

Distinction, Number, and the Empirical Filter: The Pre-Arithmetic Research Framework

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Abstract

Several research lines in mathematical physics begin from identities between arithmetic objects and partition functions, or between hierarchical structure and ultrametric geometry, and ask whether any physical system realizes the resulting structure. Each such line must answer the same two questions, and most failures in the literature come from answering them carelessly: which of these structures is being claimed as real, and what observation would show the claim false. This paper states the discipline once, as a reusable framework. It constructs a nine-level ladder from the primitive of distinction to the empirical filter of physics, with one construction operation between each pair of levels; it states two boundary rules governing any movement up the ladder — a rule against uncommitted reification of the primitive, and a rule that no mathematical isomorphism passes as a physical realization without a stated measurement protocol, a null model, and a falsification condition; and it defines a compact claim record that any research claim can carry, together with mechanical demotion rules for claims that violate the boundary rules. The framework makes no empirical claims of its own. It systematizes discipline that is currently distributed across seven published records of the QNFO program [umporr014, res021, res027, res028, res029, res030, res031], and it states the one claim it does make — that the ladder covers the published lineage without remainder — together with a falsification protocol, so the framework itself can be checked the way it asks everything else to be checked.

1. Introduction and scope

The arithmetic-statistics program at QNFO has published, during 2026, a sequence of records connecting number-theoretic structure to statistical mechanics. A gas whose modes are indexed by primes, with logarithmic single-particle energies, has partition functions that are exactly zeta objects: unrestricted occupation reproduces the Riemann zeta function, and squarefree occupation reproduces the ratio of zeta functions at argument and double argument [res027]:

$$Z_B(\beta) = \prod_p (1 - p^{-\beta})^{-1} = \zeta(\beta), \quad Z_F(\beta) = \prod_p (1 + p^{-\beta}) = \frac{\zeta(\beta)}{\zeta(2\beta)}, \quad \ln Z_{MB}(\beta) = \sum_p p^{-\beta} = P(\beta).$$

Here Z_B , Z_F , and Z_{MB} are the partition functions under unrestricted, squarefree, and Boltzmann occupation respectively, and P denotes the prime zeta function. Bounded-occupation generalizations form a continuous family between the two [res028]. A companion record consolidated the correspondence and its practitioner-facing reading [res029], and a computational study then adjudicated whether the arithmetic cut can be distinguished from non-arithmetic alternatives with matched level density [res030]. Parallel work on hierarchy distance established that the number of distinctions required to separate two objects is an ultrametric that does not depend on the realization — taxonomy, p-adic digits, or Laurent coefficients — and that this distance is the canonical finite distance of the program [umporr014].

Each of these records had to solve, independently, the same methodological problem: where a mathematical identity ends and a claim about the physical world begins. The present framework is the extracted common discipline. It is a methodological scaffold, not an empirical result: it makes no claim about what the world is like, and none of its nine levels asserts anything ontic on its own.

The framework is motivated by a simple failure mode. Two extreme positions repeatedly appear in work of this kind. One treats a combinatorial identity — a partition function equal to a zeta function — as if it were already a statement about bosons or fermions. The other treats all such identities as content-free. Both positions lose the same thing: the controlled passage from a formal structure to an empirical claim, which is the only place where the correspondence acquires content. The framework fixes what that passage requires, in advance, for every claim in the program. A reader of any QNFO record can locate each of its claims on the ladder, see the declared commitment level of each, and read exactly what observation would refute it. This is the framework's purpose: to make the map-territory boundary of an arithmetic-physics program legible, and its empirical commitments auditable, without requiring every paper to rebuild the discipline from scratch.

2. The nine-level construction ladder

The framework organizes every research object by the level at which it is constructed. The levels are ordered; each is built from the one below by a single named operation, and the ordering is strict in the sense that skipping levels is not permitted without declaring every intermediate step.

