excluded by design.

Dirac semimetals extend discrete scaling symmetry to condensed matter. Massless Dirac fermions in two and three dimensions form sequences of quasibound states in the field of an attractive charged impurity, with an energy ratio between successive states that is constant — a discrete scaling symmetry arising because the continuous scaling symmetry of the Dirac equation is broken by a short-range boundary condition, exactly as in the Efimov effect [1]. The magnetic response is regime-dependent: in two dimensions an applied magnetic field shifts the spectrum and produces magnetic oscillations that exhibit the discrete scaling symmetry through the interplay between quasibound states and Landau levels, whereas in three dimensions the magnetic oscillations do not show the symmetry before the quantum limit because different channels couple and no unique scaling factor exists; only in the quantum limit, where the system becomes quasi-one-dimensional and channels decouple for shallow quasibound states, does the magnetic dependence of the spectrum exhibit the symmetry again [1]. The phrase "lack of a unique scaling factor" in the three-dimensional pre-quantum-limit regime is a direct experimental-systems statement that the scaling factor is not an invariant of the material but a property of a regime.

The same Efimov physics in Dirac materials can be formulated as a renormalization-group limit cycle. In the field-theory treatment of an electron interacting with a static charged impurity in a Dirac semimetal, the Efimov effect is captured by limit-cycle solutions of the RG, and vacuum polarization screening — the many-body correction analogous to the Lamb shift for "relativistic hydrogen atoms" — modifies the effect; the

results connect directly to recent transport experiments on Dirac materials and apply to Weyl materials as well [5]. The RG formulation makes the structural point explicit: a limit cycle is a closed orbit in coupling space, and the cycle length — hence the scaling factor — is set by the flow, not by a universal constant.

In quantum gravity, log-periodic modulations arise generically from discrete scale invariance but with an arbitrary period. The general form of the modulation follows from a scaling argument: continuous scale invariance gives power laws $f(z) \propto z^\alpha$, and breaking the continuous symmetry to a discrete one produces a log-periodic modulation factor F_ω whose period is a free parameter; such modulations arise in all quantum gravities where the Hausdorff and/or spectral dimension vary with scale, and discrete scale invariance makes the spacetime dimension complex-valued [3]. The simplest realization is a fundamentally discrete pregeometry similar to hierarchical lattices, explicit in multifractional theories, where logarithmic oscillations appear in the deep UV; in less regular combinatorial structures such as loop quantum gravity, spin foams, and group field theory, complex dimensions may arise only in special situations [3]. The free period parameter ω in this framework is the theoretical counterpart of the system-dependent factors measured in the laboratory: discrete scale invariance forces periodicity in $\ln$ scale, but nothing in the mechanism fixes the period.

The mechanism by which continuous scaling breaks to discrete is itself informative. In conformal quantum mechanics, the $so(2,1)$ algebra generated by the Hamiltonian H , the dilatation generator G , and the special conformal generator K describes a large class of systems including molecules, black holes, graphene, and Calogero models; the continuous scaling symmetry can be broken purely quantum-mechanically when the boundary conditions — equivalently, the domain of the Hamiltonian — are not preserved by the action of the dilatation operator, and the broken symmetry can be restored by a suitable mixed state associated with a nonvanishing von Neumann entropy [8]. The breaking is therefore a property of the boundary condition and the state, not a universal constant of the algebra; different domains of the same Hamiltonian produce different discrete remnants.

Scaling symmetry in classical and gravitational settings likewise carries system-dependent parameters. In classical mechanics, variational symmetries of the action yield conservation laws while non-variational symmetries of the equations of motion — epitomized by scale-invariant hydrostatics — yield first-order non-conservation laws between invariants, from which all the familiar properties of polytropes follow, including a common core density profile and the Emden functions [9]. In matter-coupled scaling-Carroll gravity formulated as a gauge theory, the construction is based on a compensating real scalar field with a general dynamical exponent z , extending Carroll gravity at $z = 1$; after fixing the scaling symmetry, the trace of the extrinsic curvature K enters the shift transformation of the vector field b_a under local Carroll boosts [10]. The exponent z is a free parameter of the theory: anisotropic scaling with arbitrary z is consistent, so even at the level of gravity-matter couplings the scaling exponent is a dial, not a constant.

