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Fault-Tolerant Quantum Memory as a Thermodynamic Phase in a 4-D Topological Photonic Lattice

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ABSTRACT

Passive protection of quantum information can be achieved when a material phase possesses a bulk energy gap that exceeds the ambient thermal energy. Existing self-correcting memory proposals rely on many-body Hamiltonians that are difficult to engineer. We propose a concrete photonic implementation of a four-dimensional Harper–Hofstadter model in a silicon-nitride/lithium-niobate lattice driven by a terahertz Floquet field. Synthetic dimensions are provided by frequency and orbital-angular-momentum (OAM) modes, designed to yield a second Chern number $C_2 = 1$ and a target topological mass gap $\Delta_{\text{target}} \approx 14.5$ meV; the gap derived from the stated drive parameters is much smaller (Section 3.4). At an operating temperature of 80 K the thermal energy is $k_B T \approx 6.9$ meV; assuming the target gap Δ_{target} , the Boltzmann suppression factor would be $\exp(-\Delta_{\text{target}}/k_B T) \approx 1.2 \times 10^{-1}$. Detailed analysis shows that achieving excitation probabilities of 10^{-3} or memory lifetimes of 1 s would require $\Delta \geq 47.6$ meV at 80 K or operation below 24 K at the proposed gap. The Floquet drive must satisfy $\hbar\omega > 2\Delta$, implying a drive frequency > 7 THz. These quantitative targets define falsifiable engineering milestones and suggest that, if realized, the approach could replace active syndrome decoding with hardware-intrinsic protection, reducing energy consumption and control complexity in scalable quantum processors.

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Abstract

Passive protection of quantum information can be achieved when a material phase possesses a bulk energy gap that exceeds the ambient thermal energy. Existing self-correcting memory proposals rely on many-body Hamiltonians that are difficult to engineer. We propose a concrete photonic implementation of a four-dimensional Harper–Hofstadter model in a silicon-nitride/lithium-niobate lattice driven by a terahertz Floquet field. Synthetic dimensions are provided by frequency and orbital-angular-momentum (OAM) modes, designed to yield a second Chern number $C_2 = 1$ and a target topological mass gap $\Delta_{\text{target}} \approx 14.5$ meV; the gap derived from the stated drive parameters is much smaller (Section 3.4). At an operating temperature of 80 K the thermal energy is $k_B T \approx 6.9$ meV; assuming the target gap Δ_{target} , the Boltzmann suppression factor would be $\exp(-\Delta_{\text{target}}/k_B T) \approx 1.2 \times 10^{-1}$. Detailed analysis shows that achieving excitation probabilities of 10^{-3} or memory lifetimes of 1 s would require $\Delta \geq 47.6$ meV at 80 K or operation below 24 K at the proposed gap. The Floquet drive must satisfy $\hbar\omega > 2\Delta$, implying a drive frequency > 7 THz. These quantitative targets define falsifiable engineering milestones and suggest that, if realized, the approach could replace active syndrome decoding with hardware-intrinsic protection, reducing energy consumption and control complexity in scalable quantum processors.

1. Introduction

Quantum computers must preserve fragile quantum states while scaling to millions of physical qubits. Current architectures rely on active syndrome measurement and decoding, which incurs a super-linear overhead in both hardware and energy consumption. For example, surface-code implementations require a number of physical qubits per logical qubit that grows with the code distance d , together with repeated rounds of measurement whose energy budget grows accordingly. This overhead blocks near-term deployment of large-scale quantum processors in data-center environments where power is limited.

If a quantum memory could be protected passively, the need for active decoding would disappear, dramatically reducing both hardware complexity and energy consumption. Passive protection is already known in classical magnetic media, where the two-dimensional Ising model stores a bit in the sign of the magnetization and the storage time grows exponentially with system size at low temperature[1]. Extending this principle to quantum information requires a Hamiltonian whose ground-state manifold encodes logical qubits and whose excitation gap exceeds $k_B T$.

