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From Operational Axioms to Field Axioms: An Audit of the GPT-to-Wightman Reconstruction Claim for Systems with Infinitely Many Degrees of Freedom

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ABSTRACT

Operational reconstructions of quantum theory have clarified the status of finite-dimensional quantum mechanics, but no published derivation takes the operational axioms of generalized probabilistic theories (GPTs) as hypotheses and produces the Wightman axioms of relativistic quantum field theory as conclusions. This gap leaves physicists unable to say which parts of quantum field theory structure are forced by operational quantum probability and which are independent relativistic input. We formalize four GPT axioms—causality, no-signaling, local tomography, and continuous reversibility—in the operator-algebraic setting of nets of von Neumann algebras, and audit each step of the claimed reconstruction against the Wightman and algebraic field theory literature. We prove that causality and no-signaling imply microcausality for localized observables; we exhibit fermionic theories, whose local state spaces violate local tomography while satisfying the graded Wightman axioms, as counterexamples showing local tomography cannot be necessary; we show continuous reversibility yields a strongly continuous unitary Poincaré representation but not the spectrum condition, which must be added as a fifth axiom; and we show real and quaternionic Hilbert-space theories satisfy all four operational axioms, so the reconstruction selects a family of theories rather than complex QFT uniquely. Of the five structural pillars audited, 1 is derived, 2 are conditional, and 2 are underived. The claim is therefore a conjecture with one proven pillar, two conditional pillars, and two unproven pillars, and we state the lemmas whose proof or refutation would settle it.

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1. Introduction

The practical problem this paper bears on is the status of the axioms of relativistic quantum field theory. Physicists working in constructive QFT, mathematical physics, and quantum foundations currently accept the Wightman or Haag-Kastler axiom systems as the definition of what a quantum field theory is, but these axiom systems postulate relativistic structure—microcausality, a unitary representation of the Poincaré group, the spectrum condition—directly, without deriving it from anything more operational. This postulational stance has a cost: it blocks any answer to the question of which parts of QFT structure are forced by the operational content of quantum theory and which are additional independent input. That question matters for three working communities. Researchers in quantum gravity and emergent spacetime need to know which features of QFT survive when spacetime is not taken as fundamental; a derivation of relativistic structure from operational axioms would tell them exactly which features are load-bearing consequences of quantum probability and which must be supplied by the geometry. Researchers in axiomatic QFT need to know whether the Wightman axioms are in any sense minimal; a reconstruction theorem would identify redundant axioms and clarify which relaxations, such as the relaxed Gårding-Wightman framework in which field operators are merely symmetric rather than self-adjoint, are forced or permitted [5]. And researchers in quantum foundations need to know

whether the generalized probabilistic theory framework, which was built to compare quantum theory against operational alternatives, has any purchase on the relativistic regime at all; if the GPT axioms do not constrain QFT structure, the reconstruction program's scope is finite-dimensional by construction.

If the claimed reconstruction theorem holds, the decision it changes is concrete: one could replace the Wightman axioms, in textbooks and in constructive programs, by four operational postulates whose physical content is directly checkable in laboratory protocols, and one could prove that no non-relativistic or non-local operational theory with infinitely many degrees of freedom is consistent. If the claim fails, the failure analysis still delivers value: it identifies precisely which additional axiom carries the relativistic structure, which is the information needed both for constructive QFT programs that relax the axioms and for foundational programs that seek pre-geometric derivations of spacetime.

The specific gap is that the reconstruction claim exists as a program statement without a theorem: no published derivation takes the four GPT axioms as hypotheses and produces the Wightman axioms as conclusions, and no published gap analysis identifies where the derivation would need assumptions the GPT axioms do not contain. The literature supplies the ingredients on both sides—GPT axiomatics on the operational side, and Wightman-type axioms, nets of algebras, and modular theory on the field-theoretic side [1]—but the bridge between them has not been audited step by step.

Our approach is a formalization-and-audit. We state each GPT axiom as a condition on a system with infinitely many degrees of freedom, modeled as a net of operator algebras or as a field theory on a Hilbert space; we then attempt the derivation of each Wightman axiom in turn, and at each step we either complete the derivation, exhibit the missing hypothesis, or exhibit a counterexample from the literature. The audit is deliberately conservative: every claim we make about the field-theoretic side is one that the algebraic and Wightman-QFT literature states explicitly, and every claim we make about the operational side is one we derive here from the GPT axioms as we state them.

Because the source material supports two readings of the reconstruction claim—a strong reading on which the four GPT axioms suffice to derive the Wightman axioms, and a weak reading on which the claim is a conjecture whose pillars must be audited individually—we state our convention here: **we take the weak, audit reading under the convention that each pillar of the reconstruction is derived, shown conditional, or refuted on its own, and that the overall claim is a conjecture until every pillar is derived**. This convention is forced by the counterexample material of Section 4.2, which shows that one pillar cannot hold as stated.

The principal results are as follows. First, we prove (Proposition 4.1) that the GPT axioms of causality and no-signaling, in the infinite-degree-of-freedom setting, imply microcausality for localized observables, so the locality pillar of the reconstruction is derivable without additional input; the derivation uses only the order structure of the causal relation and the tensor-product factorization that local tomography or its fermionic replacement supplies. Second, we show (Proposition 4.2 and the counterexample built on the free fermion net) that local tomography is not a necessary axiom of the reconstruction: fermionic theories satisfy the Wightman axioms while violating local tomography, so a reconstruction theorem that lists local tomography among its hypotheses either excludes fermions or must weaken the axiom to a graded (super) tensor product; the free fermion model is the case where the smeared fields are bounded operators and the distinction between the field net and the von Neumann algebra net disappears [1]. Third, we show (Proposition 4.3) that continuous reversibility yields a strongly continuous unitary representation of the symmetry group but does not by itself yield the spectrum condition; the positive-energy condition is additional input, and we exhibit where it must be inserted. Fourth, we show (Proposition 4.4) that the reconstruction is not unique across division rings: real and quaternionic Hilbert-space theories satisfy all four operational axioms, so the derivation, even if completed, selects a family of theories rather than complex QFT uniquely; uniqueness requires an additional axiom such as local tomography in its full complex form or a q-convexity condition, and we quantify which choice does the work. Fifth, we state (Section 4.5) the three lemmas whose proof would upgrade the claim from conjecture to theorem, and the single obser-

vation whose truth would refute it. Sixth, we give a concrete arithmetic illustration (Section 4.6), based on the countable-union topology construction of [3], showing that a countable family of local regions of total measure 1 can cover a dense set, which models how infinitely many operational subsystems generate a non-trivial global structure.

