

Is the Harmonic Oscillator an Infrared Attractor in All Places? A Place-Resolved Analysis of the Harmonic Paradigm

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Abstract

The conjecture that the harmonic oscillator is a universal infrared (IR) attractor for bosonic quantum systems has so far been formulated and tested only over the Archimedean place, i.e., with respect to the usual real absolute value. We ask whether this attractor status survives a change of place in the sense of Ostrowski's classification of absolute values on $\mathbb{Q}$. We set up a place-resolved renormalization-group (RG) framework in which a dimensionless distance α_v from a flowing bosonic system to the oscillator fixed point is defined separately for the Archimedean place $v = \infty$ and for the p -adic places $v = p$, using a putative p -adic fixed point whose spectrum we model on the levels of the Bruhat–Tits tree of $\mathbb{Q}_p$ (an explicit modeling assumption of this paper; the adelic oscillator of [8] is established qualitatively, but its supplied summary states no level formula). We derive explicitly: (i) the Archimedean level structure $E_n = \hbar\omega(n + \frac{1}{2})$ with constant spacing; (ii) a geometric p -adic level ladder $E_n^{(p)} = E_0 p^{-n}$ with adjacent-gap ratio p , under a clearly labeled modeling assumption; (iii) an exponential RG approach $\alpha_v(\ell) = e^{-\gamma_v \ell}$, evaluated numerically; and (iv) a test of adelic linkage via the product formula, verified exactly for $x = 2/3$. We find that attraction is place-dependent in structure: constant-gap attraction in $\mathbb{R}$ coexists with geometric-gap attraction in $\mathbb{Q}_p$, and the adelic product formula constrains amplitudes, not spectra, so the two fixed points are formally compatible but not linked by any spectral identity. The attractor claim is therefore neither simply preserved nor reversed but structurally transformed.

1. Introduction

The Harmonic Paradigm proposes that the quantum harmonic oscillator (HO) is the universal infrared attractor of quantum theory: generic bosonic systems, when probed at sufficiently long wavelengths, flow toward oscillator-like behavior regardless of microscopic detail. An internal red-team assessment describes the Paradigm's ambitious scope, including an "8-rung ladder spanning from transmon anharmonicity to quantum gravity" [14], and the Paradigm has practical consequences: if the HO is the IR attractor of quantum mechanics, then bosonic encodings are argued to be the native encoding of quantum information, a claim backed by a resource comparison in which bosonic codes require 5–40 times fewer photons and roughly 100 times fewer modes than surface codes at a target logical error rate of $p_L = 10^{-6}$ [12]. The Paradigm sits inside a broader "Adelic Cross-Domain Program" that maps physical architecture onto Bruhat–Tits trees, the p -adic analogues of hyperbolic space, connecting the fine-structure constant, the renormalization group, quantum error correction, and the Efimov effect [13].

The problem this paper addresses is one of place-ambiguity. Every statement of the form "the oscillator attracts" implicitly quantifies over states, observables, and — crucially — a number field equipped with an absolute value. By Ostrowski's theorem, the nontrivial absolute values on $\mathbb{Q}$ are, up to equivalence, the usual real absolute value and the p -adic absolute values $|\cdot|_p$. The Harmonic Paradigm's attractor claim has only ever been tested under the first of these; its red-team assessment notes that although the Paradigm's bibliography invoked Ostrowski's theorem and p -adic structures, its core mechanisms were confined to the Archimedean place [14]. It is therefore natural to ask the sharp question: is the oscillator's attractor status (a) preserved, (b) reversed, or (c) structurally transformed under p -adic and adelic norms?

We answer with a framework and a diagnosis. We define a place-resolved distance α_v from a flowing bosonic system to the oscillator fixed point in each place v , derive the RG flow toward each oscillator, and test whether the adelic product formula imposes a genuine constraint linking the Archimedean and p -adic attractors. Our conclusion is option (c): attraction can exist in every place, but with structurally different spectra (constant gaps in $\mathbb{R}$, geometric gaps in $\mathbb{Q}_p$), and the adelic product formula constrains amplitudes rather than spectra, so the two attractors share a name by structural analogy, not by a spectral identity. This is a negative result for the strongest form of the Harmonic Paradigm — the place-invariant upgrade fails — but a positive result for a weaker form: the oscillator can be an IR attractor in all places, each in its own spectral dialect.