1. **Distinction.** A cut separating inside from outside. The minimal unit of structure; the framework takes it as a primitive. Following the Laws of Form lineage [spencerbrown1969], a system is constituted by the distinctions that define it. Nothing is asserted here about what the world is made of: the cut is a construction primitive of the language, not a claimed constituent of reality.

1. **Pre-arithmetic structure.** Structure without number: order, hierarchy, partition, adjacency. The distinction-count distance between two leaves of a rooted tree — the number of cuts needed to separate them — is defined at this level: with $d(a, b)$ the number of distinctions required to separate leaves a and b , the strong triangle inequality $d(a, c) \leq \max\{d(a, b), d(b, c)\}$ holds, so the structure is an ultrametric before any prime or valuation appears [umporr014]. Ultrametric structure is therefore pre-arithmetic: it exists whether or not counting exists.

1. **Arithmetic.** Counting and composition of distinctions. Concatenation of cuts gives addition; iteration across independent cut-families gives multiplication; irreducible multiplicative distinctions are the primes; unique factorization is the composition law. The construction operation of this level is counting.

1. **Number theory.** Patterns of the composition: the distribution of irreducibles, factorization statistics, and the generating functions of the composition, including L-functions. The operation is pattern discernment on the output of counting.

1. **Valuation.** Assigning size to distinctions: norms and absolute values. By Ostrowski's theorem there is one Archimedean place and one p-adic place per prime; the p-adic valuation is one realization of hierarchy distance — not its ground, which lives at level 2. The operation is sizing.

1. **Geometry.** The resulting relational form: metric and ultrametric spaces as the form taken by valued distinctions, including their rigidity properties. The operation is taking form.

1. **Information.** Distinction made operational: counting distinctions as bits, entropy, and the localization of distinctions in two-point statistics rather than one-point thermodynamic functions [res030]. The operation is operationalization.

1. **Measurement.** The finite-resolution application of valuation to observation: which observable, on which system, with which instrument, at which resolution and noise budget — what a finite observer can actually distinguish. The operation is finite resolution.

1. **Physics.** The empirical filter: falsification decides which of the structures are real. The operation is filtering by observation.

The ladder is a translation device as much as a construction: each level corresponds to a recognizable disciplinary home, which makes the framework legible outside its program of origin.

Ladder level	Standard disciplinary term
Distinction	boundary, cut (Laws of Form)
Pre-arithmetic	hierarchy, order theory, cladistics
Arithmetic	counting, factorization
Number theory	distribution of irreducibles, L-functions
Valuation	norms, places (valuation theory)
Geometry	metric and ultrametric spaces
Information	entropy, two-point statistics
Measurement	metrology: protocol, resolution, noise

Ladder level	Standard disciplinary term
Physics	falsifiability, empirical adequacy

Three structural rules complete the ladder. First, every claim, model, and interpretation declares its level or its span of levels; a claim that cannot state its level is not yet a claim. Second, work within a level is the default and needs no justification, while every cross-level move requires a declared bridge. Third, movement downward is interpretation, not derivation: a physical result may reinterpret an arithmetic object, but it does not derive it. Violations of these rules — particularly conflating two objects that live at different levels, such as prime-gap statistics and zero statistics [res030] — are the framework's primary defect class, and are adjudicated case by case in the records cited.

3. The two boundary rules

The ladder says where objects live; two boundary rules say what may be claimed about them.

3.1 Committed reification only

The primitive of distinction is methodological by default. Treating it — or anything built on it — as a constituent of the world is a further commitment, and the framework requires that commitment to be explicit, made per claim, made in advance, and accompanied by the full realization requirements of Section 3.2. A commitment made after a null result has no force; the discipline exists so that ontology is never smuggled into formalism retroactively. The framework itself commits to nothing ontic anywhere on the ladder.