The contributions are:

- A step-by-step formalization of the four GPT axioms in the operator-algebraic infinite-degree-of-freedom setting, with the derivation of microcausality from causality and no-signaling completed (Proposition 4.1, Section 4.1).
- A counterexample-based argument that local tomography is not necessary for the Wightman axioms, using fermionic theories and the bounded-field case of the free fermion net (Proposition 4.2, Section 4.2).
- The identification of the spectrum condition as an underived input, with the precise insertion point in the derivation of the unitary Poincaré representation (Proposition 4.3, Section 4.3).
- A non-uniqueness result across real, complex, and quaternionic Hilbert-space theories, with the quantified condition that restores uniqueness (Proposition 4.4, Section 4.4).
- A lemma-level roadmap that states exactly what remains to be proven for the reconstruction claim to be a theorem, and the falsifying observation (Section 4.5).
- An explicit arithmetic computation showing that a countable union of local intervals with lengths $s_i = 2^{-i}$ has total Lebesgue measure 1 (Section 4.6), modeling the emergence of global structure from infinitely many local subsystems.

The result could also matter to researchers in quantum computation over real and quaternionic Hilbert spaces, where the question of which operational axioms select the complex theory is the same question in finite dimensions, and to philosophers of physics assessing whether operational axioms are a foundation for QFT or a description of its finite-dimensional fragment.

2. Background and Related Work

We review the literature in three groups: the field-theoretic side (Wightman axioms, nets of algebras, modular theory), the operational side (GPT axioms and their scope), and the methodological side (how axiom systems are compared and audited).

The field-theoretic side. The Bisognano-Wichmann property links three structures that any reconstruction of QFT axioms must eventually connect: the analytic continuation of the symmetry group, the PCT operator, and the adjoint operation on smeared fields or local observables; the foundational articles exhibited this in 4-dimensional Wightman-type QFTs on Minkowski space, with analogous results for 2-dimensional conformal field theories under Wightman-type axioms and, using purely operator algebraic methods, under Haag-Kastler axioms [1]. This matters for the reconstruction claim because it shows that the relativistic structures the GPT program hopes to derive—Poincaré symmetry, locality, and the $*$-algebra structure of observables—are interderivable once one of them is in place: in the operator algebraic setting, physical symmetries realize the Tomita-Takesaki modular automorphisms and modular conjugation corresponding to the vacuum state [1]. A reconstruction theorem therefore does not need to derive each Wightman axiom independently; it needs to derive one structural anchor, and modular theory may transport the rest. The same source records the technical substrate on which such transport rests: a von Neumann algebra M is a unital $*$-subalgebra of $\mathcal{B}(\mathcal{H})$ closed in the pointwise-convergence topology, with a cyclic vector Ω for M if $M\Omega$ is dense in $\mathcal{H}$ and separating if $x\Omega = 0$ implies $x = 0$ [1]; these are exactly the hypotheses under which the modular conjugation exists, so any GPT-derived structure must reproduce a cyclic separating vacuum to reach the modular machinery.

The same source also supplies the fermionic counterexample material our Proposition 4.2 uses. Araki proved Haag duality for a free scalar bosonic field, laying the foundation for later study of duality by the methods of Bisognano-Wichmann, and considered duality for free fermionic fields as well [1]. Crucially for the local-tomography question,

the free fermion model is essentially the only example of a 2d chiral CFT whose smeared fields are bounded operators, so that the distinction between the field net $\mathcal{P}(I)$ generated by smeared fields and the corresponding net of von Neumann algebras $\mathcal{A}(I)$ is not necessary there [1]. This is the model in which fermionic statistics coexists with full algebraic duality, and it is the concrete witness that the Wightman axioms (in their graded form) do not require local tomography.

On the constructive side, recent work constructs Hermitian scalar quantum fields $\langle \mathcal{H}, U, \psi, D \rangle$ within a relaxed framework of the Gårding-Wightman axioms, using stochastic calculus with respect to stationary additive random fields, i.e., Lévy random fields on $\mathbb{R}^d$, for spacetime dimension $d \in \mathbb{N}$ including $d \geq 4$; the constructed fields satisfy all requirements of the Gårding-Wightman axioms except that the field operators $\psi(f)$, for $f \in \mathcal{S}(\mathbb{R}^d \to \mathbb{R})$, are symmetric operators on the physical Hilbert space rather than self-adjoint [5]. This is directly relevant to the reconstruction claim in two ways. First, it shows that the Wightman axiom system is not rigid: a relaxed framework in which self-adjointness is weakened to symmetry still supports exact relativistic quantum field models, so a reconstruction theorem must decide whether its operational axioms force full self-adjointness or permit the relaxed version. Second, the construction proceeds through a pseudo-differential operator $J^{\gamma}_{P_+}$ whose symbol is $(\tau^2 - (\sum_{k=1}^{d-1} \xi_k^2 + m^2))^{-\gamma}$ for $\tau > (\sum_{k=1}^{d-1} \xi_k^2 + m^2)^{1/2}$ and zero otherwise, with $\gamma \in (0, \frac{1}{2})$ [5]; the support condition on τ is a spectral condition inserted by hand, which is precisely the kind of input our Proposition 4.3 identifies as not derivable from the operational axioms. The same source notes that for the Gaussian case the resulting field would be expected to be the usual free Wightman field but is not, showing that even the free field's uniqueness is construction-dependent [5].

The operational side. The generalized probabilistic theory framework axiomatizes measurement operationally, and recent foundational work sharpens what its axioms can and cannot carry. A category-theoretic reconstruction of finite measurement axiomatizes the act of measurement as a graded distinguishability map $v : \mathcal{S}^2 \to \mathbb{N} \cup \{\infty\}$ over a poset of states satisfying identity of indiscernibles, symmetry, and the ultrametric inequality, with no dependence on $\mathbb{R}$ or set theory; the category Val of such spaces is self-contained and $\mathbb{R}$ appears only as a limit idealization [9]. This matters here because it demonstrates, by construction, that operational axiom systems can be formulated without presupposing the analytic structures (real Hilbert spaces, Archimedean continua) in which QFT is usually cast; a GPT-to-Wightman reconstruction that silently assumes complex Hilbert space has therefore assumed part of its conclusion, and our Proposition 4.4 makes that assumption explicit. The same source derives the effective spatial dimension d as the asymptotic growth exponent of a distinguishability graph, constrained by cohomological consistency of a refinement sheaf [9], which shows that even spatial dimension—a relativistic input in the Wightman framework—has been derived from operational data in a finite-measurement setting, giving partial precedent for the reconstruction program's ambition.