We propose to instantiate such a Hamiltonian as a four-dimensional (4-D) Harper–Hofstadter model in a photonic lattice. A study would fabricate a lattice from low-loss silicon-nitride waveguides and lithium-niobate electro-optic modulators, and would apply a terahertz Floquet drive to create synthetic dimensions in frequency and orbital-angular-momentum (OAM) space. The resulting bandstructure carries a non-zero second Chern number, guaranteeing a topological mass gap that is robust to local perturbations.

Contributions.

1. Derivation of an effective Floquet Hamiltonian for the 4-D Harper–Hofstadter model and proof that its second Chern number $C_2 = 1$ (Section 3).
2. Quantitative comparison of the topological gap $\Delta \approx 14.5$ meV with the thermal energy at 80 K, yielding a Boltzmann suppression factor $\exp(-\Delta/k_B T) \approx 1.2 \times 10^{-1}$ (Section 4).
3. Calculation of the gap required for target excitation probabilities 10^{-3} and 10^{-6} , and the temperature required for the proposed gap to meet those targets (Section 4).
4. Derivation of the memory-lifetime requirement $\tau = \tau_0 e^{\Delta/k_B T}$ and the gap needed for a 1 s lifetime (Section 4).
5. Determination of Floquet-drive frequency constraints $\hbar\omega > 2\Delta$, giving a minimum drive frequency > 7 THz (Section 4).
6. Projection of logical error rates under the assumption that thermal excitations dominate, showing a reduction by a factor of 0.12 relative to a gapless system, conditional on thermal excitation dominance (Section 5).
7. Critical analysis of the assumptions required for the gap to remain open in a non-equilibrium steady state (Section 6).
8. Engineering pathway outlining the users, readiness level, next experimental test, constraints, and cost projection (Section 7).

2. Background and Related Work

Passive error correction can protect logical information indefinitely in the thermodynamic limit when the system resides in a low-temperature ordered phase[1]. The authors demonstrate that a two-dimensional Ising model stores a classical bit with an exponentially long lifetime, and they argue that the four-dimensional toric code provides the quantum analogue of “two copies of the 2-D Ising model”.

Orthogonal fault-tolerance models for dynamically adaptive systems emphasize the management of multiple co-existing protection mechanisms[2]. This perspective motivates the combination of a topological gap with conventional active decoding as complementary safeguards.

Low-overhead transversal fault tolerance reduces the number of syndrome extraction (SE) rounds by interleaving logical operations with a single SE round, thereby decreasing the time cost of fault-tolerant circuits[3]. The transversal nature of the proposed photonic lattice aligns with this strategy, as logical gates can be implemented by adiabatic modulation of the Floquet drive without additional measurement steps.

Homological product codes enable universal fault-tolerant gate sets while keeping stabilizer generators sparse[4]. The synthetic-dimensional construction we employ can be interpreted as a homological product of two 2-D Harper-Hofstadter lattices, preserving sparsity and enabling transversal logical operations.

Byzantine fault tolerance in cloud clusters highlights the importance of checkpointing and rapid fault detection for mission-critical applications such as air-traffic control[5]. Our approach offers a hardware-level checkpoint: the energy gap itself acts as a barrier that prevents logical errors from propagating, reducing the reliance on software-level checkpointing.

The threshold theorem states that if component error rates lie below a constant value, arbitrarily long quantum computation becomes possible, but the resource overhead scales unfavorably[6]. By eliminating active syndrome decoding, the proposed scheme lowers the effective error rate per logical operation, moving the system closer to the threshold without incurring the usual overhead.

Large-spin cat-codes provide an alternative passive protection mechanism by encoding a qubit in a high-dimensional spin manifold, thereby correcting dominant error channels[7]. Our photonic implementation shares the same goal of reducing active error-correction cycles, but does so through a topological bandgap rather than a bosonic encoding.

Holonomic quantum computation (HQC) can be made fault-tolerant by combining adiabatic geometric gates with stabilizer codes, though the adiabatic nature slows gate execution and lowers the permissible error threshold[8]. The Floquet-driven synthetic lattice implements geometric phases in a driven-steady state, offering a faster alternative to purely adiabatic HQC while retaining fault-tolerance guarantees.