Throughout, "place" means an equivalence class of absolute values on $\mathbb{Q}$, and "Bruhat–Tits tree" means the locally infinite $(p+1)$ -regular tree on which $\mathrm{PGL}_2(\mathbb{Q}_p)$ acts. All quantitative claims are derived in Section 4 with every input number stated; Section 5 reports only those computed values or clearly labeled projections.

2. Background and Related Work

The harmonic oscillator has been generalized along many algebraic axes, and each generalization is evidence about which features of the oscillator are robust and which are artifacts of the ambient number structure. We review the supplied bibliography, noting for each entry what it actually supports.

Oscillators over exotic number systems. The adelic program of [8] formulates adelic quantum mechanics and considers the corresponding harmonic oscillator model; the ad harmonic oscillator exhibits interesting features, among them a softening of the uncertainty relation. This is the closest published antecedent to our question: it establishes that an oscillator can be defined adically and that even so basic a structure as the uncertainty relation is place-sensitive. The supplied summary, however, gives no spectral details, so we cannot draw on it for the level structure of the adelic oscillator; our spectral analysis in Sections 3–4 is therefore an independent construction, checked for consistency against the qualitative features (existence, well-posedness, softened uncertainty) that [8] does report. The bicomplex quantum harmonic oscillator of [6] pushes generalization further: the oscillator problem is solved over bicomplex numbers — pairs of complex numbers forming a commutative ring with zero divisors — by adapting the algebraic treatment of the standard oscillator, and eigenvalues and eigenkets are found. That the algebraic oscillator construction survives even over a ring with zero divisors supports the working hypothesis that "oscillator" names a robust algebraic fixed point, not merely a real-variable solution. A generalized harmonic oscillator on noncommutative spaces is considered in [5], where dynamical symmetries are classified and the physical equivalence of noncommutative systems sharing the same energy spectrum is investigated, with general solutions of the three-dimensional noncommutative oscillator found. The lesson is that spectrum alone does not fix physics: two systems with the same spectrum can be physically inequivalent, so a place-resolved attractor claim must specify more than eigenvalues.

Dynamics and damping. The general solution of the quantum damped harmonic oscillator is given in [1]; the supplied summary states no further detail, but dynamical completeness of the solution family is a prerequisite for fixed-point language: an attractor must be a well-defined solution family before it can be a limit of flows. Similarly, [7] treats the anti-PT-symmetric harmonic oscillator and its relation to the inverted oscillator, demonstrating that the common formal replacement $\omega \rightarrow i\omega$, used to obtain the inverted oscillator, leads to unbounded eigenvectors and involves unclear points in the redefinition. This matters because an RG flow away from a stable oscillator fixed point is often modeled by exactly such an analytic continuation; [7] warns that the "inverted" sector is not a harmless mirror of the stable one, a caution we adopt when discussing repulsion in Section 6.

Spectral and analytic structure. The characterization of quantum limits and semi-classical measures for sequences of eigenfunctions of coupled oscillators with arbitrary frequencies in [4] shows that the structure of the set of semi-classical measures depends strongly on the arithmetic relations between the frequencies of the decoupled oscillators. This is directly relevant: arithmetic relations among frequencies are precisely what changes when one changes place, so place-dependence of attractor structure is already visible in the Archimedean theory through arithmetic. On the analytic side, [3] shows that harmonic oscillator propagators and fractional Fourier transforms are essentially the same, deduces continuity and fixed-time estimates on modulation spaces, and applies these to prove Strichartz estimates for the oscillator propagator; this gives a function-analytic handle on propagation that is independent of spectral-gap structure and could be transplanted to p -adic modulation spaces in future work.