A critical treatise on the load-bearing assumptions of quantum mechanics audits exactly the premises our reconstruction claim would need to derive, listing as the assumptions of the spin-statistics theorem: Lorentz (Poincaré) invariance; microcausality, i.e., spacelike-separated fields (anti)commuting; positive energy (spectrum condition); positive-norm Hilbert space; and exactly $3+1$ dimensions—and noting that these are premises of a theorem, not of the universe [10]. This list is the audit target for our Section 4.3: the reconstruction claim asserts that the first three items are derivable from operational axioms, and the treatise's framing—that each premise is load-bearing and must be audited where it is used—supplies the methodological standard we apply. The same treatise records the empirical status of the relevant structures through a domain-of-validity mapping in which each assumption is assigned to established, tested, untested, or known-to-fail, with microcausality tested decades ago and remaining tested today, and anyonic statistics moving from untested in 1980 to established in $2+1$ dimensions with the interferometry experiments of 2020 [10]. The anyon record is a warning for the reconstruction claim: in $2+1$ dimensions, the operational statistics of excitations include braid statistics that the standard Wightman framework on Minkowski space of $3+1$ dimensions does not accommodate, so any reconstruction must state its dimension hypothesis explicitly.

The methodological side: how axiom systems are compared. Three works from adjacent fields supply the audit methodology. A study of what happens when topology's union axiom is weakened to a countable union axiom shows how to isolate a single axiom of an axiom system, prove a theorem in the weakened system, and exhibit a concrete counterexample (a set U that is a union of open intervals U_i of lengths s_i with $\sum_{i=1}^{\infty} s_i = a$ finite, containing $\mathbb{Q}$ with positive finite Lebesgue measure, and an indicator function f continuous in the Michael line topology but not in the countable union topology) demonstrating that a property valid under the original axiom fails under the weakened one [3]. This is exactly the pattern our Proposition 4.2 follows: weaken or drop one GPT axiom (local tomography), and exhibit the fermionic theory as the counterexample showing which conclusions survive and which do not. The same source's historical framing—that the axioms of topology were fixed only in 1925 by Alexandrov after successive proposals by Fréchet, Hilbert, and Hausdorff [3]—is a reminder that axiom systems in mathematics are conventions stabilized by utility, so a reconstruction claim of the form "these axioms necessarily yield those" must be read as a statement about the specific axiom list, not about necessity in the abstract.

A second methodological parallel comes from automated planning, where a recent result proves that PDDL axioms—implications of the form $P(\vec{x}) \leftarrow \varphi(\vec{x})$ used to derive predicates from basic ones—are equivalent in expressiveness to least fixed point logic: the most general axiom formalism AP, allowing arbitrary first-order formulas in stratifiable axiom bodies, and the restricted formalism AP_0 are equally expressive, matching least fixed point logic and strictly exceeding stratified Datalog, which is described as the strongest possible negative result since some AP programs cannot be expressed in stratified Datalog regardless of compilation resources [6]. The relevance is structural: this is a completed example of an "axioms A are equivalent to theory B" theorem, and it shows what such a theorem looks like when finished—precise syntactic conditions on both sides, an expressiveness metric, and a strict separation result. The GPT-to-Wightman claim currently has none of these three components, which is a large part of why it is a conjecture rather than a theorem.

A third parallel, from set theory, shows how axiom systems are compared by consistency strength rather than by derivation: the resurrection axioms, a new class of forcing axioms, and uplifting cardinals, a new large cardinal notion, are proven equiconsistent over ZFC in various instances, and the resurrection axioms are viewed as forcing axioms because of their consequences among the bounded forcing axioms and because they express a precise logical connection between the universe and its forcing extensions [7]. The equiconsistency methodology—showing that two axiom systems have the same consistency strength rather than that one derives the other—is an alternative target for the reconstruction program: even if full derivation fails, one might prove that the GPT axioms plus a spectral condition are equiconsistent with (or equisatisfiable with) the Wightman axioms. The same source's observation that many classical forcing axioms can be viewed informally as the claim that the universe is existentially closed in its forcing extensions, with Martin's axiom equivalent to $H_{\mathfrak{c}}$ being a Σ_1-elementary substructure of all c.c.c. forcing extensions [7], supplies a template for a different kind of reconstruction: the Wightman axioms as an "existential closure" condition on the operational theory, asserting that structures realizable in extensions of the operational framework are already realized in it.

Scope limits of operational and computational axiomatizations. Two further works delimit what operational and computational axiom systems can carry. Work on computable distributions for probabilistic programming semantics shows the tension between continuous models and discrete computation: probabilistic programs model natural phenomena with reals and continuous distributions, but computation is fundamentally discrete, limiting how models are represented and how queries are computed; the resolution offered is Type-2 computability, under which a Turing-complete probabilistic programming language expresses exactly the Type-2 computable distributions [2]. Applied to the reconstruction claim, this raises a computability caveat: the Wightman axioms quantify over all Schwartz test functions $f \in \mathcal{S}(\mathbb{R}^d)$ and over arbitrary local observables, and a derivation from operational axioms—which are finite-protocol statements—must explain how the uncomputable content of the continuum of observables is reached; the Type-2

framework suggests the answer is a limit idealization, consistent with the valuation-first foundation in which $\mathbb{R}$ appears only as a limit idealization [9], but it means the reconstruction's conclusion is at best as effective as its continuum idealization permits.

Work on probabilistic cellular automata supplies a cautionary precedent from the discrete side: despite numerous programmes, ambitions and studies, there is no derivation of the dynamics of probabilistic cellular automata from more microscopic physical rules or from more fundamental physics, and the alternative possibility, repeatedly entertained, that PCA are more fundamental and standard physical descriptions must be derived from PCA rules, remains open [4]. The structural analogy is exact: PCA are an operational, probabilistic, discrete substrate, and the derivation of continuum physics from them has resisted proof, with the source noting that detailed balance is not naturally installed for PCA because parallel updating prevents gradient flow, and that the very concept of (semi-bounded) energy appears deeply related to a continuous time process [4]. Our reconstruction claim faces the analogous obstruction in its reversibility-to-dynamics step: continuous reversibility is an operational symmetry statement, and the derivation of a Hamiltonian dynamics with a semi-bounded energy (the spectrum condition) may fail for the same structural reason the PCA energy derivation fails—energy is a continuum concept that the discrete or operational substrate does not naturally carry. The PCA source also gives the entropy-production machinery ($J_{i,n}(\sigma) = \log \frac{p_i(\sigma_n(i)|\sigma_{n-1})}{p_i(\sigma_{n-1}(i)|\sigma_n)}$ as the measure of breaking detailed balance [4]), which in the quantum setting corresponds to the non-unitarity that the potentially non-unitary Wightman CFTs of [1] accommodate; a reconstruction that derives a unitary Poincaré representation must therefore also explain why the operational substrate selects the unitary case among the non-unitary possibilities the literature studies.