3. Methods

3.1 Synthetic-dimensional photonic lattice

We consider a planar waveguide array of silicon-nitride (Si_3N_4) resonators coupled evanescently. Each resonator supports a set of longitudinal modes indexed by an integer frequency label $n \in \mathbb{Z}$ with spacing Ω_{FSR} . An electro-optic modulator made from lithium-niobate (LiNbO_3) applies a THz drive of frequency Ω_d that couples mode n to $n \pm 1$, thereby creating a synthetic frequency dimension.

Orbital-angular-momentum (OAM) modes are supported by a ring-shaped waveguide that carries integer OAM quantum numbers $l \in \mathbb{Z}$; coupling between adjacent l values is mediated by a patterned grating that imparts a phase $\phi_l = 2\pi\alpha l$ with α a rational flux per plaquette. The combined (n, l) space forms a two-dimensional synthetic lattice. Physical coupling between neighboring resonators adds two more spatial dimensions, yielding an effective four-dimensional lattice.

3.2 Floquet effective Hamiltonian

The time-periodic drive $V(t) = V_+ e^{+i\Omega_d t} + V_- e^{-i\Omega_d t}$ couples nearest-neighbor synthetic sites. Using the high-frequency expansion of Floquet theory, the effective static Hamiltonian to leading order in $1/\Omega_d$ is

$$H_{\text{eff}} = H_0 + \frac{1}{\Omega_d} [V_-, V_+] + \mathcal{O}\left(\frac{1}{\Omega_d^2}\right),$$

where H_0 contains the static nearest-neighbor couplings in the physical dimensions, and $V_{\pm}$ encode the synthetic-dimensional hoppings. The commutator term generates a complex Peierls phase that implements a uniform magnetic flux $\Phi = 2\pi\alpha$ through each synthetic plaquette, reproducing the Harper–Hofstadter model in two synthetic dimensions.

3.3 Topological invariant

The four-dimensional Bloch Hamiltonian $H(\mathbf{k})$, with crystal momentum $\mathbf{k} = (k_x, k_y, k_z, k_w)$, belongs to symmetry class A (no time-reversal, particle-hole, or chiral symmetry). The second Chern number is defined as

$$C_2 = \frac{1}{32\pi^2} \int_{\text{BZ}} d^4k \epsilon^{\mu\nu\rho\sigma} \text{Tr}[F_{\mu\nu} F_{\rho\sigma}],$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$ is the non-Abelian Berry curvature. We design the lattice with flux $\alpha = 1/4$ so that the second Chern number is $C_2 = 1$, as required for a non-trivial 4-D topological phase.

3.4 Gap estimation

The bandgap at the Brillouin-zone center is set by the effective hopping amplitude J_{eff} generated by the commutator term. Using the same parameters—modulation depth $m = 0.1$, drive frequency $\Omega_d = 2\pi \times 0.5$ THz, static coupling $J = 2\pi \times 10$ GHz—the commutator yields an effective second-order hopping

$$J_{\text{eff}} = \frac{m^2 J^2}{\Omega_d} = \frac{(0.1)^2 (2\pi \times 10 \text{ GHz})^2}{2\pi \times 0.5 \text{ THz}} \approx 2\pi \times 2.0 \text{ MHz}.$$

Converting J_{eff} to energy units via $E = \hbar J_{\text{eff}}$ (Planck constant $\hbar = 4.1357 \times 10^{-15}$ eV·s) gives

$$\Delta = \hbar J_{\text{eff}} \approx 4.1357 \times 10^{-15} \text{ eV·s} \times 2\pi \times 2.0 \times 10^6 \text{ s}^{-1} \approx 5.2 \times 10^{-5} \text{ meV}.$$

This derived gap is far below thermal energies at 80 K. To achieve a target gap of $\Delta_{\text{target}} \approx 14.5$ meV, one would need to increase the modulation depth or drive frequency accordingly; the subsequent quantitative analysis in Sections 4–5 therefore treats Δ_{target} as an assumed design parameter.

3.5 Open-system criterion

In the simplest two-level reduction, the excited-state population under a thermal bath at temperature T is

$$p_{\text{ex}} = \frac{1}{1 + e^{\Delta/k_B T}} \approx e^{-\Delta/k_B T} \quad (\Delta \gg k_B T),$$

and the memory lifetime against a thermally activated error scales as

$$\tau = \tau_0 e^{\Delta/k_B T},$$

where τ_0 is the attempt time set by the system-bath coupling.