Thermodynamics formulated geometrically makes the same point from a different direction. In the symplectic approach, the thermodynamic phase space is expanded by a global scale variable Z , and extensive and intensive quantities scale with arbitrary weights r_i, r, ρ associated with the variables; fixing the extended global scale variable recovers the standard scale-invariant thermodynamic description, and the formalism covers systems whose homogeneity degrees differ, illustrated by the diffeomorphism between the Lagrangian submanifolds of the ideal and van der Waals gases [7]. Arbitrary scaling weights are built into the formalism; a universal weight is not.

Finally, the dyadic factor 2 does appear pervasively — but as a convention of analysis rather than a law of physics. In harmonic analysis, Besov spaces stem from a dyadic covering of frequency space and modulation spaces from a uniform covering, connected by the α -modulation spaces of Gröbner at intermediate $\alpha \in [0, 1]$; one can construct modulation function spaces with a scaling symmetry in which a dyadic scale $\lambda = 2^{j_0}$ is fixed by choice, and the scaling property of the spaces follows from how the translation operator acts on the vector weight under this chosen dyadic scale [11]. The factor 2 here is the base of the chosen partition of unity. This distinction — factor 2 as the base of a chosen decomposition versus factor 2 as a physical period — is exactly the confusion the factor-2 universality conjecture risks, and it motivates the falsification criterion of Section 5. Frameworks that model reality as a hyperdimensional fractal manifold with scale-dependent spectral dimensions likewise predict scale-dependent structure without fixing a universal scaling ratio [14].

3. Methods

This paper is an analytical reanalysis of published scaling factors; it performs no experiment, measurement, or simulation. The method has three parts.

Model. Every confirmed discrete-scale-invariant spectrum considered here is log-periodic: successive states obey

$$E_{n+1} = \frac{E_n}{\lambda}, \quad E_n = E_0 \lambda^{-n},$$

where $\lambda > 1$ is the scaling factor. Equivalently, in terms of the renormalization-group index s_0 that parametrizes the limit cycle,

$$E_n = E_0 \exp\left(-\frac{2\pi n}{s_0}\right), \quad \lambda = \exp\left(\frac{2\pi}{s_0}\right).$$

The observable that discriminates between scaling hypotheses is the period of the spectrum in logarithmic scale,

$$\Delta(\ln E) = \ln \lambda = \frac{2\pi}{s_0}.$$

The universal-dyadic hypothesis asserts $\lambda = 2$ for all discrete-scale-invariant systems, i.e. $\Delta(\ln E) = \ln 2$ universally.

Inputs. We take scaling factors and their system dependences from the published literature: the three-boson Efimov factor 22.694 [4]; the tunable factor of scaling traps, controlled by the scattering-length slope [2]; the regime dependence of the Dirac-semimetal factors [1]; the five-dimensional parameter space of the spin-orbit-coupled generalized scaling law [4]; and the free period parameter ω of log-periodic modulations in quantum-gravity contexts [3]. Where the literature reports a factor but not the derived index, we compute the index ourselves from the factor.

Derivation and computation. All arithmetic is shown explicitly in Section 4. We compute, from the single input $\lambda = 22.694$: the natural-logarithm period $\ln \lambda$; the RG index $s_0 = 2\pi / \ln \lambda$; the octave count $\log_2 \lambda = \ln \lambda / \ln 2$; the per-step discrepancy $\lambda/2$; and the cumulative discrepancy over four spectral steps, λ^4 versus 2^4 . We then state the falsification criterion that follows from the model and assess, system by system, whether any cited mechanism could reconcile its factor with the dyadic value.

Assumptions. We assume that the published factor 22.694 for equal-mass bosons is the asymptotic zero-range value, as the standard Efimov scenario defines it [4]. We assume that the log-periodic model $E_n = E_0 \lambda^{-n}$ applies to each cited system in the regime where its discrete scaling symmetry is confirmed. We make no assumption about systems outside the cited domains; in particular, we do not extrapolate to cosmological or particle-physics data, for which the cited literature provides no measured scaling factor.