3.2 From isomorphism to realization

A mathematical isomorphism — however exact — is a map, not a territory. No claim may pass from one to the other without three items stated in advance. First, a measurement protocol: which observable, on which system, with which instrument, at which resolution. Second, a null model: what the data would look like if the structure were absent. The canonical pattern in this program is the matched-level-density null, in which synthetic spectra carry the same smoothed level density as the target but none of its arithmetic structure [res030]. Third, a falsification condition: the observation, specified before data collection, that would refute the claim. The framework states these three items as one requirement because a protocol without a null model cannot say what a positive result means, and a null model without a falsification condition cannot end an inquiry.

Two reporting rules accompany the requirement. Quantitative claims report effect sizes, not only significance levels: a large deviation in one spectral window can carry little information about the rest of the spectrum, and the reportable object is the full discriminating curve, not a single threshold crossing [res030]. And claims whose observables are derived from one underlying two-point function — pair correlation, spectral form factor, number variance — are one channel, not three independent confirmations, and are corrected accordingly.

3.3 The claim record

Every claim that follows the framework carries eight fields: its ladder level; its carrier (whether it is definitional, formal, computational, empirical, or engineered); its ontic commitment (methodological by default, heuristic, or ontic); its map-territory status (map, bridge, or territory); and, whenever the claim reaches for physical reality, the protocol, null model, and falsification condition of Section 3.2. Claims about the framework itself are permitted and carry the same record with level marked as meta-level.

The record is made auditable by mechanical demotion rules. Any bridge or territory claim missing one of the three mandatory items — protocol, null model, or falsification condition — is demoted one step per missing item, from territory to bridge to map, and no further than map. An ontic commitment without the full triple is demoted to heuristic. Where two violations apply to one claim, both demotions apply and the lower status wins. The rules are total: every claim state has exactly one outcome and every demotion terminates, which makes the record's integrity checkable by machine — a property the deposited verification suite exercises exhaustively.

4. Relationship to the published records

The framework restates none of the results it systematizes. The hierarchy distance and its realization independence are established in [umporr014]. The exact partition-function correspondence — unrestricted occupation giving the zeta function, squarefree occupation giving its double-argument ratio — is established in [res027] and extended to bounded occupation in [res028]. The consolidated map and its practitioner-facing reading are given in [res029]. The computational adjudication of the arithmetic cut against matched-density

nulls, including the location of the arithmetic information in two-point statistics, is given in [res030]. The companion consolidation record [res031] audits the dictionary and its five-level interpretive ladder at paper level; the present framework generalizes that ladder to the nine levels above, which re-partition its upper half rather than extending it, and the two records are kept consistent on that point.

The framework's level assignments to the published lineage are stated in the accompanying source archive. Two assignments that the first version flagged as provisional — the measurement-level reading of [res029] and the span assigned to [res021] — have been adjudicated: the measurement-level reading was withdrawn in favor of an operational reading, and the span is stated with its declared bridges. The adjudications, with per-record evidence, accompany this version.

5. Verification

The framework's quantitative and mechanical content is checked by a deposited script that (1) verifies the Euler-product identity underlying level 3 numerically, including the squarefree ratio and the bounded-occupation endpoint; (2) verifies the ultrametric triangle for the distinction-count distance on random hierarchies; (3) exhaustively checks that the demotion rules of Section 3.3 terminate and assign exactly one outcome to every claim state; and (4) checks the integrity of the ladder and of the cross-level assignment table. The script is deterministic, dependency-free, and re-runnable from the deposited layout; its full output is deposited alongside this paper.

6. The framework's own claim

The framework makes exactly one claim: that the nine-level ladder covers the published lineage of the program without remainder — that every published object assigns to at least one level or declared span. The claim fails if a published object cannot be assigned to any level or span; it fails in a second, independent way if a defect already adjudicated in the lineage turns out not to be adjudicable by the two boundary rules, which would show a third rule type is needed. Both failure modes are specified in advance, the level assignments are listed record by record in the source archive, and the provisional assignments are named above. The framework asks of itself only what it asks of every claim that passes through it.

References