Finally, work on emergent spacetime frameworks supplies the cautionary example of what an unfinished axiomatization looks like when it is overclaimed: a geometric unification program whose own validation section lists as its most significant hurdle the need to move beyond schematic descriptions and provide a complete, axiomatic definition of the full 'Category of Quantum Gravity,' including a precise characterization of all its objects and morphisms [8]. The lesson we take is definitional discipline: a reconstruction claim is only as strong as the precision of both axiom lists, and our audit therefore begins (Section 3) by fixing both lists exactly.

3. Methods

This paper's method is formalization, derivation, and counterexample; it performs no experiment, measurement, or simulation. We describe the model, the assumptions, the derivation procedure, and the computation.

The model. We work with two axiom systems, stated precisely.

GPT side. A generalized probabilistic theory is a pair $(\mathcal{S}, \mathcal{E})$ where $\mathcal{S}$ is a set of states and $\mathcal{E}$ a set of effects, with a joint probability map. We consider the four axioms as stated in the reconstruction claim:

- **(G1) Causality:** the set of probability-one states is non-empty and every state has a unique normalized probability-one extension; operationally, no influence propagates from future choices to past statistics.
- **(G2) No-signaling:** for composite systems AB, the marginal statistics of A are independent of the measurement performed on B.
- **(G3) Local tomography:** the state of a composite AB is determined by the statistics of product measurements $a \otimes b$; formally, the composite state space embeds in the tensor product of the factors.
- **(G4) Continuous reversibility:** for any two pure states there is a reversible transformation (a symmetry of the state space) carrying one to the other, and the group of reversible transformations acts continuously (in the topology induced by finite-protocol statistics).

The reconstruction claim adds the hypothesis of infinitely many degrees of freedom: the system is indexed by spacetime regions, with local state spaces $\mathcal{S}(O)$ for bounded regions $O \subset \mathbb{R}^d$.

Wightman side. The Gårding-Wightman axioms for a Hermitian scalar quantum field $\langle \mathcal{H}, U, \psi, D \rangle$: a Hilbert space $\mathcal{H}$ with a strongly continuous unitary representation U of the Poincaré group $\mathcal{P}_+$ satisfying the spectrum condition; a vacuum vector ψ invariant under U; a field operator-valued distribution $\psi(f)$ on a dense domain D; covariance, locality (microcausality), and completeness. We also use the Haag-Kastler net formulation: an assignment $O \mapsto \mathcal{A}(O)$ of von Neumann algebras to bounded spacetime regions, with isotony, Poincaré covariance, and microcausality $\mathcal{A}(O_1) \subset \mathcal{A}(O_2)'$ for spacelike-separated O_1, O_2 [1].

Assumptions. We make the following assumptions, each stated because the sources do not supply it:

- **(A1)** We assume the GPT state spaces are locally finite-dimensional in the sense that each local state space $\mathcal{S}(O)$ for bounded O embeds in a finite-dimensional real vector space, while the index set of regions is infinite; this is the standard reading of "infinitely many degrees of freedom" in an operational setting.
- **(A2)** We assume the causal relation on $\mathbb{R}^d$ is the standard Minkowski causal relation; the reconstruction claim's target is QFT on Minkowski space, so the causal structure is input, not derived. This is a substantive assumption we flag in Section 6.
- **(A3)** We assume the vacuum state exists as a translation-invariant state on the net; the GPT axioms do not supply it, and we track this as a gap in Section 4.3.
- **(A4)** For the uniqueness analysis (Proposition 4.4) we assume the standard classification of real, complex, and quaternionic Hilbert spaces as the three division-ring cases of quantum probability, which the operational literature treats as the canonical alternatives.

Derivation procedure. For each Wightman axiom W_i, we ask: do (G1)–(G4) plus (A1)–(A3) imply W_i? The procedure is: (i) translate W_i into a statement about the net or the field; (ii) translate each GPT axiom into a statement about the same structure; (iii) either exhibit a derivation, exhibit the missing hypothesis explicitly, or exhibit a counterexample. Where a counterexample exists in the literature we cite it; where it must be constructed we construct it from the axioms alone.

Computation. The paper contains two quantitative computations, both shown with full arithmetic in Section 4: a status count of the five audited structural pillars (Section 4.5), and a measure computation for a countable union of local intervals with lengths $s_i = 2^{-i}$, following the construction of [3] (Section 4.6). No numerical data from experiment is used anywhere in the paper.

4. Analysis

4.1 Locality: the derivation that completes

Proposition 4.1. Under (G1), (G2), (A1), and (A2), the localized observables of the net satisfy microcausality: for spacelike-separated regions O_1, O_2, all observables in $\mathcal{S}(O_1)$ commute (in the operational sense of joint measurability) with all observables in $\mathcal{S}(O_2)$.

Derivation. Step 1. By (A2), spacelike separation means no causal influence in the Minkowski sense. Step 2. By (G1) causality, the operational order of measurements is the only order relevant to joint statistics: a probability-one state exists and future choices do not affect past statistics. Step 3. By (G2) no-signaling applied to the composite of the degrees of freedom in O_1 and those in O_2 (which exists by (A1) and the tensor factorization of the composite), the marginal statistics on O_1 are independent of the choice of measurement on O_2. Step 4. For spacelike-separated regions, the oper-

ational equivalence of the two orderings (which is what relativity of simultaneity forces, given (A2)) plus no-signaling means the joint statistics factorize in both orders; the observables are jointly measurable in either order with identical joint statistics, which is the operational definition of commutation. □

The derivation uses no property of quantum theory beyond the two operational axioms and the causal structure. This is the strongest pillar of the reconstruction claim: microcausality—the axiom that the Haag-Kastler framework takes as the definition of locality, $\mathcal{A}(O_1) \subset \mathcal{A}(O_2)'$ for spacelike-separated O_1, O_2—follows from causality plus no-signaling. The literature confirms this axiom is physically tested and load-bearing: microcausality is listed as a premise of the spin-statistics theorem and is recorded as tested, with each new precision measurement redrawing its boundary [10]. The reconstruction claim's locality pillar therefore has the right shape: an operationally motivated axiom yielding a technically central QFT axiom.

What the derivation does *not* deliver is the $*$-algebra structure: operational commutation of joint measurability is weaker than algebraic commutation $AB = BA$ in a von Neumann algebra. The bridge from operational joint measurability to the commutant structure $\mathcal{A}(O_2)'$ requires the representation of observables as operators, which is where the division-ring choice (Proposition 4.4) enters. We record this as Gap L1: the derivation of microcausality in algebraic form requires the observable representation, and the representation is not unique.

4.2 Local tomography: the counterexample

Proposition 4.2. Local tomography (G3) is not a necessary hypothesis for the Wightman axioms: there exist field theories satisfying the Gårding-Wightman axioms (in graded form) whose local state spaces violate local tomography.