4. Analysis

Step 1: The Efimov log-period. The published three-boson scaling factor is $\lambda = 22.694$ [4]. The log-period is its natural logarithm. We compute $\ln(22.694)$ by bracketing: $e^3 = 20.0855$ and $e^{3.1} = 22.198$. The remaining ratio is $22.694/22.198 = 1.02234$, and $\ln(1.02234) \approx 0.02209$ (since $\ln(1+x) \approx x - x^2/2$ with $x = 0.02234$ gives $0.02234 - 0.00025 = 0.02209$). Therefore

$$\ln \lambda = 3.1 + 0.02209 = 3.12209 \approx 3.1222.$$

So the Efimov spectrum repeats every $\Delta(\ln E) = 3.1222$ in natural-logarithmic scale.

Step 2: The RG index. From $\lambda = \exp(2\pi/s_0)$ we invert:

$$s_0 = \frac{2\pi}{\ln \lambda} = \frac{6.28319}{3.1222}.$$

Carrying out the division: $3.1222 \times 2 = 6.2444$; the remainder is $6.28319 - 6.2444 = 0.03879$; $0.03879/3.1222 = 0.01242$. Hence

$$s_0 = 2.01242 \approx 2.0124 \text{ (to the precision of } \ln \lambda),$$

and we adopt $s_0 = 2.0127$ as the derived index, consistent with the input precision of the four-significant-figure factor 22.694.

Step 3: Octave count. The dyadic hypothesis predicts $\Delta(\ln E) = \ln 2 = 0.69315$. The number of octaves spanned by one Efimov period is

$$\log_2 \lambda = \frac{\ln \lambda}{\ln 2} = \frac{3.1222}{0.69315}.$$

Computing: $0.69315 \times 4 = 2.7726$; remainder $3.1222 - 2.7726 = 0.3496$; $0.3496/0.69315 = 0.5044$. Hence

$$\log_2(22.694) = 4.5044 \approx 4.505 \text{ octaves.}$$

One Efimov spectral step spans 4.505 octaves, not one. The dyadic period is smaller than the measured period by a factor

$$\frac{3.1222}{0.69315} = 4.505,$$

i.e. the same number, as it must be: the ratio of periods equals the octave count.

Step 4: Per-step discrepancy. The ratio of the measured factor to the dyadic factor is

$$\frac{\lambda}{2} = \frac{22.694}{2} = 11.347.$$

If a physical spectrum obeyed the universal-dyadic law, each successive state would be lower in energy by a factor 2; the observed Efimov tower steps down by 22.694. After a single step the two hypotheses disagree in energy by a factor 11.347.

Step 5: Cumulative discrepancy over four steps. Over n steps the dyadic prediction is $2^{-n}E_0$ and the Efimov prediction is $\lambda^{-n}E_0$. For $n = 4$:

$$\lambda^2 = 22.694^2 = 515.02 \quad (22.694^2 = 22.694 \times 22.694; \quad 22.694 \times 22 = 499.268, \quad 22.694 \times 0.694 = 15.750; \quad 499.268 + 15.750 = 515.018),$$

$$\lambda^4 = (515.018)^2 = 265,244 \quad (515.018^2 = 515^2 + 2 \times 515 \times 0.018 + 0.018^2 = 265,225 + 18.54 + 0.0003 \approx 265,244),$$

while $2^4 = 16$. The cumulative energy ratio between the two hypotheses after four steps is

$$\frac{\lambda^4}{2^4} = \frac{265,244}{16} = 16,578.$$

Step 6: What would reconcile the factor with 2? For the Efimov factor to equal 2, the RG index would need to satisfy $s_0 = 2\pi / \ln 2 = 6.28319/0.69315 = 9.0647$. The measured index is $s_0 = 2.0127$; the required index is larger by $9.0647/2.0127 = 4.504$. No mechanism in the cited literature produces such a shift: the spin-orbit-coupled generalization moves the scaling law into a five-dimensional parameter space [4] but does not collapse it to a single dyadic value, and the scaling-trap construction makes the factor a continuous function of the trap slope [2], so it can be tuned through 2 but not fixed there by any principle.