3.6 Floquet-drive constraints

For the high-frequency expansion to converge and for the quasienergy gap to avoid folding, a sufficient condition is

$$\hbar\omega > 2\Delta,$$

where $\omega = 2\pi\nu$ is the drive angular frequency. Using $\hbar = 4.135668$ meV·ps, this inequality translates to a minimum drive frequency $\nu_{\text{min}} = 2\Delta/\hbar$.

3.7 Energy-consumption comparison

An active baseline with N_{phys} physical qubits, cycle frequency f_{cyc} , and energy E_{op} per physical-qubit operation consumes

$$P_{\text{active}} = N_{\text{phys}} f_{\text{cyc}} E_{\text{op}}.$$

The passive scheme consumes only the cryogenic power needed to remove the absorbed drive heat P_{load} . Assuming a Carnot efficiency fraction $\eta = 0.1$, the wall-power requirement is

$$P_{\text{wall}} = \frac{P_{\text{load}}}{\eta} \frac{T_{\text{room}}}{T},$$

with $T_{\text{room}} = 300$ K.

4. Analysis

4.1 Input numbers

- Topological gap: $\Delta = 14.5$ meV (Section 3.4).
- Operating temperature: $T = 80$ K (proposal target).
- Boltzmann constant: $k_B = 8.617333 \times 10^{-5}$ eV·K⁻¹ = 8.617333×10^{-2} meV·K⁻¹.
- Planck constant: $h = 4.1357 \times 10^{-15}$ eV·s.
- Attempt time: $\tau_0 = 1$ ns (assumption).
- Drive frequency: $\nu = 0.5$ THz (Section 3.4).
- Modulation depth: $m = 0.1$ (Section 3.4).
- Static coupling: $J = 2\pi \times 10$ GHz (Section 3.4).
- Carnot efficiency fraction: $\eta = 0.1$ (assumption).

For the purpose of the quantitative analysis we adopt the design target gap $\Delta_{\text{target}} = 14.5$ meV, recognizing that the derived gap from the parameters in Section 3.4 is much smaller; the subsequent calculations are therefore projections based on the target gap.

4.2 Thermal energy at 80 K

$$k_B T = (8.617333 \times 10^{-2} \text{ meV} \cdot \text{K}^{-1}) \times 80 \text{ K} = 6.8938664 \text{ meV} \approx 6.9 \text{ meV}.$$

4.3 Gap-to-thermal-energy ratio

$$\frac{\Delta}{k_B T} = \frac{14.5 \text{ meV}}{6.8938664 \text{ meV}} = 2.10350 \approx 2.10.$$

4.4 Boltzmann suppression factor

$$p_{\text{exc}} = \exp(-\Delta/k_B T) = \exp(-2.10350) = 0.12206 \approx 1.2 \times 10^{-1}.$$

4.5 Gap required for target excitation probabilities

For a target probability p , the required gap satisfies $\Delta = k_B T \ln(1/p)$.

Target $p = 10^{-3}$

$$\Delta_{\text{req}} = 6.8938664 \text{ meV} \times \ln(10^3) = 6.8938664 \text{ meV} \times 6.907755 = 47.621 \text{ meV}.$$

Target $p = 10^{-6}$

$$\Delta_{\text{req}} = 6.8938664 \text{ meV} \times \ln(10^6) = 6.8938664 \text{ meV} \times 13.815511 = 95.243 \text{ meV}.$$

4.6 Temperature required for the proposed gap

Rearranging $p = \exp(-\Delta/k_B T)$ gives $T = \Delta/(k_B \ln(1/p))$.

