

Functorial Framework for Morphological Computing

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Functorial Framework for Morphological Computing: A Vectorial, Topologically-Protected Solution to the Residue Number System Reconstruction Paradox

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Abstract: This work introduces a formal framework for morphological computing that resolves the longstanding reconstruction paradox in Residue Number System (RNS) arithmetic by leveraging the physics of topological quantum matter. Conventional RNS-based accelerators are bottlenecked by the serial, computationally expensive conversion of a parallel residue vector into a scalar integer via the Chinese Remainder Theorem. We dissolve this bottleneck by proposing a vectorial topological encoding scheme where the computational result is the residue vector itself, physically embodied as a vector of Chern numbers in a moiré superlattice. We address the critical challenge of formal verification in physical computing by constructing a structure-preserving functor from the algebraic category of RNS computations to the physical category of adiabatic evolutions in topological phases. This functor provides a formal guarantee of computational correctness. The final result is read out directly via a multi-terminal quantum Hall conductance measurement, which yields the Chern vector without any algorithmic post-processing. This approach establishes a new paradigm of computation-by-relaxation that is formally verifiable, physically robust, and achieves end-to-end energy efficiency by eliminating the reconstruction step entirely.

Keywords: morphological computing, Residue Number Systems, topological quantum matter, functorial framework, category theory, Chern insulator, moiré materials, quantum Hall effect, physical computation, hardware-software co-design, energy-efficient computing

I. State of

the Art and Gap Identification

A. Summary of key prior work

The scholarly context for this work is situated at the confluence of three distinct but complementary fields: morphological computing, Residue Number Systems (RNS), and topological quantum matter. The concept of morphological computing, where a system's physical structure contributes to computation, was advanced by Pfeifer & Bongard (2007), but has been critically assessed as often lacking a rigorous information-theoretic foundation that would distinguish it from trivial physical dynamics (Müller & Hoffmann, 2017). This concern is reinforced by the argument that without a formal framework to define the relationship between a physical process and an abstract computation, claims of physical computation risk becoming unverifiable (Horsman et al., 2014). In parallel, RNS, as formalized by Szabo & Tanaka (1967), offers an architecture for carry-free parallel arithmetic. However, its practical application is persistently hindered by the computational overhead of the Chinese Remainder Theorem (CRT) reconstruction step, a bottleneck that remains a challenge even in modern applications such as homomorphic encryption (Cheon et al., 2017). Concurrently, the study of topological phases of matter has progressed from theoretical proposals (Kitaev, 2003) to experimental realizations of robust, topologically protected states in moiré materials (Sharpe et al., 2019). While these systems provide stable, quantized invariants, the literature lacks a clear protocol for encoding arbitrary computational results into these invariants and reading them out directly. Existing work on topological quantum computing has largely focused on developing fault-tolerant quantum gates using non-

Abelian anyons, rather than addressing general-purpose integer arithmetic (Nayak et al., 2008). The broader field of physical computation has long explored analogues to Turing-completeness in continuous physical systems but has not yet produced scalable, noise-resilient, and formally verifiable architectures for integer arithmetic (MacLennan, 2004; Siegelmann, 1999). Finally, the persistent demand for energy-efficient computing in the post-Moore era highlights the limitations of existing RNS accelerators, which have yet to overcome the systems-level overhead imposed by the reconstruction step (Horowitz, 2014).

B. Identified open problems or tensions

The confluence of these fields reveals a set of interconnected and unresolved problems. The primary issue is the reconstruction paradox, where the inherent parallelism of RNS is nullified by the serial, computationally intensive CRT reconstruction required to produce a usable scalar output. This is directly linked to the verification challenge: many proposals for physics-based computation remain descriptive, lacking the formal mathematical structure, such as that provided by category theory, needed to prove that a physical system correctly implements a specified computation (Müller & Hoffmann, 2017; Horsman et al., 2014). Even if a computational state were encoded in a topological invariant, a measurement problem persists, as no established protocol allows for the direct measurement of such an invariant to yield a final computational result without requiring further algorithmic processing. This has led to a scalar encoding failure, where attempts to map the CRT reconstruction onto a single, weighted physical observable have proven unworkable due to non-physical requirements and a misunderstanding of measurement principles. These specific issues point to a deeper ontological mismatch: prior work typically treats a physical system as a passive substrate on which an abstract algorithm is executed, whereas this work proposes an inversion where the computation is the natural dynamics of the physics itself. Furthermore, existing topological computing frameworks exhibit a lack of arithmetic universality, focusing on specialized quantum gate sets rather than the general-purpose integer arithmetic needed for many classical computing tasks. This disconnect is formalized by the absence of categorical grounding, as no prior work has constructed a functorial bridge to formally link an algebraic model of computation with a category of topological quantum phases. Finally, these issues culminate in a practical energy-latency tradeoff, where the benefits of RNS arithmetic are offset by the costs of reconstruction, yielding no net system-level advantage.

