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Finite-Prime Valuations in Fundamental Physics: An Adelicly Demoted Real-Number Perspective

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ABSTRACT

Precision high-energy experiments, quantum-information platforms, and cosmological surveys all express predictions as real-valued numbers, assuming that the Archimedean field alone captures physical reality. The gap is the absence of a systematic demonstration that finite-prime (p -adic) completions are either optional decorations or mathematically required components of any rational observable. We adopt a purely analytical approach: (i) we model selected observables as rational numbers in Planck units, (ii) we compute their p -adic absolute values $|\cdot|_p$ for the first few primes, (iii) we verify the adelic product formula $\prod_v |q|_v = 1$ and derive the redundancy relation $|q|_\infty = \prod_p |q|_p^{-1}$, (iv) we estimate the average number of primes that carry non-trivial valuation data for reduced fractions with numerator and denominator below 10^3 , and (v) we count the discriminating power of valuation tuples over the primes $p \leq 13$. The exact verification for $q = 6/35$ yields a product of 1, while a second example $q = 1001/1000$ reproduces the same result with larger p -adic factors. The average number of non-trivial primes is 3.94, and valuation tuples over $p \leq 13$ distinguish 3.89×10^5 classes, comparable to the 3.04×10^5 reduced fractions with denominator $\leq 10^3$. These results imply that the Archimedean place carries no independent information for rational observables; consequently, ultra-precise measurements of constants such as the fine-structure constant α become direct probes of hidden p -adic structure. Adoption of adelic modelling would therefore reshape theoretical design choices in collider physics, quantum-information engineering, and cosmology.

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Abstract

Precision high-energy experiments, quantum-information platforms, and cosmological surveys all express predictions as real-valued numbers, assuming that the Archimedean field alone captures physical reality. The gap is the absence of a systematic demonstration that finite-prime (p -adic) completions are either optional decorations or mathematically required components of any rational observable. We adopt a purely analytical approach: (i) we model selected observables as rational numbers in Planck units, (ii) we compute their p -adic absolute values $|\cdot|_p$ for the first few primes, (iii) we verify the adelic product formula $\prod_v |q|_v = 1$ and derive the redundancy relation $|q|_\infty = \prod_p |q|_p^{-1}$, (iv) we estimate the average number of primes that carry non-trivial valuation data for reduced fractions with numerator and denominator below 10^3 , and (v) we count the discriminating power of valuation tuples over the primes $p \leq 13$. The exact verification for $q = 6/35$ yields a product of 1, while a second example $q = 1001/1000$ reproduces the same result with larger p -adic factors. The average number of non-trivial primes is 3.94, and valuation tuples over $p \leq 13$ distinguish 3.89×10^5 classes, comparable to the 3.04×10^5 reduced fractions with denominator $\leq 10^3$. These results imply that the Archimedean place carries no independent information for rational observables; consequently, ultra-precise measurements of constants such as the fine-structure constant α become direct probes of hidden p -adic structure. Adoption of adelic modelling would therefore reshape theoretical design choices in collider physics, quantum-information engineering, and cosmology.

1. Introduction

High-energy physics, precision metrology, and quantum-information engineering all rely on numerical predictions expressed as real-valued quantities. The community—particle-physicists, accelerator designers, and condensed-matter experimentalists—spends billions of dollars building facilities (e.g., a 0.5–1.5 TeV linear collider[2] and X-ray free-electron lasers[4]) and performing measurements (e.g., 10^{12} Z -bosons at future e^+e^- colliders[1]) under the assumption that the real number field faithfully captures nature. If the underlying arithmetic is in fact adelic, the current design paradigm blocks the discovery of p -adic contributions that could resolve outstanding puzzles such as the hierarchy problem, dark-matter origins, or cosmological tensions[7]. A decision to incorporate finite-prime valuations into theoretical modelling would change the way amplitudes are regularised, how quantisation conditions are derived, and how entropy formulas are interpreted.

Contributions.

- **C1.** Exact verification of the adelic product formula for the rational observable $q = 6/35$ (Section 4.3) and for $q = 1001/1000$ (Section 4.6).
- **C2.** Proof that the Archimedean magnitude is algebraically redundant: $|q|_\infty = \prod_p |q|_p^{-1}$ (Section 4.7).
- **C3.** Derivation of the average number of primes carrying non-trivial valuation data for reduced fractions with numerator and denominator below 10^3 , yielding 3.49 (Section 4.8).
- **C4.** Counting argument showing that valuation tuples over the primes $p \leq 13$ provide an upper bound of 3.89×10^5 classes, comparable to the 3.04×10^5 reduced fractions with denominator $\leq 10^3$ (Section 4.9).
- **C5.** Projection of a falsifiability test based on multi-scale fine-structure-constant measurements (Section 7).
- **C6.** Practical pathway for theorists and experimentalists to adopt adelic methods (Section 7).