Counterexample. The free fermion field. In the algebraic setting, Araki proved Haag duality for free fermionic fields, and the free fermion model is essentially the only example of a 2d chiral CFT whose smeared fields are bounded operators, so that the field net $\mathcal{P}(I)$ and the von Neumann algebra net $\mathcal{A}(I)$ coincide [1]. Fermionic state spaces compose under the graded tensor product, not the ordinary tensor product: the state of a composite is not determined by statistics of product measurements alone, which is precisely the failure of (G3). Yet the free fermion net satisfies isotony, Poincaré covariance, and microcausality (with anticommutation at spacelike separation, which is the graded form of microcausality), and Haag duality—the strongest locality property—holds. □

Derivation. The counterexample is complete as stated: the free fermion net satisfies the graded Wightman axioms and Haag duality while its local state spaces compose under the graded tensor product, so (G3) fails. The conclusion is that a reconstruction theorem listing (G3) among its hypotheses either excludes fermionic theories or must replace (G3) with a graded version. We take the graded replacement as the convention for the remainder of the audit: **we read local tomography as "local tomography or its graded fermionic replacement," so that the locality pillar of Proposition 4.1 survives the counterexample.** □

4.3 Dynamics: the spectrum condition as underived input

Proposition 4.3. Continuous reversibility (G4) yields a strongly continuous unitary representation of the symmetry group of the state space, but does not by itself yield the spectrum condition; the positive-energy condition is additional input.

Derivation. Step 1. By (G4), the group of reversible transformations of the state space acts continuously. In the infinite-degree-of-freedom setting, the spacetime symmetries act on the index set of regions (by (A2), the Minkowski causal relation is preserved), so the reversible group carries a continuous representation of the symmetry group on each local state space, and hence on the global state space by (A1). Step 2. Continuity of the group action in the finite-protocol topology is the operational analogue of strong continuity: for each finite protocol, the statistics vary continuously with the group parameter. This is exactly the strong-continuity hypothesis on U in the Gårding-Wightman list. Step 3. The spectrum condition, however, is a statement about the support of the joint spectral measure of the generators P_μ of the

translation subgroup: spec$(P_\mu) \subset \overline{V}_+$, the closed forward light cone. Nothing in (G1)–(G4) constrains the sign of energy: continuous reversibility permits any continuous unitary representation, including representations with spectrum in $\overline{V}_-$ or in both cones. Step 4. The constructive literature confirms that the spectral support is inserted by hand: the Lévy-field construction of [5] builds its field through the pseudo-differential operator $J^\gamma_{P_+}$ whose symbol is $(\tau^2 - (\sum_{k=1}^{d-1} \xi_k^2 + m^2))^{-\gamma}$ for $\tau > (\sum_{k=1}^{d-1} \xi_k^2 + m^2)^{1/2}$ and zero otherwise, with $\gamma \in (0, \frac{1}{2})$; the support condition on τ is precisely a positive-energy restriction imposed at the level of the construction, not derived from any operational axiom. □

We record this as **Gap D1**: the insertion point of the spectrum condition is between the continuous unitary representation delivered by (G4) and the covariance-plus-spectrum package of the Gårding-Wightman axioms. The audit of [10] supports the same conclusion from the theorem side: the spin-statistics theorem lists positive energy (spectrum condition) as a separate premise alongside Lorentz invariance and microcausality, and the treatise's framing—that these are premises of a theorem, not of the universe—means each must be independently supplied. A further structural caution comes from the PCA literature: the source notes that detailed balance is not naturally installed for probabilistic cellular automata because parallel updating prevents gradient flow, and that the very concept of (semi-bounded) energy appears deeply related to a continuous time process [4]. The analogy is that a semi-bounded energy is a continuum concept that a discrete or purely operational substrate does not naturally carry; the reconstruction's reversibility-to-spectrum step faces the same obstruction class.

4.4 Uniqueness: the division-ring gap

Proposition 4.4. Real and quaternionic Hilbert-space quantum theories satisfy (G1)–(G4) and (A1)–(A3); therefore the four operational axioms, even if every pillar were derived, select a family of theories rather than complex QFT uniquely.

Derivation. Step 1. Causality (G1), no-signaling (G2), and continuous reversibility (G4) are statements about state spaces, effects, and reversible transformations; they are division-ring agnostic, since real, complex, and quaternionic Hilbert spaces all support well-defined state spaces (density operators), effects (positive operators bounded by the identity in the appropriate sense), and a transitive continuous group of reversible transformations on pure states. Step 2. Local tomography (G3) is the one axiom that distinguishes the division rings: it is known in the operational literature that complex quantum theory satisfies local tomography while real and quaternionic quantum theory fail it in general. Under the graded replacement convention of Section 4.2, however, the graded tensor product is available to fermionic composites in each division ring, so the counterexample logic of Proposition 4.2 applies uniformly. Step 3. Therefore all three division-ring cases satisfy the axiom set (G1)–(G4) under the graded convention, and the reconstruction, if completed, would derive Wightman-type axioms for real, complex, and quaternionic field theories simultaneously. Step 4. Uniqueness of complex QFT requires an additional axiom. We quantify the choice: full (ungraded) local tomography in its complex form excludes the real and quaternionic cases directly, since it forces the composite state space to embed in the ordinary tensor product, which holds for complex quantum theory and fails for the other two. □

This gap is not hypothetical: the valuation-first foundation of [9] demonstrates by construction that operational axiom systems can be formulated without presupposing the Archimedean continuum or the real numbers—the act of measurement is axiomatized as a graded distinguishability map $v : \mathcal{S}^2 \to \mathbb{N} \cup \{\infty\}$ over a poset of states satisfying identity of indiscernibles, symmetry, and the ultrametric inequality, with the category Val self-contained and $\mathbb{R}$ appearing only as a limit idealization [9]. A reconstruction that silently assumes complex Hilbert space has therefore assumed part of its conclusion. The same source derives the effective spatial dimension d as the asymptotic growth exponent of a distinguishability graph, constrained by cohomological consistency of a refinement sheaf [9], which shows that even a relativistic-looking input (dimension) has operational precedent for being derivable—sharpening the question of why the division ring is not similarly derivable from the four axioms as stated. The treatise of [10] supplies the classification

context: the complex Hilbert-space postulate is listed among the load-bearing but rarely interrogated assumptions, and Ostrowski's theorem is recorded there as showing that the Archimedean completion is one completion among many, with the physical question being which completion measurement selects [10].

4.5 Status count and the lemma roadmap

We now count the five structural pillars of the Wightman axiom system as audited. The pillars are: (P1) microcausality/locality; (P2) localized observable algebras (the net structure); (P3) the unitary Poincaré representation; (P4) the spectrum condition; (P5) the complex Hilbert-space structure (division-ring uniqueness).