Attractors and universality claims. The study of what attracts to attractors in [9] examines, for systems relevant to heavy-ion collisions, whether, how, and to what extent solutions become insensitive to aspects of their initial conditions, in Israel–Stewart theory and kinetic theory where a universal attractor solution governs the approach. This supplies the template for our $\alpha_v(\ell)$ construction: an attractor is meaningful exactly when a quantifiable distance from the universal solution decays along the flow, largely independently of initial conditions. In the same spirit, [11] finds a universal mass scale for all q -form fields in multi-brane worlds, with ultralight modes, via a covariant multi-localization of the Lagrangians; the supplied summary is truncated, so we cite it only for the existence of a universal bosonic mass scale, a parallel universality claim in the bosonic domain.

Applications as stress tests. QHO battery models are, per the entry of [2], experimentally realizable, have high ergotropy, and can store more than one quantum of energy; the entry states that the models are reinvestigated to answer fundamental questions about benefit and unbounded charging, but the summary is truncated mid-question, so we use it only for the established points. Meanwhile [10] documents a cautionary episode: a claimed procedure using a scaled Fourier transform to beat the standard Heisenberg value of $1/2$ in simultaneous position–momentum resolution was in fact invalid for quantum mechanics. This is a warning against overclaiming oscillator universality, which we heed in Section 6.

The Harmonic Paradigm and its audit. The resource-commensurable comparison of bosonic codes [12] reports that at $p_L = 10^{-6}$ bosonic codes require 5–40 times fewer photons and roughly 100 times fewer modes than surface codes, and states the novel claim that the oscillator as the quantum-mechanical IR attractor implies bosonic codes are the native encoding; our results bear on the *premise* of that implication, not on its internal resource arithmetic. The Adelic Cross-Domain Program [13] maps the architecture of the Standard Model onto Bruhat–Tits trees — the p -adic analogues of hyperbolic space — connecting the fine-structure constant, the renormalization group, quantum error correction, the Efimov effect, and the Standard Model mass spectrum; this is the source for the tree-geometric setting of our p -adic analysis. Finally, the red-team assessment [14] states that the Harmonic Paradigm (versions V1.0–V4.0, 2026) proposed the oscillator as the universal IR attractor of quantum theory with the 8-rung ladder noted above, and that although its bibliography invoked Ostrowski's theorem and p -adic structures, its core mechanisms were confined to the Archimedean place — the entry text is cut off at exactly the point that matters, so we use only what is stated: the Paradigm's own red-team flagged the Archimedean confinement of its mechanisms. Our paper directly addresses the open question thereby identified.

3. Methods

3.1 Places and absolute values

A place of $\mathbb{Q}$ is an equivalence class of absolute values. We write $v = \infty$ for the Archimedean place with $|x|_\infty$ the usual absolute value, and $v = p$ for the p -adic places, where $|p^k m|_p = p^{-k}$ for m coprime to p . The adelic product formula states that for every nonzero rational x ,

$$\prod_v |x|_v = |x|_\infty \prod_p |x|_p = 1.$$

3.2 The two oscillator fixed points

Archimedean fixed point. The standard quantum harmonic oscillator has Hamiltonian $H_\infty = \frac{\hat{P}^2}{2m} + \frac{m\omega^2 \hat{X}^2}{2}$ and spectrum

$$E_n^{(\infty)} = \hbar\omega \left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, \dots$$

with constant spacing $\Delta E_\infty = \hbar\omega$ and nondegenerate levels.

p -adic fixed point. The adelic oscillator of [8] is established qualitatively (existence, softened uncertainty relation), but its supplied summary states no level formula and names no spectrum. We therefore adopt, as a stated modeling assumption of this paper, a putative p -adic fixed point whose spectrum is organized on the levels of the Bruhat–Tits tree of $\mathbb{Q}_p$, which we take to be the $(p+1)$ -regular tree on which $\mathrm{PGL}_2(\mathbb{Q}_p)$ acts; this tree structure is our adopted definition, not a result imported from [8] or [13]. We model the level- n energy as

$$E_n^{(p)} = E_0 p^{-n}, \quad n = 0, 1, 2, \dots$$

This is a modeling assumption, stated as such: the supplied literature [8] establishes the existence and qualitative features of the adelic oscillator but its summary does not state a level formula, so we adopt the geometric ladder as the canonical tree-organized spectrum and flag all conclusions that depend on it. The spacing is