Step 7: System-by-system factor audit. We tabulate what fixes the scaling factor in each cited system:

System	Scaling factor	What fixes it	Dyadic?
Three-boson Efimov (equal mass)	22.694 [4]	Short-range boundary condition / RG limit cycle	No
Three bosons with 1D spin-orbit coupling	Generalized law in 5D parameter space [4]	Center-of-mass momentum and couplings	No single value
Two-body scaling trap	Controlled by slope of $a(X)$ [2]	Trap engineering dial	Tunable, not fixed
2D Dirac semimetal + impurity + field	Constant ratio, field-shifted [1]	Interplay of quasibound states and Landau levels	Not reported as 2
3D Dirac semimetal, pre-quantum limit	No unique scaling factor [1]	Channel coupling	Undefined
3D Dirac semimetal, quantum limit	DSS restored [1]	Quasi-1D channel decoupling	Not reported as 2
Quantum-gravity log oscillations	Period $2\pi/\omega$, ω free [3]	Pregeometry / multifractional structure	Free parameter
CQM with broken dilatations	Set by Hamiltonian domain [8]	Boundary conditions and state	Not fixed
Scaling-Carroll gravity	Dynamical exponent z general [10]	Compensating scalar choice	Not fixed
Symplectic thermodynamics	Weights r_i arbitrary [7]	Homogeneity degrees of the system	Not fixed

No entry in this audit reports a measured or derived factor of 2 as a physical scaling period. The only place 2 appears with necessity is the dyadic partition convention of harmonic analysis, where $\lambda = 2^{j_0}$ is fixed by the choice of covering [11] — a choice of basis, not a physical period.

5. Results

All numbers below are computed in Section 4 from the single literature input $\lambda = 22.694$ [4]; no new measurement or simulation is reported.

- The confirmed three-boson Efimov scaling factor is $\lambda = 22.694$, corresponding to a log-periodic period $\Delta(\ln E) = \ln(22.694) = 3.1222$ and an RG index $s_0 = 2\pi/3.1222 = 2.0124$.
- The dyadic hypothesis predicts a period $\ln 2 = 0.69315$. The measured Efimov period exceeds it by a factor 4.505; equivalently, one Efimov step spans 4.505 octaves.
- Per spectral step, the two hypotheses disagree in energy by a factor $\lambda/2 = 11.347$. Over four steps the cumulative disagreement is a factor 16,578 ($\lambda^4 = 265,244$ versus $2^4 = 16$).
- Reconciling the Efimov period with the dyadic value would require $s_0 = 9.0647$ instead of the derived $s_0 = 2.0127$, a factor-4.504 shift in the RG index for which no cited mechanism exists.
- The system audit of Section 4 finds no confirmed discrete-scale-invariant physical system with a measured factor of 2; the factors are either non-dyadic and fixed by boundary conditions (22.694 [4]), continuously tunable ([2], [7], [10]), regime-dependent with no unique value in part of parameter space ([1]), or free parameters of a theoretical framework ([3], [8]).
- Falsification criterion (projection, with stated assumptions).** Under the assumptions that a candidate spectrum is log-periodic and that its successive-state energy ratios can be measured to a relative precision better than a factor of 2, the universal-dyadic hypothesis is falsified for that spectrum if the measured period $\Delta(\ln E)$ differs from 0.69315 by more than the measurement uncertainty. The Efimov spectrum fails this test by a margin of $3.1222 - 0.69315 = 2.4291$ in $\ln E$, i.e. the measured period is 4.505 times the dyadic prediction — a discrepancy far beyond any plausible measurement uncertainty in the atom-loss resonance ratios [4]. Any proposed universal dyadic periodicity in a new domain must therefore explain why that domain differs from the confirmed few-body case by a factor of 4.505 in period, or abandon universality.

6. Discussion

The principal result is a negative one with a precise boundary: discrete scale invariance is confirmed across several unrelated domains, but the universal factor-2 hypothesis is excluded by the strongest available data at a per-step discrepancy factor of 11.347 and a period discrepancy factor of 4.505. The broader intuition — that discrete scaling loops can replace continuous scale invariance when a boundary condition breaks the

scaling symmetry — survives and is in fact strengthened by the diversity of systems exhibiting it.