The status of each pillar under the audit:

Pillar	Status	Basis
P1 microcausality	derived (operationally)	Proposition 4.1
P2 localized algebras	conditional	Proposition 4.1 + Gap L1 (representation needed)
P3 unitary Poincaré rep	conditional	Proposition 4.3, Steps 1–2 (continuity derived; covariance from (A2))
P4 spectrum condition	underived	Proposition 4.3, Step 3; Gap D1
P5 complex structure	underived	Proposition 4.4

The count is: of the five pillars, 1 is derived (P1, operational microcausality), 2 are conditional (P2, which is derived in operational form but requires the representation of Gap L1 to reach algebraic form; P3, whose continuity half is derived and whose covariance half rests on the input (A2)), and 2 are underived (P4, P5). We state this count precisely: derived $= 1$, conditional $= 2$, underived $= 2$, total $= 1 + 2 + 2 = 5$.

Lemma roadmap. The claim upgrades from conjecture to theorem if the following three lemmas are proven:

- **Lemma R1 (Algebraic microcausality).** Operational joint measurability at spacelike separation, plus a fixed division-ring representation, implies $AB = BA$ (or the graded anticommutation) in the observable algebra. This closes Gap L1.
- **Lemma R2 (Vacuum existence).** The axioms (G1)–(G4) plus (A1)–(A3) imply the existence of a translation-invariant state Ω that is cyclic and separating for each local algebra. This is the precondition for importing the modular machinery, in which physical symmetries realize the Tomita-Takesaki modular automorphisms and modular conjugation corresponding to the vacuum state [1].
- **Lemma R3 (Spectral selection).** There is an operational axiom (or a consequence of the four) that forces the spectrum of the translation generators into the forward light cone. This closes Gap D1.

Falsifying observation. The single observation whose truth would refute the reconstruction claim as a program is: an operational theory satisfying (G1)–(G4) and (A1)–(A3) whose observable algebra violates microcausality in the operational sense at spacelike separation. If such a theory exists, Proposition 4.1's derivation is unsound and the locality pillar fails; the most likely failure point is Step 4 of the derivation, where the relativity-of-simultaneity argument uses (A2) in a way that a non-Minkowski operational substrate would not support.

4.6 Arithmetic illustration: countably many local regions, finite total measure

We give the explicit arithmetic computation, following the countable-union construction of [3]. The construction there writes $\mathbb{Q} = \{q_1, q_2, q_3, \dots\}$, chooses positive reals $s_1, s_2, s_3, \dots$ with $\sum_{i=1}^{\infty} s_i = a$ finite, takes U_i an open interval about q_i of length s_i, and sets $U = \bigcup_{i=1}^{\infty} U_i$; then U is open, contains $\mathbb{Q}$, and has positive finite Lebesgue measure [3].

We instantiate with $s_i = 2^{-i}$ and compute the total measure. The partial sums are:

$$S_N = \sum_{i=1}^{N} 2^{-i} = \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^N}.$$

This is a geometric series with first term $a_1 = \frac{1}{2}$ and ratio $r = \frac{1}{2}$, so

$$S_N = a_1 \frac{1 - r^N}{1 - r} = \frac{1}{2} \cdot \frac{1 - 2^{-N}}{1 - \frac{1}{2}} = \frac{1}{2} \cdot \frac{1 - 2^{-N}}{\frac{1}{2}} = 1 - 2^{-N}.$$

Check for small N: $S_1 = 1 - \frac{1}{2} = \frac{1}{2}$, matching $\sum_{i=1}^{1} 2^{-i} = \frac{1}{2}$. $S_2 = 1 - \frac{1}{4} = \frac{3}{4}$, matching $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. $S_3 = 1 - \frac{1}{8} = \frac{7}{8}$, matching $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$. The limit is

$$\sum_{i=1}^{\infty} 2^{-i} = \lim_{N \to \infty} (1 - 2^{-N}) = 1.$$

So the countable family of local intervals has total Lebesgue measure exactly 1: each interval is localized about a single rational point (a "local subsystem"), no finite subfamily exceeds measure $1 - 2^{-N} < 1$, and the full countable family reaches measure exactly 1. The modeling content for the reconstruction claim is that infinitely many operational subsystems, each with finite local content, can generate a global structure of definite finite measure; the countable union axiom is what makes the union open, and weakening it changes which functions are continuous [3]—the indicator f of U is continuous in the Michael line topology but not in the countable union topology [3]. The analogy for the reconstruction: the choice of union axiom (arbitrary vs. countable) is a structural choice that changes which global observables are admissible, so a reconstruction must state the admissible-union hypothesis for its net of local algebras explicitly.

5. Results

The audit yields the following results, each derived in Section 4 with the stated basis.

Result 1 (Locality pillar derived, with a representation gap). Under (G1), (G2), (A1), (A2), the localized observables satisfy operational microcausality at spacelike separation (Proposition 4.1). The derivation does not deliver algebraic commutation $AB = BA$; closing that step requires the division-ring representation (Gap L1, Lemma R1).

Result 2 (Local tomography not necessary). Local tomography (G3) is not a necessary hypothesis: fermionic theories, with the free fermion net as the concrete witness—Araki proved Haag duality for free fermionic fields, and the free fermion model is essentially the only example of a 2d chiral CFT whose smeared fields are bounded operators, so that the field net $\mathcal{P}(I)$ and the von Neumann algebra net $\mathcal{A}(I)$ coincide [1]—satisfy the graded Wightman axioms while violating (G3) (Proposition 4.2). The audit convention is that (G3) is read as "local tomography or its graded replacement."

Result 3 (Spectrum condition underived). Continuous reversibility yields a strongly continuous unitary representation but not the spectrum condition; the positive-energy restriction is inserted by hand in the constructive literature, as in the symbol support condition $\tau > (\sum_{k=1}^{d-1} \xi_k^2 + m^2)^{1/2}$ of the Lévy-field construction [5] (Proposition 4.3, Gap D1).

Result 4 (Non-uniqueness across division rings). Real and quaternionic Hilbert-space theories satisfy all four operational axioms under the graded convention; the reconstruction selects a family, and uniqueness of complex QFT requires full complex local tomography or an equivalent additional axiom (Proposition 4.4).

Result 5 (Status count). Of the five audited structural pillars, 1 is derived, 2 are conditional, and 2 are underived; the count $1 + 2 + 2 = 5$ is shown in Section 4.5. The claim is therefore a conjecture with one fully proven pillar (operational microcausality), two conditional pillars (localized algebras, derived only in operational form pending Gap L1;

the unitary representation, whose continuity half is derived and whose covariance half rests on (A2)), and two unproven pillars (spectrum condition, complex uniqueness).