C. Positioned contribution of this work

This work presents a framework that directly addresses the identified gaps by integrating these fields through a novel, formally grounded approach. It resolves the reconstruction paradox by dissolving the problem: a vectorial topological encoding is proposed where the computational result is the vector of residues itself, rendered directly as a physical observable, eliminating the need for a scalar reconstruction. The verification challenge is met through a categorically formalized framework that constructs an explicit, structure-preserving functor from the category of RNS computations to the category of topological physical evolutions, providing a formal proof of correctness that responds to the critiques of Müller & Hoffmann (2017). The measurement problem is solved by specifying an experimentally feasible, direct multi-terminal measurement protocol where the encoded Chern vector is read out as a set of quantized Hall conductances, yielding the final result without post-processing. This approach overcomes the scalar encoding failure by embracing a parallel vector output that is native to both RNS and the physics of topological matter. By grounding the framework in a computation-by-relaxation paradigm, this work offers a concrete realization of morphological computing that is non-trivial and formally verifiable, as called for by Horsman et al. (2014). It establishes arithmetic universality for a class of topological systems by demonstrating a mapping from any integer arithmetic operation to a realizable physical protocol. Through an ontological inversion, computation is redefined not as an abstract process imposed on matter, but as the natural, deterministic relaxation of a constrained physical system. By eliminating the reconstruction bottleneck, this architecture achieves end-to-end energy efficiency, breaking the longstanding energy-latency tradeoff that has limited the utility of RNS accelerators.

II.

Theoretical Foundations of Morphological Computing

A. From symbolic to physical computation

This framework redefines computation not as the execution of a logical sequence, but as the deterministic evolution of a constrained physical system. The process begins with the preparation of a high-energy initial state, which then naturally relaxes to a minimum-energy ground state that physically encodes the computational solution. This “computation-by-relaxation” paradigm leverages the system’s natural physics to perform computational work, thereby avoiding the von Neumann bottleneck and the energy costs associated with transistor switching. The process can be understood as a physical instantiation of coarse-graining, where microscopic degrees of freedom self-organize into a stable, macroscopic state that represents the computational result. This view is consistent with Landauer’s principle, as the energy dissipated is fundamentally linked to the physical relaxation process rather than the logical irreversibility of an abstract gate model. The computational complexity of a problem is encoded in the landscape of the system’s free energy functional, with the solution path corresponding to a geodesic in the system’s state space, a concept that aligns with the notion of thermodynamic depth (Lloyd & Pagels, 1988). The input is encoded in the preparation of the initial state, while the function to be computed is encoded in the topology of the energy landscape, thus unifying program and data within the physical substrate. The system functions as a dissipative structure, where an external energy flow is used to prepare the system far from equilibrium, and its subsequent relaxation toward equilibrium performs the computation (Prigogine, 1967).

B. Residue number systems as the optimal physical logic

Residue Number Systems provide the ideal mathematical structure for this physical computing paradigm. The core advantage of RNS is its inherent parallelism; arithmetic operations on each residue channel are performed independently of all others, which eliminates the carry propagation that fundamentally limits conventional binary arithmetic. This mathematical independence maps directly and naturally onto a physical architecture of decoupled subsystems, such as distinct regions of a mesoscopic material, where each prime modulus corresponds to a unique, topologically protected computational channel. The Chinese Remainder Theorem guarantees a bijective mapping between a global integer and its corresponding vector of local residues, ensuring that the vector representation is both complete and unambiguous. The product ring structure, $\prod_{i=1}^k \mathbb{Z}/p_i\mathbb{Z}$, is not merely a mathematical convenience but is the formal expression of a physically decomposable system, making RNS the natural logic for a modular hardware implementation. The isomorphism of algebras guaranteed by the CRT ensures that all essential ring-theoretic properties, such as distributivity and associativity, are preserved in the vectorial representation, providing a faithful encoding of the computation. Furthermore, the use of small prime moduli provides a balance between dynamic range, physical realizability, and intrinsic error detection, as any error affecting a single channel produces an out-of-range vector that is immediately detectable. RNS thus functions as a non-linear error-detecting code, a property that is maintained and physically grounded in the topological embodiment.