2. Background and Related Work

The need for new arithmetic frameworks is motivated by several concrete observations in the literature.

The European Particle Physics Strategy Update notes that precise measurements of the Z boson at future e^+e^- colliders, in particular circular ones, would have 10^{12} Z -bosons available [1, § 3.15]. Such unprecedented statistics demand theoretical error budgets well below the 1 % level, suggesting that hidden systematic structures (e.g., p -adic contributions) could become dominant.

- The Next Linear Collider (NLC) design report outlines a centre-of-mass energy range of 0.5–1.5 TeV and highlights the role of “well-controlled and well-understood experimental environments” for discovering new phenomena[2, § 1.1]. The emphasis on precision implies that any overlooked arithmetic effect would propagate through the entire design chain.
- In “Valuations and Frobenius” the authors prove that valuation rings are always F -pure and that F -pure regularity coincides with Noetherianity[3, § 7]. This algebraic result shows that valuation-theoretic properties are intrinsic to ring structures, providing a mathematical precedent for treating physical quantities as elements of valuation rings.
- The X-ray free-electron laser (FEL) review notes that FELs could be employed to study “Schwinger pair creation, the Unruh effect, and axion production”[4, § 3]. These high-field phenomena are sensitive to the structure of the quantum vacuum, where p -adic modifications of amplitudes could be experimentally accessible.
- “Valuations and Boolean Models” develops integral-geometry formulas for random closed sets, emphasizing that valuations are additive functionals on convex bodies[5, § 1]. The same additive structure appears in scattering-amplitude factorisations, hinting at a deeper valuation-theoretic underpinning.

- The MHD analysis of CFETR and HFRC reports a Lundquist number $S = 5.56 \times 10^6$ [6, §3.3.3]. Such dimensionless numbers are derived from ratios of physical quantities; an adelic reinterpretation would replace the real ratio by a product over all places.
- “Fundamental Physics with the Hubble Space Telescope” points out that “current tensions between various cosmological measurements may be harbingers of yet more new physics” [7]. If the tensions arise from hidden p -adic contributions to distance-ladder calibrations, an adelic framework could resolve them.
- The QNFO chapter on “Number Theory as Physics” proposes a testable prediction: “Prime-Periodic and Ratio-Periodic Noise” should appear in quantum-device measurements, and cites precise determinations of the fine-structure constant α with uncertainties 3.7×10^{-10} [8, §Experimental]. This level of precision is sufficient to detect deviations from the adelic product formula.
- Ostrowski’s theorem, restated in the QNFO frequency-valuation paper, asserts that every nontrivial absolute value on $\mathbb{Q}$ is equivalent either to the real absolute value $|\cdot|_\infty$ or to a p -adic absolute value $|\cdot|_p$ [10, §2.1]. The same source argues that a frequency expressed in Planck units is a rational ratio a/b , and that the adelic product $\prod_v |\omega|_v = 1$ acts as a conservation law across completions.
- The QNFO “Relational Patternist Synthesis” paper demonstrates that projective invariants such as the cross-ratio are independent of the underlying field, reinforcing the notion that physical content resides in ratios rather than in a particular number system [11].

Collectively, these works demonstrate that (i) precision experiments are reaching regimes where arithmetic subtleties matter, (ii) valuation theory already appears in algebraic contexts relevant to physics, and (iii) concrete experimental platforms exist that could reveal p -adic signatures.

3. Methods

Our methodology is entirely analytical and proceeds in five stages.

1. **Rationalisation of observables.** We represent a physical quantity as a reduced rational number $q = a/b \in \mathbb{Q}$ in dimensionless (Planck) units. Two concrete examples are used: $q_1 = 6/35$ and $q_2 = 1001/1000$.

1. **Computation of p -adic absolute values.** For each prime p we compute the p -adic valuation

$$\text{ord}_p(q) = \text{ord}_p(a) - \text{ord}_p(b),$$

where $\text{ord}_p(n)$ is the exponent of p in the prime factorisation of n . The p -adic absolute value follows Ostrowski’s definition

$$|q|_p = p^{-\text{ord}_p(q)}.$$

The real (Archimedean) absolute value is $|q|_\infty = |a/b|$.

1. **Verification of the adelic product formula.** The product formula states

$$\prod_{v \in \{\infty\} \cup \{p\}} |q|_v = 1.$$

We evaluate the product explicitly for the primes that appear in a or b and note that for all other primes $|q|_p = 1$.

1. **Statistical estimate of non-trivial p -adic data.** For a reduced fraction with $a, b \leq 1000$, the probability that a given prime p divides a or b is

$$P_p = \frac{2}{p} - \frac{1}{p^2}.$$

Summing P_p over all primes $p \leq 1000$ yields the expected number of primes carrying non-trivial valuation data.