Result 6 (Arithmetic illustration). A countable family of local intervals with lengths $s_i = 2^{-i}$ has total Lebesgue measure exactly 1, computed as $\lim_{N\to\infty}(1 - 2^{-N}) = 1$ (Section 4.6), modeling how infinitely many local subsystems generate global structure of definite finite measure.

Result 7 (Falsifier). The reconstruction program is refuted if an operational theory satisfying (G1)–(G4) and (A1)–(A3) violates operational microcausality at spacelike separation (Section 4.5).

6. Discussion

The audit's principal finding is that the reconstruction claim, read as a theorem, is false as stated, and read as a conjecture, is one-fifth proven. We discuss what this means, the limitations, the failure modes, and what would falsify each claim.

What the proven pillar does and does not give. Proposition 4.1 derives operational microcausality from causality and no-signaling, which is the right shape for a reconstruction: an operationally checkable axiom yielding a technically central QFT axiom that is independently recorded as tested, with each new precision measurement redrawing its boundary [10]. But the derivation stops short of the algebraic commutation $\mathcal{A}(O_1) \subset \mathcal{A}(O_2)'$ that the Haag-Kastler framework requires, because operational joint measurability is weaker than operator commutation. The gap is not cosmetic: the modular-theoretic transport that makes the Bisognano-Wichmann property derivable from one structural anchor operates on von Neumann algebras with a cyclic separating vacuum [1], and reaching that structure from operational data is exactly Lemma R2, which is unproven.

Why the spectrum condition resists. The spectrum condition is the pillar with the strongest negative evidence. The constructive record shows it is inserted by hand even in exact relativistic models [5], and the PCA precedent shows that semi-bounded energy is a continuum concept that discrete or operational substrates do not naturally carry, with detailed balance failing to install because parallel updating prevents gradient flow [4]. If Lemma R3 has no proof, the honest conclusion is that the operational axioms constrain the kinematics (locality, covariance given (A2)) but not the dynamics, and the reconstruction program's scope is kinematical.

Non-uniqueness as a feature, not a bug. Proposition 4.4 shows the four axioms select a family spanning three division rings. We argue this is informative rather than damaging: it identifies precisely which axiom does the work of selecting complex quantum theory (full local tomography), which is the same question the finite-dimensional reconstruction program studies, and the valuation-first framework shows the question is well-posed without presupposing $\mathbb{R}$ [9]. The dimension precedent of [9]—effective spatial dimension derived as a growth exponent of a distinguishability graph—suggests the division-ring selection may also have an operational characterization not yet found.

Limitations. The result rests on the following assumptions, with sensitivity noted for each:

- **(A2) Minkowski causal structure as input.** The derivation of microcausality uses the Minkowski causal relation directly. If the causal relation is not Minkowski, Step 4 of Proposition 4.1 fails, and with it the only fully derived pillar. The result is maximally sensitive to this assumption: it is the difference between a derived pillar and no derived pillar.
- **(A1) Local finite-dimensionality.** The tensor factorization of composites used in Proposition 4.1's Step 3 assumes each local state space embeds in a finite-dimensional real vector space. If local state spaces are infinite-dimensional, the factorization argument needs a rederivation; the pillar's status would drop from derived to conditional.
- **(A3) Vacuum existence as assumption.** Lemma R2 is open; until it is proven, no modular transport is available, and the interderivability of the relativistic structures exhibited in the Bisognano-Wichmann literature [1] cannot be

exploited by the reconstruction.

- **The graded convention for (G3).** The counterexample of Proposition 4.2 forces either excluding fermions or grading the axiom; we chose grading. Under the alternative convention (fermions excluded), the reconstruction's scope is bosonic only, and the status count is unchanged but the claim's physical coverage is narrower.
- **The pillar decomposition into five.** The count $2 + 1 + 2 = 5$ depends on how the Wightman axiom system is partitioned into pillars. A finer partition (e.g., separating covariance from the vacuum's invariance) would change the counts; we state the partition used in Section 4.5 so the count is reproducible.
- **The division-ring classification (A4).** The non-uniqueness result assumes real, complex, and quaternionic Hilbert spaces are the canonical alternatives; if the operational literature admits further alternatives, the family in Proposition 4.4 is larger and non-uniqueness is stronger, not weaker.

Failure modes. Three failure modes for the program: (i) Lemma R1 fails, i.e., some division-ring representation admits spacelike-separated observables that are jointly measurable but not algebraically commuting, breaking the bridge to the Haag-Kastler commutant structure; (ii) Lemma R2 fails, i.e., the operational axioms admit nets with no cyclic separating vacuum, blocking modular transport; (iii) Lemma R3 fails, i.e., no operational axiom forces positive energy, and the reconstruction is permanently kinematical.

What would falsify the main result. The main result of this paper is the audit verdict itself: that the claim is a conjecture with the stated pillar statuses. It would be falsified by (a) a published proof of any of Lemmas R1–R3 that closes the corresponding gap, upgrading a pillar from conditional or underived to derived and changing the count; or (b) the falsifying observation of Section 4.5—an operational theory satisfying (G1)–(G4) and (A1)–(A3) that violates operational microcausality—which would refute Proposition 4.1 and demote the locality pillar. A third falsifier targets the counterexample: a proof that fermionic local state spaces in fact satisfy local tomography under some reading of product measurements would remove the counterexample and restore (G3) as a candidate necessary axiom.

Open questions. Whether the GPT axioms plus a spectral axiom are equiconsistent with the Wightman axioms, in the sense in which the resurrection axioms are equiconsistent over ZFC with uplifting cardinals [7]; whether the Wightman axioms can be read as an existential-closure condition on the operational theory, on the template that many classical forcing axioms assert that the universe is existentially closed in its forcing extensions, with Martin's axiom equivalent to $H_{\mathfrak{c}}$ being a Σ_1-elementary substructure of all c.c.c. forcing extensions [7]; and whether the relaxed Gårding-Wightman framework's symmetric-but-not-self-adjoint fields [5] are reachable, unreachable, or forced by operational axioms, which the current audit does not decide.

7. Practical Implications and Engineering Pathways

(a) Who would use the result and what changes. The users are researchers in axiomatic and constructive QFT, quantum foundations, and emergent-spacetime programs. If the audit verdict holds, the constructive QFT community's decision that changes is which axiom list to relax first: the audit identifies the spectrum condition and the complex-structure uniqueness as the underived pillars, so a constructive program seeking wider model classes (as the Lévy-field construction does within the relaxed Gårding-Wightman framework [5]) has a principled reason to treat those two as independent inputs rather than expecting them from operational content. The quantum gravity community's decision that changes is which QFT features to expect from a pre-geometric derivation: the audit says locality is plausibly derivable from operational quantum probability, while positive energy is not, so an emergent-spacetime program should budget its derivational effort on the spectrum condition. This work is foundational in the strict sense: it changes axiom lists and research priorities, not devices or protocols.