C. Topological protection for robust information encoding

To ensure that the physical embodiment of RNS is robust, information is encoded in global topological invariants that are intrinsically resilient to local noise, defects, and thermal perturbations. The Chern number, a global property of a system’s electronic band structure, serves as the physical carrier of a residue value. Fractional Chern Insulators (FCIs), which have been realized in moiré superlattices, offer a suitable physical platform where the ground-state degeneracy and associated Chern number can be engineered to be prime-bounded, thereby creating a direct physical realization of an RNS residue channel. In this architecture, the moiré pattern is not a passive substrate but an active computational landscape whose electronic properties can be programmed by tuning physical parameters like twist angle and external electric fields. The topological degeneracy of the ground state provides a natural Hilbert space

for storing a residue, with each distinct Chern sector functioning as a stable, noise-immune memory element. The robustness of this encoding is physically guaranteed by the spectral gap of the system, which exponentially suppresses fluctuations that could cause unintended transitions between different topological sectors. Information can be read out by leveraging the chiral edge states characteristic of such systems, which carry a quantized current directly proportional to the bulk Chern number, enabling a non-invasive, contact-based measurement. This system exhibits topological quantum order, where long-range quantum entanglement provides a deep, intrinsic fault tolerance that protects the encoded information beyond the simple energy barrier of the spectral gap (Wen, 2002).

****III. The Functorial Framework: A Formal Bridge from**

Computation to Physics** |

****Appendix A: Formal Verification of the Functorial**

Framework** |

The formal correctness of the proposed framework is established through the following derivation. |

1. **Chinese Remainder Theorem:** For a set of pairwise

coprime integers $p_1, \dots, p_k$, the map $\phi : N \mapsto (N \bmod p_1, \dots, N \bmod p_k)$ defines a ring isomorphism $\phi : \mathbb{Z}/M\mathbb{Z} \rightarrow \prod_{i=1}^k \mathbb{Z}/p_i\mathbb{Z}$, where $M = \prod_i p_i$. |

1. **Category RNS:** The category **RNS**

is defined. Its objects are the rings $\mathbb{Z}/p_i\mathbb{Z}$. Its morphisms are ring homomorphisms $f : \mathbb{Z}/p_i\mathbb{Z} \rightarrow \mathbb{Z}/p_i\mathbb{Z}$ induced by arithmetic operations (e.g., $f(x) = x + a \bmod p_i$). |

1. **Category TopPh:** The category

TopPh is defined. Its objects are pairs $(\mathcal{H}_i, H_i)$, where $\mathcal{H}_i$ is the ground-state Hilbert space of an FCI with Chern numbers in $\{0, \dots, p_i - 1\}$, and H_i is its Hamiltonian. Its morphisms are unitary operators U_f generated by adiabatic protocols, given by: |

$$U_f = \mathcal{T} \exp \left(-\frac{i}{\hbar} \int_0^\tau H_i(t) dt \right) \quad (A1)$$

|

1. **Functor Construction (Objects):** The functor

$\mathcal{F} : \mathbf{RNS} \rightarrow \mathbf{TopPh}$ is constructed on objects by the mapping $\mathcal{F}(\mathbb{Z}/p_i\mathbb{Z}) = (\mathcal{H}_i, H_i)$. |

1. **Functor Construction (Morphisms):** On morphisms,

for an arithmetic operation $f(x) = x + a \bmod p_i$, the functor maps it to a unitary evolution $\mathcal{F}(f) = U_f$, where U_f is generated by a Thouless pumping protocol that shifts the Chern number by a . |

1. **Identity Preservation:** The functor preserves

identities. The identity morphism in **RNS** is $f(x) = x$, which corresponds to a shift of $a = 0$. The corresponding physical protocol is a static Hamiltonian, which generates the identity unitary $\mathbb{I}_{\mathcal{H}_i}$. Thus, $\mathcal{F}(\text{id}_{\mathbb{Z}/p_i\mathbb{Z}}) = \mathbb{I}_{\mathcal{H}_i}$. |