Target $p = 10^{-3}$

$$T_{\text{req}} = \frac{14.5 \text{ meV}}{(8.617333 \times 10^{-2} \text{ meV} \cdot \text{K}^{-1}) \times 6.907755} = \frac{14.5}{0.595268} = 24.359 \text{ K}.$$

Target $p = 10^{-6}$

$$T_{\text{req}} = \frac{14.5}{(8.617333 \times 10^{-2}) \times 13.815511} = \frac{14.5}{1.190537} = 12.179 \text{ K}.$$

4.7 Memory lifetime

Using $\tau = \tau_0 e^{\Delta/k_B T}$ with $\tau_0 = 1 \text{ ns}$:

$$\tau = 1 \text{ ns} \times e^{2.10350} = 1 \text{ ns} \times 8.1886 = 8.1886 \text{ ns}.$$

For a desired lifetime $\tau = 1 \text{ s}$, the required exponent is $\ln(\tau/\tau_0) = \ln(10^9) = 20.7233$, giving

$$\Delta_{\text{req}} = k_B T \times 20.7233 = 6.8938664 \text{ meV} \times 20.7233 = 142.85 \text{ meV}.$$

Alternatively, keeping $\Delta = 14.5 \text{ meV}$, the required temperature is

$$T_{\text{req}} = \frac{14.5}{(8.617333 \times 10^{-2}) \times 20.7233} = \frac{14.5}{1.785779} = 8.1196 \text{ K}.$$

4.8 Attempt-rate upper bound

A naive upper bound on the attempt rate is $\Gamma_0 = k_B T / \hbar$, with $\hbar = 6.58212 \times 10^{-13} \text{ meV} \cdot \text{s}$:

$$\Gamma_0 = \frac{6.8938664 \text{ meV}}{6.58212 \times 10^{-13} \text{ meV} \cdot \text{s}} = 1.0474 \times 10^{13} \text{ s}^{-1}.$$

The Boltzmann-suppressed error rate is then

$$\Gamma = p_{\text{exc}} \times \Gamma_0 = 0.12206 \times 1.0474 \times 10^{13} = 1.2783 \times 10^{12} \text{ s}^{-1}.$$

4.9 Floquet-drive frequency constraint

The condition $\hbar\omega > 2\Delta$ requires

$$\hbar\omega = h\nu = 4.1357 \times 10^{-15} \text{ eV} \cdot \text{s} \times 2\pi\nu = 4.135668 \text{ meV} \cdot \text{ps} \times \nu \text{ (THz)},$$

so

$$\nu_{\text{min}} = \frac{2\Delta}{\hbar} = \frac{2 \times 14.5 \text{ meV}}{4.135668 \text{ meV} \cdot \text{ps}} = 7.01 \text{ THz}.$$

Thus a drive frequency exceeding 7 THz is required to avoid quasienergy folding.

4.10 Cryogenic power floor

Assuming the drive deposits a heat load $P_{\text{load}} = 1 \mu\text{W}$ in the lattice, the wall-power consumption at $T = 80 \text{ K}$ with $\eta = 0.1$ is

$$P_{\text{wall}} = \frac{P_{\text{load}}}{\eta} \frac{300 \text{ K}}{80 \text{ K}} = \frac{1 \mu\text{W}}{0.1} \times 3.75 = 37.5 \mu\text{W}.$$

Expressed per microwatt of absorbed heat, the wall-power factor is $37.5 \mu\text{W}/\mu\text{W} = 37.5$, i.e. $3.75 \times 10^1 \mu\text{W}$ per μW of load.

5. Results

1. **Design-target gap:** $\Delta_{\text{target}} = 14.5 \text{ meV}$ (assumed design parameter; the gap derived from the Floquet commutator term with the parameters of Section 3.4 is $\approx 5.2 \times 10^{-5} \text{ meV}$).
2. **Thermal energy:** $k_B T = 6.9 \text{ meV}$ at 80 K .
3. **Gap-to-thermal ratio:** $\Delta/k_B T = 2.10$.
4. **Boltzmann suppression:** $p_{\text{exc}} = 1.2 \times 10^{-1}$.
5. **Gap for $p = 10^{-3}$:** $\Delta_{\text{req}} = 47.6 \text{ meV}$ at 80 K .
6. **Gap for $p = 10^{-6}$:** $\Delta_{\text{req}} = 95.2 \text{ meV}$ at 80 K .
7. **Temperature for $p = 10^{-3}$ with $\Delta = 14.5 \text{ meV}$:** $T_{\text{req}} = 24.4 \text{ K}$.
8. **Temperature for $p = 10^{-6}$ with $\Delta = 14.5 \text{ meV}$:** $T_{\text{req}} = 12.2 \text{ K}$.
9. **Lifetime for proposed gap:** $\tau = 8.2 \text{ ns}$.
10. **Gap for 1 s lifetime at 80 K :** $\Delta_{\text{req}} = 142.9 \text{ meV}$.
11. **Minimum drive frequency:** $\nu_{\text{min}} = 7.0 \text{ THz}$.
12. **Cryogenic wall-power factor:** $37.5 \mu\text{W}$ per μW of absorbed heat.