$$\Delta E_n^{(p)} = E_n^{(p)} - E_{n+1}^{(p)} = E_0 p^{-n} \left(1 - \frac{1}{p}\right),$$

so adjacent gaps shrink geometrically: the ratio of the gap at level n to the gap at level $n+1$ is p . The degeneracy of level n is the number of tree vertices at distance n from a base vertex: $g_p(0) = 1$ and $g_p(n) = (p+1)p^{n-1}$ for $n \geq 1$.

3.3 Place-resolved RG flow and attractor distance

For each place v , let $\omega_v(\ell)$ be the effective oscillator parameter of a flowing bosonic system at RG scale ℓ , and let $\omega_{v,*}$ be the fixed-point value. We posit the linearized flow

$$\frac{d\omega_v}{d\ell} = -\gamma_v (\omega_v - \omega_{v,*}),$$

with solution $\omega_v(\ell) = \omega_{v,*} + (\omega_v(0) - \omega_{v,*}) e^{-\gamma_v \ell}$. The dimensionless distance from the attractor is

$$\alpha_v(\ell) \equiv \frac{|\omega_v(\ell) - \omega_{v,*}|}{\omega_{v,*}} = \alpha_v(0) e^{-\gamma_v \ell}.$$

Attraction in place v means $\gamma_v > 0$; repulsion means $\gamma_v < 0$; structural transformation means the fixed-point spectra differ in kind (constant vs. geometric gaps) even when both $\gamma_v > 0$.

3.4 Adelic linkage test

The strongest form of place-invariance would be a constraint of product-formula type linking the place-wise attractor distances, e.g., $\prod_v \alpha_v = \text{const}$ along the flow. We test whether any such identity is forced by the adelic structure, using the product formula as the only adelic input, and we also identify where the product formula *does* bind (amplitudes).

4. Analysis

All numbers in this section are derived here from stated inputs; nothing is imported from simulation or measurement.

A1. Product formula verification. Input: $x = 2/3$, with $p = 2$ and $p = 3$ (standard definitions of $|\cdot|_p$, Section 3.1). Steps:

$$|2/3|_2 = \frac{|2|_2}{|3|_2} = \frac{2^{-1}}{1} = \frac{1}{2}, \quad |2/3|_3 = \frac{|2|_3}{|3|_3} = \frac{1}{3^{-1}} = 3, \quad |2/3|_\infty = \frac{2}{3}.$$

Product:

$$\frac{1}{2} \times 3 \times \frac{2}{3} = \frac{3}{2} \times \frac{2}{3} = 1.$$

The product formula holds exactly, as required.

A2. Bruhat–Tits tree vertex counts (under stated assumptions). We adopt as a modeling assumption (Section 3.2) that the Bruhat–Tits tree of $\mathbb{Q}_p$ is $(p+1)$ -regular with the number of vertices at level n equal to $(p+1)p^{n-1}$ for $n \geq 1$; this structure and count formula are definitions adopted by this paper, not results sourced from any cited work. Under these assumptions, for $p = 2$:

$$N_1 = 3, \quad N_2 = 3 \times 2 = 6, \quad N_3 = 3 \times 4 = 12, \quad N_4 = 3 \times 8 = 24.$$

Cumulative vertices through level $n = 4$: $1 + 3 + 6 + 12 + 24 = 46$. For $p = 3$: $N_1 = 4$, $N_2 = 12$, $N_3 = 36$; cumulative through level 3: $1 + 4 + 12 + 36 = 53$. The growth rate is p per level, so the density of p -adic oscillator levels grows without bound, in contrast with the uniformly spaced Archimedean ladder. In closed form the cumulative count through level N is

$$S_p(N) = 1 + (p+1) \frac{p^N - 1}{p - 1};$$

for $p = 2$ this gives $S_2(N) = 1 + 3(2^N - 1) = 3 \cdot 2^N - 2$, and the check $S_2(4) = 3 \cdot 16 - 2 = 46$ agrees with the direct sum above.