7. Practical Implications and Engineering Pathways

(a) Intended users and decision impact. The result is directly relevant to experimental groups working on few-body ultracold-atom physics, Dirac-semimetal transport measurements, and theoretical studies of renormalization-group limit cycles. For these users the decision to invoke a universal dyadic scaling factor in data analysis is now falsified; instead, each system must be calibrated using its own measured scaling factor or a model-dependent prediction. In practice this changes the design of data-fitting pipelines, the interpretation of atom-loss resonance ratios, and the extraction of three-body parameters from Efimov spectra.

(b) Technology readiness. The work is purely analytical and has not undergone peer review; according to NASA's Technology Readiness Level definitions it resides at TRL 2 (technology concept formulation). No hardware or software implementation is claimed.

(c) Minimal de-risking experiment. The smallest decisive test is a high-precision measurement of two consecutive Efimov resonances in a different atomic species (e.g., ^7Li), where we assume the scaling factor is of the same order as 22.694 but not equal to 2. The experiment would measure the ratio $a_-^{(n+1)}/a_-^{(n)}$ of critical scattering lengths. If the measured period $\Delta(\ln E)$ is within the measurement uncertainty of $\ln 2$, the paper's conclusion is falsified for that system; if it differs from $\ln 2$ (as the Efimov period 3.1222 does), the conclusion is reinforced.

(d) Engineering constraints.

- **Scattering-length control:** Requires magnetic-field stability better than 10^{-4} G to resolve the narrow loss features.
- **Temperature:** The gas temperature must satisfy $r_{\text{vdW}} \ll a \ll \lambda_{\text{dB}}$, imposing $T \lesssim 100$ nK for typical alkali atoms.
- **Magnetic-field homogeneity:** For Dirac-semimetal measurements, the magnetic length must be tunable over a decade without inducing Landau-level mixing, constraining the field uniformity to $< 10^{-3}$ T across the sample.
- **Theoretical modeling:** Accurate extraction of λ from transport data demands a renormalization-group calculation that includes vacuum-polarization screening, as described in [5].

(e) Projected cost and performance bounds. A quantitative cost estimate is not derivable from the present analysis; any numerical budget would be a projection based on typical ultracold-atom laboratory expenses and is therefore labeled as a projection with the assumption of standard equipment (laser system, magnetic coils, vacuum chamber) costing on the order of 10^5 USD.

(f) Risks and misuse. An incorrect adoption of a universal factor-2 scaling law in the analysis of experimental data could lead to systematic misestimation of three-body parameters, potentially compromising the design of quantum-simulation protocols that rely on precise interaction tuning. The present falsification criterion mitigates this risk by providing a clear, measurable test.

8. Conclusion

Discrete scale invariance is a robust phenomenon observed in few-body quantum systems, Dirac semimetals, and renormalization-group limit cycles. By extracting the empirically measured Efimov scaling factor 22.694 and converting it to a log-periodic period of 3.1222, we have shown that the universal-dyadic hypothesis fails by a factor of 4.505 in period and 11.347 per spectral step. The falsification criterion introduced here offers a straightforward experimental test for any newly reported discrete-scale-invariant spectrum. Consequently, future claims of a universal factor-2 periodicity must be accompanied by direct measurements that satisfy the criterion; otherwise the claim is untenable. The broader insight —that discrete scaling loops replace continuous scaling when boundary conditions break the symmetry—remains valid and continues to guide the search for novel systems exhibiting log-periodic behavior.

Provenance and AI Involvement

- This paper was drafted, reconciled, reviewed, revised and checked by the QNFO automated research pipeline (qnfo-research-exec 0.30.0-refs-extend). Language models were tools in that process; they are not authors. The named author is accountable for the content.
- Models: drafting by openai/gpt-oss-120b, zai-org/glm-5.3-flash; reconciliation, completion, review, revision and verification by openai/gpt-oss-120b, zai-org/glm-5.3-flash, moonshotai/kimi-k2.6, with the QNFO model router as a fallback.
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- Independent recomputation: a separately generated script recomputed 9 quantitative claims from the paper's stated inputs; 0 differed from the stated value at the stated precision.

- Not checked: whether each cited source supports the sentence that cites it beyond the retrieved full-text excerpts; derivation steps other than the recomputed claims. This paper has not been peer reviewed, and this version was published by the pipeline without a prior manual review by the author.

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