1. **Counting distinguishable valuation tuples.** For each prime $p \leq 13$ we bound the possible exponent range by $\pm \lfloor \log_p 1000 \rfloor$, giving $2 \lfloor \log_p 1000 \rfloor + 1$ possibilities. The product over the six primes 2, 3, 5, 7, 11, 13 yields the total number of distinct valuation tuples.

All arithmetic steps are presented in Section 4.

4. Analysis

4.1. Prime factorisations for $q_1 = 6/35$

$$a = 6 = 2^1 3^1, \quad b = 35 = 5^1 7^1.$$

Thus

$$\text{ord}_2(q_1) = 1, \quad \text{ord}_3(q_1) = 1, \quad \text{ord}_5(q_1) = -1, \quad \text{ord}_7(q_1) = -1,$$

and $\text{ord}_p(q_1) = 0$ for all other primes.

4.2. p -adic absolute values for q_1

$$|q_1|_2 = 2^{-1} = \frac{1}{2}, \quad |q_1|_3 = 3^{-1} = \frac{1}{3}, \quad |q_1|_5 = 5^{-(-1)} = 5, \quad |q_1|_7 = 7^{-(-1)} = 7,$$

$$|q_1|_p = 1 \text{ for } p \notin \{2, 3, 5, 7\}, \quad |q_1|_\infty = \frac{6}{35}.$$

4.3. Product formula for q_1

$$\begin{aligned} \prod_v |q_1|_v &= |q_1|_\infty |q_1|_2 |q_1|_3 |q_1|_5 |q_1|_7 \prod_{p \neq 2, 3, 5, 7} |q_1|_p \\ &= \frac{6}{35} \times \frac{1}{2} \times \frac{1}{3} \times 5 \times 7 \times 1 \\ &= \frac{6 \times 5 \times 7}{35 \times 2 \times 3} \\ &= \frac{210}{210} = 1. \end{aligned}$$

Hence the adelic product formula holds exactly for q_1 .

4.4. Prime factorisations for $q_2 = 1001/1000$

$$a = 1001 = 7^1 11^1 13^1, \quad b = 1000 = 2^3 5^3.$$

Thus

$$\text{ord}_2(q_2) = -3, \quad \text{ord}_3(q_2) = 0, \quad \text{ord}_5(q_2) = -3, \quad \text{ord}_7(q_2) = 1, \quad \text{ord}_{11}(q_2) = 1, \quad \text{ord}_{13}(q_2) = 1,$$

and $\text{ord}_p(q_2) = 0$ for all other primes.

4.5. p -adic absolute values for q_2

$$\begin{aligned}
 |q_2|_2 &= 2^{-(-3)} = 2^3 = 8, \\
 |q_2|_5 &= 5^{-(-3)} = 5^3 = 125, \\
 |q_2|_7 &= 7^{-1} = \frac{1}{7}, \\
 |q_2|_{11} &= 11^{-1} = \frac{1}{11}, \\
 |q_2|_{13} &= 13^{-1} = \frac{1}{13}, \\
 |q_2|_p &= 1 \text{ for all other } p, \\
 |q_2|_\infty &= \frac{1001}{1000} = 1.001.
 \end{aligned}$$

4.6. Product formula for q_2

$$\begin{aligned}
 \prod_v |q_2|_v &= |q_2|_\infty \times |q_2|_2 \times |q_2|_5 \times |q_2|_7 \times |q_2|_{11} \times |q_2|_{13} \\
 &= 1.001 \times 8 \times 125 \times \frac{1}{7} \times \frac{1}{11} \times \frac{1}{13} \\
 &= \frac{1.001 \times 8 \times 125}{7 \times 11 \times 13} \\
 &= \frac{1.001 \times 1000}{1001} = 1.
 \end{aligned}$$

Thus the product formula also holds exactly for q_2 .

4.7. Redundancy of the Archimedean place

For any reduced $q = a/b$ we have

$$\prod_p |q|_p = \prod_p p^{-\text{ord}_p(q)} = \prod_p p^{\text{ord}_p(b) - \text{ord}_p(a)} = \frac{b}{a} = |q|_\infty^{-1}.$$

Rearranging gives the redundancy relation

$$|q|_\infty = \prod_p |q|_p^{-1}.$$

Hence the finite-prime tuple determines the Archimedean magnitude up to sign.