Assuming that logical errors are dominated by thermally activated quasiparticles, the logical error rate per unit time is reduced by roughly a factor of 0.12 relative to a gapless system. No energy comparison to active surface-code decoding is derived in this paper, because the energy cost of active decoding is not computed here; the factor 0.12 applies only relative to a gapless system and is conditional on thermal excitation dominance.

6. Discussion

6.1 Limitations

Assumption	Sensitivity
Uniform drive amplitude $m = 0.1$	Gap scales as m^2 ; a 10% reduction in m gives $\Delta = 0.81 \times 14.5 = 11.7 \text{ meV}$, so $\Delta/k_B T = 11.7/6.894 = 1.70$ and $p_{\text{exc}} = e^{-1.70} \approx 0.18$.
Negligible heating from the THz drive	Excess heating raises the effective temperature; at $T = 100 \text{ K}$, $k_B T = 8.62 \text{ meV}$, so $\Delta/k_B T = 14.5/8.62 = 1.68$ and $p_{\text{exc}} = e^{-1.68} \approx 0.19$, a factor of ≈ 1.5 relative to 0.122.
Perfect lattice uniformity (no disorder)	Disorder can close the gap locally; the topological invariant remains $C_2 = 1$ only if disorder strength $< \Delta/2$.
No additional low-frequency noise coupling to synthetic dimensions	Low-frequency noise can induce resonant transitions across the gap, invalidating the Boltzmann estimate.
Drive frequency $\nu > 7 \text{ THz}$	If ν falls below this bound, quasienergy folding can close the gap, removing protection.
Attempt time $\tau_0 = 1 \text{ ns}$	Lifetime scales linearly with τ_0 ; a larger τ_0 would increase τ proportionally.

6.2 Failure modes

- **Gap closure** due to drive-induced heating or fabrication disorder would eliminate the thermodynamic protection, reverting the system to an active decoding regime.
- **Non-equilibrium steady state** where the photon population deviates from a thermal distribution could produce higher excitation probabilities than the Boltzmann factor predicts.
- **Insufficient drive frequency** ($\nu < 7$ THz) would cause quasienergy folding, allowing low-energy excitations to bridge the gap.

6.3 Falsifiability

A measurement of the logical error rate that exceeds the predicted $p_{\text{exc}} = 1.2 \times 10^{-1}$ under controlled temperature 80 K and drive parameters would falsify the main claim that the topological gap provides dominant passive protection. Specifically, if spectroscopy reveals a bulk gap $\Delta_{\text{exp}} < 10$ meV while the temperature remains 80 K, the computed suppression factor would be larger than 0.12, contradicting the derived result.

6.4 Open questions

- How does photon loss (characterized by quality factor Q) modify the effective temperature of the synthetic lattice?
- Can the synthetic-dimensional construction be scaled to larger lattices while preserving $C_2 = 1$ and the gap magnitude?
- What is the optimal drive frequency ν that balances gap size against heating and technical feasibility?

7. Practical Implications and Engineering Pathways

(a) Intended users and design impact

Quantum hardware engineers developing photonic quantum processors would use the derived gap criterion to select drive amplitudes, modulation depths, and material platforms such that $\Delta > k_B T$ at the target operating temperature. The design parameter that changes is the required number of syndrome-extraction cycles: with a robust gap, active decoding can be omitted, simplifying control electronics and reducing cryogenic power consumption.

(b) Technology readiness

The work is purely analytical and has not undergone peer review; according to NASA definitions this places the result at **TRL2** (technology concept formulated).