A3. Spectral gap comparison. Archimedean: $\Delta E_n^{(\infty)} = \hbar\omega$ for all n ; with $\hbar\omega$ normalized to 1, gaps are $(1, 1, 1, \dots)$. p -adic (model of Section 3.2, $E_0 = 1$, $p = 2$):

$$\Delta E_0^{(2)} = 1 - \frac{1}{2} = \frac{1}{2}, \quad \Delta E_1^{(2)} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}, \quad \Delta E_2^{(2)} = \frac{1}{8}, \quad \Delta E_3^{(2)} = \frac{1}{16}.$$

Gap ratio check: $\Delta E_0^{(2)} / \Delta E_1^{(2)} = (1/2) / (1/4) = 2 = p$, and $\Delta E_1^{(2)} / \Delta E_2^{(2)} = (1/4) / (1/8) = 2 = p$. For $p = 3$, $E_0 = 1$: gaps are $1 - 1/3 = 2/3$, then $1/3 - 1/9 = 2/9$, then $2/27$; ratio $(2/3) / (2/9) = 3 = p$. The two fixed points therefore have spectra of different kind: constant gaps versus geometrically shrinking gaps with unbounded level density.

A4. RG approach to each attractor. Input: $\gamma_v = 0.1$ (illustrative coupling, stated as an assumption), $\alpha_v(0) = 1$, $\ell = 20$. Then

$$\alpha_v(20) = e^{-0.1 \times 20} = e^{-2} \approx 0.135335283.$$

At $\ell = 40$: $\alpha_v(40) = e^{-4} \approx 0.018315639$. At $\ell = 60$: $\alpha_v(60) = e^{-6} \approx 0.002478752$. Each successive interval of 20 RG steps reduces the distance by the factor $e^{-2} \approx 0.135335$, i.e., by roughly 86.466%. This holds in any place with the same γ_v ; the place-dependence of attraction resides in γ_v and in the fixed-point spectrum, not in the exponential form.

A5. Adelic linkage test. Hypothesis $\mathcal{H}$: the flow obeys a product constraint $\prod_v \alpha_v(\ell) = c$ for all ℓ . Take two places, $v = \infty$ and $v = 2$, with equal $\gamma = 0.1$ and $\alpha_\infty(0) = \alpha_2(0) = 1$, so $\alpha_\infty(\ell) = \alpha_2(\ell) = e^{-0.1\ell}$ and

$$\prod_v \alpha_v(\ell) = e^{-0.2\ell},$$

which depends on ℓ ; hence c is not constant unless the γ_v satisfy a fine-tuned compensation. For the product to be ℓ -independent we would need $\sum_v \gamma_v = 0$, e.g., $\gamma_\infty = +0.1$ and $\gamma_2 = -0.1$ (attraction in one place, repulsion in the other). Nothing in the adelic product formula (A1) forces $\sum_v \gamma_v = 0$: the product formula constrains the norms of rational *numbers* $|x|_v$, not the RG couplings γ_v of *dynamical systems* in each place. We conclude that adelic linkage of the attractor distances is not imposed; it would require an additional dynamical principle not currently available.

A6. Amplitude-level adelic constraint. Where the product formula *does* bind is on amplitudes. If an adelic wavefunction $\Psi(x)$ has rational evaluation points $x \in \mathbb{Q}$, then the local amplitudes $|\Psi(x)|_v$ can be normalized so that $\prod_v |\Psi(x)|_v = 1$ pointwise, mirroring A1. This is an amplitude constraint, not a spectral one: it can hold simultaneously with constant Archimedean gaps and geometric p -adic gaps, since it fixes no eigenvalue. This is the precise sense in which the two attractors are compatible but unlinked.

5. Results

R1 (exact). The adelic product formula is verified for $x = 2/3$: $\frac{1}{2} \times 3 \times \frac{2}{3} = 1$ (A1).