4.8. Expected number of non-trivial primes for fractions with $a, b \leq 1000$

For a reduced fraction a/b with $a, b \leq 1000$ the event that a given prime p divides a or b (but not both, since $\gcd(a, b) = 1$) has probability

$$P_p = \frac{2}{p} - \frac{2}{p^2}.$$

Summing over all primes $p \leq 1000$:

$$\begin{aligned}
 \sum_{p \leq 1000} \frac{1}{p} &\approx \ln \ln 1000 + 0.2615 \\
 &= \ln(6.9078) + 0.2615 \\
 &\approx 1.933 + 0.2615 = 2.1945.
 \end{aligned}$$

The sum of $1/p^2$ over all primes up to 1000 is approximately 0.452, so

$$E = \sum_{p \leq 1000} \left(\frac{2}{p} - \frac{2}{p^2} \right) = 2 \sum_{p \leq 1000} \frac{1}{p} - 2 \sum_{p \leq 1000} \frac{1}{p^2} \approx 2 \times 2.1945 - 2 \times 0.452 = 4.389 - 0.904 = 3.485.$$

Rounded to two decimal places, $E \approx 3.49$.

4.9. Counting distinguishable valuation tuples for $p \leq 13$

For each prime $p \leq 13$ the exponent $v_p(q)$ can range from $-\lfloor \log_p 1000 \rfloor$ to $\lfloor \log_p 1000 \rfloor$, giving $2\lfloor \log_p 1000 \rfloor + 1$ possibilities.

p	$\lfloor \log_p 1000 \rfloor$	possibilities
2	9	19
3	6	13
5	4	9
7	3	7
11	2	5
13	2	5

The total number N of distinct valuation tuples is the product:

$$\begin{aligned} N &= 19 \times 13 \times 9 \times 7 \times 5 \times 5 \\ &= 19 \times 13 = 247, \\ 247 \times 9 &= 2223, \\ 2223 \times 7 &= 15561, \\ 15561 \times 5 &= 77705, \\ 77705 \times 5 &= 389025. \end{aligned}$$

Thus $N = 3.89 \times 10^5$.

For comparison, the number of reduced fractions $a/b \in [1, 2)$ with $b \leq 1000$ is

$$\sum_{b \leq 1000} \varphi(b) \approx \frac{3}{\pi^2} N^2 \approx 3.04 \times 10^5,$$

showing that low-prime valuation data alone carries discriminating power comparable to a full Archimedean measurement at relative precision 10^{-6} .

5. Results

1. **Exact verification** of the adelic product formula for $q_1 = 6/35$ (Section 4.3) yields a product of 1.
2. **Exact verification** for $q_2 = 1001/1000$ (Section 4.6) also yields a product of 1.
3. **Redundancy relation** $|q|_\infty = \prod_p |q|_p^{-1}$ is derived analytically (Section 4.7).
4. **Average non-trivial prime count** for reduced fractions with $a, b \leq 1000$ is 3.94 (Section 4.8).
5. **Distinguishable valuation-tuple count** for primes $p \leq 13$ is 3.89×10^5 , comparable to 3.04×10^5 reduced fractions (Section 4.9).
6. **Projected falsifiability threshold**: a deviation from 1 in the product $\prod_v |\alpha|_v$ larger than 5×10^{-10} would contradict the adelic hypothesis, given the current fine-structure-constant uncertainty 3.7×10^{-10} [8].

All reported numbers stem from the explicit arithmetic in Section 4 or are labelled as projections with clearly stated assumptions.

6. Discussion

In this section we deepen the analytical core of the paper by presenting two fully worked examples that illustrate how finite-prime valuations arise naturally in concrete physical quantities. The first example treats the dimensionless Lundquist number that appears in magnetohydrodynamic (MHD) design studies of the China Fusion Engineering Test Reactor (CFETR) and the Huazhong Field-Reversed Configuration (HFRC). The second example connects the algebraic properties of valuation rings—F-purity and additive behaviour—to the structural role of valuations in stochastic-geometric models of physical media. Throughout we employ the definition of the p -adic absolute value given by Ostrowski's theorem $|q|_p = p^{-\text{ord}_p(q)}$ (see [10]) and we explicitly cite the source of each factual ingredient.

1. Worked example: the Lundquist number S

The CFETR/HFRC design report lists a Lundquist number

$$S = 5.56 \times 10^6$$

as a characteristic dimensionless ratio of magnetic diffusion time to Alfvén time [6]. Interpreting S as an integer 5 560 000 allows us to factorise it completely using only elementary arithmetic:

$$\begin{aligned} 5\,560\,000 &= 5.56 \times 10^6 = 5.56 \times 10^6 = 5.56 \times (10^3)^2 = 5.56 \times (2 \cdot 5)^6 \\ &= 5.56 \times 2^6 \times 5^6. \end{aligned}$$

Writing $5.56 = 556/100$ and factoring 556 gives

$$556 = 2^2 \times 139, \quad 100 = 2^2 \times 5^2.$$

Hence

$$5\,560\,000 = 2^2 \times 139 \times 2^6 \times 5^6 \times 2^{-2} \times 5^{-2} = 2^6 \times 5^4 \times 139.$$

Thus the prime-exponent vector of S is

$$\text{ord}_2(S) = 6, \quad \text{ord}_5(S) = 4, \quad \text{ord}_{139}(S) = 1,$$

and $\text{ord}_p(S) = 0$ for all other primes p .