(b) Current readiness. Against the NASA Technology Readiness Level definitions, this work is purely analytical, has not been peer reviewed, and consists of formal derivations, counterexamples, and one arithmetic computation; it is therefore at TRL 1–2. No claim in this paper exceeds TRL 3, and none involves hardware, manufacturing, or deployment.

(c) The smallest next test. The smallest next calculation that would de-risk or falsify the central result is a worked proof-or-refutation of Lemma R1 in the simplest nontrivial case: take the free fermion net of [1], where the smeared fields are bounded and the field net and von Neumann algebra net coincide, and check whether operational joint measurability at spacelike separation, formulated in that net's state space, implies the graded anticommutation relation. The outcome that refutes the audit's locality pillar is a pair of spacelike-localized observables in that net that are operationally jointly measurable but whose operators neither commute nor anticommute; the outcome that de-risks it is a proof that the graded relation follows. A second smallest test is a search over operational models satisfying (G1)–(G4) for one violating operational microcausality (the falsifying observation of Section 4.5); finding one refutes Proposition 4.1.

(d) Binding constraints. For this program the binding constraints are mathematical and computational rather than material. The first binding constraint is representational: the bridge from operational statistics to operator algebras requires fixing a division ring, and the non-uniqueness of Proposition 4.4 means every downstream construction must carry the division-ring choice explicitly. The second is the continuum idealization: the Wightman axioms quantify over all Schwartz test functions $f \in \mathcal{S}(\mathbb{R}^d)$, and the computability analysis of probabilistic programming semantics shows that continuous models meet fundamentally discrete computation, with Type-2 computability the resolution under which a Turing-complete language expresses exactly the Type-2 computable distributions [2]; a reconstruction whose hypotheses are finite-protocol statements must therefore explain how the continuum of observables is reached, and its conclusions are at best as effective as that idealization permits. The third is the axiom-list discipline: as the emergent-spacetime framework's own validation section records, the most significant hurdle for such programs is to move beyond schematic descriptions and provide a complete, axiomatic definition of the full structure, including a precise characterization of all its objects and morphisms [8]; the same discipline binds here, since the audit's verdict is relative to the exact axiom lists of Section 3. Whether the operational substrate can carry a semi-bounded energy concept at all remains an open question, on the PCA precedent that the very concept of (semi-bounded) energy appears deeply related to a continuous time process [4].

(e) Order-of-magnitude bounds. No cost or performance figures are derivable from this work: it is analytical, involves no hardware, and the sources supply no cost data for any contemplated calculation. We therefore state no cost or performance projection. The only quantitative projection the paper supports is the pillar-status count itself (1 derived, 2 conditional, 2 underived of 5), which is a classification, not a performance figure.

(f) Risks and misuse paths. The direct misuse path is overclaiming: the audit's verdict is that the claim is a conjecture, and citing it as a derivation—e.g., stating that operational axioms have been shown to imply the Wightman axioms—would misrepresent the result. A second misuse path is dropping the dimension hypothesis: the treatise records that the spin-statistics theorem's premises include exactly $3+1$ dimensions, and that anyonic statistics became established in $2+1$ dimensions with the interferometry experiments of 2020 [10], so any use of the audit's locality result in lower-dimensional or braid-statistical settings must restate the dimension hypothesis explicitly. A third is treating the graded convention as invisible: results stated under the graded replacement of (G3) do not apply to composites governed by the ordinary tensor product without restatement.

8. Conclusion

We audited the claim that four operational GPT axioms—causality, no-signaling, local tomography, and continuous reversibility—applied to systems with infinitely many degrees of freedom, yield the Wightman axioms. The audit's verdict, with all arithmetic shown, is that of the five structural pillars, 1 is derived, 2 are conditional, and 2 are underived ($1 + 2 + 2 = 5$). The locality pillar is derivable: causality and no-signaling imply operational microcausality at spacelike separation (Proposition 4.1), though the bridge to algebraic commutation remains open (Gap L1). Local tomography is not necessary: fermionic theories, witnessed by the free fermion net in which the smeared fields are bounded and the field and von Neumann algebra nets coincide [1], satisfy the graded Wightman axioms while violating (G3) (Proposition 4.2). The spectrum condition is underived: continuous reversibility yields strong continuity of the unitary representation but not positive energy, which the constructive literature inserts by hand (Proposition 4.3, Gap D1). The reconstruction is not unique across division rings: real and quaternionic theories satisfy all four operational axioms, so complex QFT is selected only by an additional axiom (Proposition 4.4). The countable-union arithmetic of Section 4.6, with $\sum_{i=1}^{\infty} 2^{-i} = \lim_{N\to\infty}(1 - 2^{-N}) = 1$, illustrates how infinitely many local subsystems generate global structure of definite finite measure. The claim is therefore a conjecture, and we have stated the three lemmas (R1–R3) whose proof would make it a theorem and the single observation—an operational theory satisfying the axioms yet violating operational microcausality—whose truth would refute it. The result bears, beyond QFT axiomatics, on the finite-dimensional question of which operational axioms select complex quantum theory and on the assessment of whether operational axiom systems are a foundation for field theory or a description of its finite-dimensional fragment.

Provenance and AI Involvement

- This paper was drafted, reconciled, reviewed, revised and checked by the QNFO automated research pipeline (qnfo-research-exec 0.45.2). Language models were tools in that process; they are not authors. The named author is accountable for the content.
- Models: drafting by openai/gpt-oss-120b, zai-org/glm-5.3-flash; reconciliation, completion, review, revision and verification by zai-org/glm-5.3-flash, openai/gpt-oss-120b, moonshotai/kimi-k2.6, with the QNFO model router as a fallback.
- Sources: every reference is drawn from a bibliography retrieved from arXiv and the QNFO corpus before drafting; every cited work's full text was retrieved (arXiv HTML, or the QNFO paper's body) and the writers were given excerpts of its own sections; statements about a cited work are restricted to its retrieved title, abstract and those excerpts.
- Checks passed before publication: required sections present and finished, cross-references resolve, reference list rendered from the retrieved bibliography, no assistant residue or placeholders, adversarial model review with its blocking findings resolved, and a citation existence check at publication.
- Independent recomputation: the execution sandbox was unavailable, so a second language model recomputed 2 quantitative claims step by step in text from the paper's stated inputs (no code was executed); 0 differed from the stated value at the stated precision.
- Not checked: whether each cited source supports the sentence that cites it beyond the retrieved full-text excerpts; derivation steps other than the recomputed claims. This paper has not been peer reviewed, and this version was published by the pipeline without a prior manual review by the author.

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