R2 (exact, model-based). Under the stated geometric model $E_n^{(p)} = E_0 p^{-n}$, the p -adic oscillator has gap ratio exactly p between adjacent levels, computed for $p = 2$ (gaps $1/2, 1/4, 1/8, 1/16$) and $p = 3$ (gaps $2/3, 2/9, 2/27$), against the constant Archimedean gap $\hbar\omega$ (A3). The fixed points are spectrally distinct in kind.

R3 (exact, model-based). Bruhat–Tits vertex counts grow as $(p+1)p^{n-1}$: for $p = 2$, levels 1–4 carry 3, 6, 12, 24 vertices (46 cumulative with the base); for $p = 3$, levels 1–3 carry 4, 12, 36 (53 cumulative) (A2).

R4 (exact given stated assumptions). With $\gamma_v = 0.1$ and $\alpha_v(0) = 1$, the attractor distance falls as $\alpha_v(\ell) = e^{-0.1\ell}$: $\alpha_v(20) \approx 0.135335$, $\alpha_v(40) \approx 0.018316$, $\alpha_v(60) \approx 0.002479$ (A4). These are projections conditional on the assumed linearized flow and coupling; they are not empirical measurements.

R5 (exact). The adelic linkage hypothesis $\prod_v \alpha_v = \text{const}$ fails for generic γ_v : with two places and equal $\gamma = 0.1$, the product is $e^{-0.2\ell}$, ℓ -dependent; linkage requires the fine-tuned condition $\sum_v \gamma_v = 0$, which the product formula does not impose (A5).

R6 (structural). The adelic product formula constrains amplitudes (A6), not spectra; therefore attraction in $\mathbb{R}$ coexists with attraction in $\mathbb{Q}_p$ (both $\gamma_v > 0$ possible) without any spectral identity linking the two fixed points. The verdict on the conjecture is option (c): structural transformation.

6. Discussion

Limitations. The weakest link is the p -adic level formula $E_n^{(p)} = E_0 p^{-n}$. The supplied literature establishes that the adelic oscillator exists and softens the uncertainty relation [8], but its summary does not state the spectrum; our geometric ladder is a principled guess consistent with tree-level organization [13], and every quantitative result that depends on it (R2, and the spectral interpretation of R3) inherits that status. A direct computation of the Vladimirov–Volovich–Zelenov spectrum would either confirm or replace this ansatz, and the qualitative conclusion — geometric rather than constant gaps — should be rechecked against the exact spectrum. Second, the RG flow is linearized; far-from-fixed-point behavior may be non-exponential, and the coupling $\gamma_v = 0.1$ is illustrative, not derived. Third, we tested linkage only for two places and a product-of-distances hypothesis; richer adelic constraints (e.g., involving local L -factors) are unexplored.

Failure modes and self-critique. If the true p -adic spectrum turned out to have constant gaps in an appropriate normalization, R2 would collapse and the "structural transformation" verdict would weaken toward (a), preservation. Conversely, if $\gamma_p < 0$ generically — if generic p -adic bosonic systems flow *away* from the oscillator — the verdict would move toward a mixed preservation/repulsion scenario, closer to (b). The warning from [7] that the inverted oscillator involves unclear points in the $\omega \rightarrow i\omega$ redefinition suggests that repulsive sectors are genuinely delicate and should not be assumed mirror-symmetric. The episode documented in [10], where an oscillator-based claim of beating the Heisenberg value $1/2$ proved invalid, is a reminder that universality claims about the oscillator have a history of overreach; our R5 is deliberately a negative result and we resist upgrading "compatible but unlinked" into any stronger adelic dogma. The noncommutative-oscillator result that same-spectrum systems can be physically inequivalent [5] further cautions that even a confirmed spectral match across places would not establish full physical equivalence of the attractors.

What would falsify the claims. (i) An exact p -adic spectrum with constant gaps would falsify R2. (ii) A derivation forcing $\sum_v \gamma_v = 0$ from adelic dynamics would falsify R5 and revive strong place-invariance. (iii) Evidence that the semi-classical measure structure of [4], which depends on arithmetic frequency relations, becomes place-independent would undercut our premise that arithmetic — hence place — matters for attractor structure.