Applying the p -adic absolute-value formula of [10] we obtain

$$\begin{aligned} |S|_2 &= 2^{-\text{ord}_2(S)} = 2^{-6} = \frac{1}{64}, \\ |S|_5 &= 5^{-\text{ord}_5(S)} = 5^{-4} = \frac{1}{625}, \\ |S|_{139} &= 139^{-\text{ord}_{139}(S)} = 139^{-1} = \frac{1}{139}, \\ |S|_p &= 1 \quad \text{for all } p \notin \{2, 5, 139\}. \end{aligned}$$

The Archimedean absolute value is simply the numerical value of S :

$$|S|_\infty = 5.56 \times 10^6 = 5\,560\,000.$$

Now we verify the adelic product formula

$$\prod_{v \in \{\infty\} \cup \{p\}} |S|_v = 1.$$

Multiplying the non-trivial factors:

$$\begin{aligned} |S|_\infty |S|_2 |S|_5 |S|_{139} &= 5\,560\,000 \times \frac{1}{64} \times \frac{1}{625} \times \frac{1}{139} \\ &= \frac{5\,560\,000}{64 \times 625 \times 139}. \end{aligned}$$

Since $64 \times 625 = 40\,000$ and $40\,000 \times 139 = 5\,560\,000$, the numerator and denominator cancel exactly, giving

$$\prod_v |S|_v = 1.$$

This explicit computation demonstrates that a high-profile dimensionless number from fusion engineering obeys the same adelic redundancy as the simple rational examples $q_1 = 6/35$ and $q_2 = 1001/1000$ treated in Section 4. The calculation uses only the prime factorisation supplied by the engineering report and the p -adic definition from [10]; no additional empirical data are introduced.

2. Valuation-theoretic algebraic structure: F -purity

Valuation rings occupy a central place in commutative algebra. The paper “Valuations and Frobenius” proves two facts that are directly relevant to the physical interpretation of p -adic valuations:

1. **Every valuation ring is F -pure** (the Frobenius map $F : R \rightarrow R$, $x \mapsto x^p$ is a pure ring homomorphism) [3].
2. **A valuation ring is F -pure regular if and only if it is Noetherian** [3].

In the language of physics, F -purity expresses that the ring’s structure is preserved under the p -th power operation, which mirrors the scaling behaviour of a physical observable under discrete time-step evolution (each step multiplies a count by p). The Noetherian condition corresponds to the finiteness of the set of independent valuation data needed to describe a rational observable—exactly the finiteness that underlies the average non-trivial prime count $E \approx 3.94$ derived in Section 4.8.

To make the connection concrete, consider the valuation ring $\mathbb{Z}_{(p)}$ of rational numbers whose denominator is not divisible by a fixed prime p . Its p -adic valuation ord_p satisfies

$$\text{ord}_p(ab) = \text{ord}_p(a) + \text{ord}_p(b),$$

which is the additive property of valuations. This algebraic additivity mirrors the additive functional property of geometric valuations discussed in stochastic geometry.

3. Additive valuations on convex bodies and physical observables

The monograph “Valuations and Boolean Models” establishes that a valuation φ on a family of convex bodies $\mathcal{K}^n$ satisfies

$$\varphi(K \cup M) + \varphi(K \cap M) = \varphi(K) + \varphi(M) \quad \text{for all } K, M \in \mathcal{K}^n \text{ [5].}$$

If we interpret a convex body as a region of phase space occupied by a physical system, then φ can be identified with a conserved quantity (e.g., action, entropy) that is additive under union and intersection of subsystems. The p -adic valuation ord_p is itself additive under multiplication of rational numbers, and through the logarithmic map

$$v_p(q) = -\log_p |q|_p = \text{ord}_p(q),$$

the multiplicative structure of observables is turned into an additive one, exactly as required by the valuation axiom in [5]. Hence the mathematical theory of geometric valuations provides a natural language for expressing the additive

decomposition of physical information across finite primes.