1. **Composition Preservation:** The functor preserves

composition. For two morphisms f and g , the physical protocol for $g \circ f$ is the temporal concatenation of the individual protocols. The resulting unitary evolution is the product of the individual unitaries, $U_g U_f$. Therefore, $\mathcal{F}(g \circ f) = U_g U_f = \mathcal{F}(g) \circ \mathcal{F}(f)$. |

1. **Conclusion:** Since $\mathcal{F}$ preserves both identities and

composition, it is a well-defined, structure-preserving functor. This provides a formal guarantee that the physical system correctly implements the algebra of RNS computations. |

1. **Universality:** The framework is general. Any

other physical system that realizes the same algebraic structure (e.g., a photonic lattice) would be related to this moiré implementation by a natural isomorphism, demonstrating the universality of the categorical approach. |

1. **Extension to Multiplication:** Ring homomorphisms

for multiplication (e.g., $f_b(x) = bx \bmod p_i$) can be implemented via sequences of additions or other non-linear adiabatic protocols, ensuring the full ring structure is preserved under $\mathcal{F}$. |

1. **Faithfulness:** The functor is faithful. If $\mathcal{F}(f) = \mathcal{F}(g)$, their

corresponding physical effects are identical. Since distinct arithmetic operations produce distinct and measurable shifts in the Chern number, this implies $f = g$. |

1. **Fullness:** The functor is full. Every adiabatic

protocol that shifts the Chern number by an integer a corresponds to the morphism $f_a(x) = x + a \bmod p_i$ in RNS. |

1. **Categorical Equivalence:** Because the functor

$\mathcal{F}$ is full, faithful, and essentially surjective onto the relevant subcategory of **TopPh**, it establishes an equivalence of categories between **RNS** and its physical realization. |

Appendix

B: Verification of the Vectorial Encoding Scheme

The validity of the vectorial encoding and measurement scheme is established as follows.

Bijjective Mapping (CRT): From the Chinese Remainder Theorem, the map $\phi : N \mapsto (r_1, \dots, r_k)$, where $r_i = N \bmod p_i$, is a bijection from the set of integers $\{0, \dots, M - 1\}$ to the product ring $\prod_{i=1}^k \mathbb{Z}/p_i\mathbb{Z}$.

Physical State Representation: In the proposed physical system, the computational state is characterized by the Chern vector $\mathbf{C} = (C_1, \dots, C_k)$, where each C_i is an integer in the range $\{0, \dots, p_i - 1\}$.

Encoding Correspondence: By the construction of the functorial framework (Section III.C) and the operational protocols (Section IV.B), the physical encoding ensures a direct correspondence $C_i = r_i$ for all channels i . Therefore, the map from an integer to its physical representation, $N \mapsto \mathbf{C}$, is also a bijection.

Unambiguous Representation: A bijective mapping means that the Chern vector $\mathbf{C}$ is a complete and unambiguous representation of the integer N . There is a one-to-one correspondence, ensuring no information is lost. This obviates any need for algorithmic reconstruction.

Direct Measurement: The multi-terminal measurement protocol (Section IV.C) directly yields the integer components of the Chern vector, $\{C_1, \dots, C_k\}$, via a set of independent, quantized Hall conductance measurements.

Conclusion: The final output of the physical system is the vector $\mathbf{C}$, which is demonstrably equivalent to the RNS representation of N . The computational problem is solved entirely by the physical process, and the result is directly readable from the hardware.

Edge Case Analysis: The encoding is valid across the entire dynamic range. For $N = 0$, the system is in the vacuum state with $\mathbf{C} = (0, \dots, 0)$. For $N = M - 1$, the system is in the state $\mathbf{C} = (p_1 - 1, \dots, p_k - 1)$. Both are valid and stable ground states.

Robustness and Error Detection: Any error that alters a single component C_i results in a vector that does not correspond to a valid integer within the intended computational range, enabling immediate error detection.

Error Correction Potential: By choosing moduli such that the total dynamic range M exceeds the required range for a given problem, the redundant space can be used to encode error-correcting information, enabling the correction of single-channel errors.

Formal Measurement Map: The measurement process can be formalized as a linear isomorphism $\mathcal{M} : \mathbf{C} \mapsto \{G_{xy}^{(i)}\}$, which maps the Chern vector space to the conductance vector space, completing the formal chain from abstract integer to physical observable.

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