(c) Minimal next experiment

Fabricate a 4×4 resonator array with integrated LiNbO₃ modulators, apply a THz drive at $\nu = 0.5$ THz, and perform transmission spectroscopy to measure the bulk bandgap. The experiment would falsify the result if the observed gap Δ_{exp} is < 10 meV under the same drive conditions.

(d) Engineering constraints

- **Material loss:** Si₃N₄ waveguides must exhibit propagation loss < 0.1 dB/cm to keep photon lifetimes longer than $1/\Delta$.
- **Drive power:** THz source must deliver peak electric fields $> 10^4$ V/m without inducing dielectric breakdown in LiNbO₃.
- **Phase stability:** Synthetic-dimensional hopping phases require sub-degree stability over the experiment duration.

- **Thermal management:** Cryogenic cooling to 80 K must remove drive-induced heating estimated at $< 1 \mu\text{W}$ per resonator.
- **Frequency requirement:** To satisfy $\hbar\omega > 2\Delta$, the drive frequency must exceed 7 THz; current THz sources at 0.5 THz would need redesign or frequency multiplication.

(e) Order-of-magnitude cost projection (assumption-based)

Assuming a foundry cost of $\backslash 5\text{k per } 1\text{cm}^2 \text{ photonic chip}$, plus $\backslash 2\text{k}$ for THz drive electronics and $\backslash 1\text{k}$ for cryogenic packaging, the total prototype cost is projected at $\approx \backslash 8\text{k}$. This projection assumes standard wafer-scale processes and off-the-shelf THz sources; deviations in fabrication yield or custom drive design could increase cost by a factor of two.

(f) Risks and misuse

If the gap is mischaracterized and a system is deployed at temperatures where $\Delta \leq k_B T$, logical errors would increase dramatically, potentially leading to undetected data corruption in quantum communication links. Careful verification of the gap before integration is therefore essential.

8. Conclusion

We have presented a proposed photonic architecture designed to reach a four-dimensional topological bandgap with a target mass gap $\Delta_{\text{target}} \approx 14.5 \text{ meV}$, which would exceed the thermal energy at 80 K by a factor of 2.1. The resulting Boltzmann suppression factor $p_{\text{exc}} \approx 1.2 \times 10^{-1}$ indicates that logical errors become thermally activated excitations rather than events requiring active syndrome decoding. Detailed quantitative analysis shows that achieving fault-tolerant suppression levels ($p \leq 10^{-3}$) or 1 s memory lifetimes would require either a larger gap ($\geq 47.6 \text{ meV}$) or lower operating temperatures ($\leq 24 \text{ K}$). The Floquet-drive frequency must exceed 7 THz to avoid quasienergy folding. The next experimental step is a spectroscopic verification of the gap in a small synthetic lattice, which will directly test the central quantitative claim. If successful, this pathway could reshape the design of scalable quantum processors, moving the field toward hardware-intrinsic protection mechanisms.

Provenance and AI Involvement

- This paper was drafted, reconciled, reviewed, revised and checked by the QNFO automated research pipeline (qnfo-research-exec 0.33.9-codeagent). Language models were tools in that process; they are not authors. The named author is accountable for the content.
- Models: drafting by openai/gpt-oss-120b, zai-org/glm-5.3-flash; reconciliation, completion, review, revision and verification by openai/gpt-oss-120b, zai-org/glm-5.3-flash, moonshotai/kimi-k2.6, with the QNFO model router as a fallback.
- Sources: every reference is drawn from a bibliography retrieved from arXiv and the QNFO corpus before drafting; every cited work's full text was retrieved (arXiv HTML, or the QNFO paper's body) and the writers were given excerpts of its own sections; statements about a cited work are restricted to its retrieved title, abstract and those excerpts.
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- Independent recomputation: a separately generated script recomputed 10 quantitative claims from the paper's stated inputs; 0 differed from the stated value at the stated precision.

- Not checked: whether each cited source supports the sentence that cites it beyond the retrieved full-text excerpts; derivation steps other than the recomputed claims. This paper has not been peer reviewed, and this version was published by the pipeline without a prior manual review by the author.

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