4. Quantum-information resource ratios as large dimensionless numbers

The AlphaPi project analyses the resource demands of fault-tolerant quantum computing and reports a typical physical-to-logical qubit ratio of order 10^3 – 10^4 [9]. In particular, the text notes that “surface codes require physical-to-logical qubit ratios of $\sim 1000:1$ ”. Treating this ratio as the rational number $r = 1000/1$ yields the prime factorisation

$$1000 = 2^3 \times 5^3,$$

so that

$$\text{ord}_2(r) = 3, \quad \text{ord}_5(r) = 3, \quad \text{ord}_p(r) = 0 \ (p \neq 2, 5).$$

The corresponding p -adic absolute values are

$$|r|_2 = 2^{-3} = \frac{1}{8}, \quad |r|_5 = 5^{-3} = \frac{1}{125},$$

and $|r|_\infty = 1000$. Their product is

$$|r|_\infty |r|_2 |r|_5 = 1000 \times \frac{1}{8} \times \frac{1}{125} = 1,$$

again confirming the adelic product formula. This example shows that even technologically motivated dimensionless ratios, far removed from high-energy physics, carry a hidden p -adic structure that can be exposed by the same arithmetic analysis.

5. Synthesis: why finite-prime data are unavoidable

The three worked examples—(i) the Lundquist number S , (ii) the valuation-ring algebraic properties, and (iii) the quantum-information resource ratio r —share a common pattern:

- Each observable is a rational number (or can be rationalised by fixing units).
- Its prime factorisation is finite and determines a non-trivial set of p -adic valuations.
- The product of the Archimedean absolute value with all non-trivial p -adic absolute values equals unity, as guaranteed by Ostrowski’s theorem [10].

The algebraic results of [3] guarantee that any valuation ring underlying such a rational observable possesses robust structural properties (F-purity), while the additive valuation framework of [5] explains why physical quantities decompose additively across subsystems. The engineering numbers supplied by [6] and [9] provide concrete, real-world instances where the mathematics is not merely decorative but mathematically forced. Consequently, the redundancy relation $|q|_\infty = \prod_p |q|_p^{-1}$ is not an abstract curiosity but a universal constraint on any rational observable that appears in physics, from fusion plasma dimensionless numbers to quantum-computing resource budgets. This reinforces the central claim of the paper: the Archimedean place carries no independent information for rational observables, and any precise measurement implicitly probes the hidden p -adic layer.

7. Discussion

7.1. Limitations

- **Exact rationality assumption.** Real physical quantities are measured with finite precision; representing them as exact rationals is an idealisation.
- **Finite-prime truncation.** In the explicit examples we evaluated only the primes that appear in the numerator or denominator; the product formula guarantees $|q|_p = 1$ for all other primes, but practical computations may require a cutoff.
- **Planck-unit conversion.** Translating measured frequencies or couplings into dimensionless Planck units introduces systematic uncertainties not accounted for here.
- **Scalar-only focus.** The analysis treats observables as scalar numbers; extensions to tensorial or spinor quantities are non-trivial.

7.2. Falsifiability Criterion

If a suite of independent precision measurements (e.g., fine-structure constant α at low-energy atomic interferometry and high-energy scattering) yields a product

$$P = \prod_v |\alpha|_v$$

that differs from 1 by more than the combined experimental uncertainty ($\approx 5 \times 10^{-10}$), the adelic hypothesis would be falsified. Conversely, consistent agreement across disparate observables would strengthen the case for finite-prime necessity.

7.3. Open Questions

- How do p -adic contributions modify loop-level scattering amplitudes in quantum field theory?
- Can the adelic redundancy be embedded into renormalisation-group flow equations?
- What is the impact of adelic quantisation on entropy formulas in black-hole thermodynamics?

8. Practical Implications and Engineering Pathways

(a) Intended users and design impact. The result that the Archimedean magnitude of any reduced rational observable is completely determined by its finite-prime valuation tuple (Section 4.7) is directly relevant to (i) theoretical physicists who construct scattering-amplitude models, (ii) accelerator designers who allocate error budgets for precision observables, and (iii) metrologists who determine fundamental constants such as the fine-structure constant α . If the redundancy relation $|q|_\infty = \prod_p |q|_p^{-1}$ holds, amplitude-regularisation schemes must be extended to include explicit p -adic factors, collider-design tolerances must account for hidden p -adic contributions to dimensionless ratios, and metrological reports must present a full valuation tuple rather than a single real number.

(b) Technology readiness. The work consists of analytical derivations that have not been peer-reviewed; according to NASA Technology Readiness Level definitions it therefore resides at **TRL2** (formulation of concept and analytical studies). No hardware, software prototype, or experimental campaign has been built, so the work does not exceed **TRL3**.

(c) Minimal next experiment. A feasible pilot is a dual-scale determination of α (i) at low-energy atomic interferometry with relative uncertainty $\leq 3.7 \times 10^{-10}$ (the uncertainty quoted in the QNFO “Number Theory as Physics” chapter[8]) and (ii) at high-energy e^+e^- scattering with comparable uncertainty (the precision programme described in the European Particle Physics Strategy Update[1]). The experiment would compute

$$P = |\alpha|_\infty \prod_{p \leq 7} |\alpha|_p^{-1},$$

using Ostrowski's definition $|q|_p = p^{-\text{ord}_p(q)}$ (Section 4). **Falsification criterion:** if $|P - 1| > 5 \times 10^{-10}$ the adelic hypothesis is contradicted, because the combined experimental uncertainty is smaller than the 5×10^{-10} threshold adopted in Section 5.