Open questions. Does the propagator–fractional-Fourier equivalence of [3] admit a p -adic analogue on p -adic modulation spaces? Do the damped-oscillator solutions of [1] possess p -adic counterparts with the same dynamical completeness? Does the universal bosonic mass scale of [11] admit a place-resolved version? And does the resource advantage of bosonic codes [12] — 5–40 times fewer photons at $p_L = 10^{-6}$ — survive if the native-encoding argument must be restated place by place?

7. Conclusion

We asked whether the harmonic oscillator's status as an IR attractor is place-invariant. Within a place-resolved RG framework, with explicit derivations and clearly labeled modeling assumptions, we find that the oscillator can attract in each place but with structurally different spectra — constant gaps in $\mathbb{R}$, geometric gaps in $\mathbb{Q}_p$ organized on Bruhat–Tits tree levels — and that the adelic product formula constrains amplitudes, not spectra, imposing no linkage between the place-wise attractor distances. The Harmonic Paradigm's attractor thesis is therefore not an artifact of the Archimedean place (attraction is not reversed), but neither is it place-invariant in any strong spectral sense: it is structurally transformed. The strongest open task is the exact computation of the p -adic oscillator spectrum, which would convert our model-based results into theorems.

References

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Appendix A. Divergence report

The constituent drafts diverged on three substantive points; each was resolved by an explicit convention, documented here.

D1. p -adic spectrum model. Drafts A and C used the geometric ladder $E_n^{(p)} = E_0 p^{-n}$ (energies decreasing with tree level). Draft B used $E_p(n) = E_0 p^{\gamma n}$ with a free exponent γ , set to $\gamma = 1$ for its linkage test (energies increasing with level). The disagreement is a convention about whether tree level indexes energy upward or downward; neither draft derived its law from the Vladimirov–Volovich–Zelenov Hamiltonian, and the supplied bibliography contains no entry stating the p -adic spectrum. *Convention chosen:* the main text adopts the A/C ladder $E_n^{(p)} = E_0 p^{-n}$, because it makes the gap-shrinking (IR) direction of the geometric series explicit and was used by two of three drafts. B's qualitative conclusion (gap structure of a different kind than the Archimedean constant spacing) is invariant under the choice, since both models produce geometric gaps with ratio p .

D2. Verdict on the attractor trichotomy. Draft A concluded that the p -adic oscillator is *repelled* from the QHO fixed point, based on an assigned negative RG coefficient $c_{\mathbb{Q}_p} = -0.05$ that its own Methods section labeled an ad-hoc illustrative choice. Drafts B and C concluded *structural transformation* (option (c)): attraction is possible in each place, but the fixed-point spectra differ in kind. *Convention chosen:* the main text adopts the B/C verdict, because A's repulsion followed from an assumed coefficient sign rather than a derivation, whereas B and C separated the (assumed) flow dynamics from the (derived, model-based) spectral comparison. A's repulsion scenario is retained in Section 6 as an explicit failure mode conditional on $\gamma_p < 0$.

D3. Adelic linkage test design. Draft A tested linkage by forming the numerical product of matched Archimedean and p -adic energy values and checking it against unity ($P = 0.234375$ for its chosen normalization, $x = 2/3$ -style inputs), treating failure of the product to equal 1 as evidence against place-invariance. Drafts B and C instead tested linkage at the level of the RG flow, asking whether a product constraint $\prod_v \alpha_v(\ell) = c$ on the attractor distances could hold along the flow, and found it requires the fine-tuned condition $\sum_v \gamma_v = 0$ that the adelic

product formula does not impose. *Convention chosen:* the main text adopts the B/C product-of-distances test (Section 3.4, A5, R5), because the object whose linkage is conjectured by the Paradigm is the attractor property (a flow statement), not a numerical product of eigenvalues; a product of energies mixes dimensionful quantities whose normalization is arbitrary, so Draft A's numeric check ($P = 0.234375 \neq 1$) tests a convention-dependent identity and was not adopted. Draft A's observation is preserved in spirit by A6: the adelic product formula does bind, but on amplitudes of rational evaluation points, not on spectra or RG couplings.