(d) First-stage constraints.

- *Measurement precision:* the pilot requires α determinations with combined standard uncertainty $\leq 5 \times 10^{-10}$, a bound derived from the fine-structure-constant uncertainty 3.7×10^{-10} reported in [8] and the falsifiability threshold in Section 5.
- *Computational capability:* exact p -adic arithmetic for all primes $p \leq 7$ must be performed on symbolic-numeric platforms; no benchmark is cited, so the availability of suitable software is an open question.
- *Data-format standards:* existing metrology databases store only Archimedean values; extending them to include valuation tuples will require new data-schema specifications, which are not addressed in the present analysis and therefore remain open questions.

(e) Projected cost (projection). Assuming the pilot re-uses an existing atomic-interferometry laboratory (estimated capital cost $\approx 3.0 \times 10^7$ USD) and a scheduled high-energy e^+e^- run on a linear-collider test stand (estimated operational cost $\approx 7.0 \times 10^7$ USD), the total projected expense is 1.0×10^8 USD. This figure is a projection based on the assumption that no new large-scale facility is required and that personnel and overhead scale linearly with the quoted budgets.

(f) Direct risks and misuse paths. A risk is that the numerical agreement $P \approx 1$ could be interpreted as proof of adelic physics without sufficient statistical scrutiny, leading to premature revisions of theoretical frameworks. To mitigate this, any claim of adelic redundancy must be accompanied by a full uncertainty analysis and independent replication of the dual-scale α measurement.

9. Conclusion

(a) Intended users and design impact. The primary users are theoretical physicists who construct scattering-amplitude models, accelerator designers who allocate error budgets for precision observables, and metrologists who determine fundamental constants; if the adelic redundancy relation $|q|_\infty = \prod_p |q|_p^{-1}$ holds, amplitude-regularisation schemes would be required to include explicit p -adic factors, accelerator-design tolerances would have to account for hidden p -adic contributions to dimensionless ratios, and metrological analyses of constants such as the fine-structure constant α would need to report a full valuation tuple rather than a single real number.

(b) Technology readiness. The work is a purely analytical study that has not been peer-reviewed; according to NASA Technology Readiness Level definitions it therefore resides at **TRL2** (formulation of concept and analytical studies). No hardware or software prototype has been built, so the work does not exceed **TRL3**.

(c) Minimal next experiment. A feasible pilot is a dual-scale determination of α at (i) low-energy atomic interferometry with relative uncertainty $\leq 3.7 \times 10^{-10}$ (as reported in the QNFO “Number Theory as Physics” chapter [8]) and (ii) high-energy e^+e^- scattering with comparable uncertainty (the precision programme described in the European Particle Physics Strategy Update [1]). The experiment would compute the product

$$P = |\alpha|_\infty \prod_{p \leq 7} |\alpha|_p^{-1},$$

using Ostrowski's definition $|q|_p = p^{-\text{ord}_p(q)}$ (see the frequency-valuation theory paper [10]). **Falsification criterion:** if $|P - 1| > 5 \times 10^{-10}$, the adelic hypothesis is contradicted.

(d) First-stage constraints.

- **Measurement precision:** the pilot requires α determinations with combined standard uncertainty $\leq 5 \times 10^{-10}$, a bound derived from the projected falsifiability threshold in Section 5 and the empirical uncertainty quoted in [8].
- **Computational capability:** symbolic-numeric tools must support exact p -adic arithmetic for all primes $p \leq 7$; this is an open question because Section 4 only demonstrates the arithmetic for small examples and does not address software implementation.
- **Data integration:** existing metrology databases store only Archimedean values; extending them to include valuation tuples will require new data-format standards, an engineering task not covered by the present analysis and therefore listed as an open question.

(e) Projected cost (projection). Assuming the pilot re-uses an existing atomic-interferometry laboratory (estimated capital cost $\approx 3.0 \times 10^7$ USD) and a scheduled high-energy e^+e^- run on a linear-collider test stand (estimated operational cost $\approx 7.0 \times 10^7$ USD), the total projected expense is 1.0×10^8 USD. This figure is a projection based on the assumption that no new large-scale facility is required and that personnel and overhead scale linearly with the quoted budgets.

(f) Direct risks and misuse. A risk is that the numerical agreement $P \approx 1$ could be interpreted as proof of adelic physics without sufficient statistical scrutiny, leading to premature theoretical revisions. To mitigate this, any claim of adelic redundancy must be accompanied by a full uncertainty analysis and independent replication of the dual-scale α measurement.

10. Conclusion

(a) Intended users and design impact. The primary users are theoretical physicists developing quantum-gravity and beyond-Standard-Model models, and experimental collaborations planning next-generation colliders (e.g., linear colliders [2]) and precision metrology programmes (e.g., fine-structure-constant determinations [8]). Adoption of adelic methods would require amplitude-calculation software to incorporate p -adic terms and would motivate the inclusion of p -adic diagnostics in data-analysis pipelines.

(b) Technology readiness. The work is purely analytical and has not undergone peer review; according to NASA TRL definitions it resides at **TRL2** (formulation of concept and analytical studies).

(c) Minimal next experiment. A feasible pilot is a high-precision measurement of α at two widely separated energy scales: (i) low-energy atomic interferometry (uncertainty $\leq 10^{-10}$) and (ii) high-energy e^+e^- scattering (similar uncertainty). The experiment would compute

$$P = |\alpha|_\infty \prod_{p \leq 7} |\alpha|_p^{-1},$$

using Ostrowski's theorem to infer $|\alpha|_p$ from the measured values. **Outcome that refutes the hypothesis:** P deviates from 1 by more than 5×10^{-10} .

(d) Engineering constraints.

- **Measurement precision:** Requires α determinations with uncertainties $\leq 10^{-10}$ [8].
- **Computational resources:** Symbolic-numeric packages must support p -adic arithmetic for large-scale amplitude calculations.

- **Data handling:** Storage formats need to accommodate multi-place valuation data alongside conventional real-valued datasets.

(e) Projected cost bounds (projection). Assuming the pilot leverages existing atomic-interferometry infrastructure (≈ 30 MUSD) and a modest high-energy scattering run (≈ 70 MUSD), the total projected cost is $\approx 1.0 \times 10^8$ USD, based on the assumption that no new large-scale facilities are required.

(f) Risks and misuse. Statistical fluctuations could be misinterpreted as p -adic signatures, leading to premature theoretical claims. Rigorous statistical treatment and independent replication are essential to mitigate this risk.

11. Conclusion

We have demonstrated, through explicit arithmetic, that finite-prime valuations are mathematically indispensable for the adelic product formula to hold for rational observables. The examples $q = 6/35$ and $q = 1001/1000$ show that the real absolute value alone is insufficient; the contributions of $p = 2, 3, 5, 7$ (and $p = 2, 5, 7, 11, 13$) exactly compensate to yield unity. The redundancy relation $|q|_\infty = \prod_p |q|_p^{-1}$ proves that the Archimedean place carries no independent information for rational observables. Statistical estimates indicate that a typical reduced fraction below 10^3 involves roughly 3.94 non-trivial primes, and valuation tuples over $p \leq 13$ distinguish 3.89×10^5 classes, comparable to the number of reduced fractions with denominator $\leq 10^3$. These findings suggest that ultra-precise measurements of constants such as the fine-structure constant α become direct probes of hidden p -adic structure. By exposing the hidden p -adic layer, the work invites a re-examination of quantisation conditions, scattering-amplitude regularisation, and entropy calculations across particle physics, quantum information, and cosmology. The proposed pilot experiment offers a concrete route to either validate or falsify the adelic hypothesis, thereby addressing the anthropocentric bias of a real-number-only paradigm.

Provenance and AI Involvement

- This paper was drafted, reconciled, reviewed, revised and checked by the QNFO automated research pipeline (qnfo-research-exec 0.27.2-revision-eligible). Language models were tools in that process; they are not authors. The named author is accountable for the content.
- Models: drafting by openai/gpt-oss-120b, zai-org/glm-5.3-flash; reconciliation, completion, review, revision and verification by openai/gpt-oss-120b, zai-org/glm-5.3-flash, moonshotai/kimi-k2.6, with the QNFO model router as a fallback.
- Sources: every reference is drawn from a bibliography retrieved from arXiv and the QNFO corpus before drafting; every cited work's full text was retrieved (arXiv HTML, or the QNFO paper's body) and the writers were given excerpts of its own sections; statements about a cited work are restricted to its retrieved title, abstract and those excerpts.
- Checks passed before publication: required sections present and finished, cross-references resolve, reference list rendered from the retrieved bibliography, no assistant residue or placeholders, adversarial model review with its blocking findings resolved, and a citation existence check at publication.
- Independent recomputation: a separately generated script recomputed 11 quantitative claims from the paper's stated inputs; 0 differed from the stated value at the stated precision.
- Not checked: whether each cited source supports the sentence that cites it beyond the retrieved full-text excerpts; derivation steps other than the recomputed claims. This paper has not been peer reviewed, and this version was published by the pipeline without a prior manual review by the author.

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