

Non-Anthropocentric Natural Units for Area and Information in Holographic Entropy Bounds

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Abstract

Information-theoretic bounds in gravitational physics are conventionally expressed in bits, a unit that presupposes two human choices: the practice of counting states by integers and the selection of logarithm base 2. This paper asks whether the holographic entropy bound can instead be stated entirely in observer-independent quantities. Working in Planck natural units ($c = \hbar = G = k_B = 1$), in which area is measured in Planck areas ℓ_P^2 and is therefore dimensionless, we take the Bekenstein–Hawking value $S = A/4$ (in nats) as the definition of the entropy of a region's horizon. We show that the bound $D \leq e^{A/4}$ on the Hilbert-space dimension D of a finite region contains no base-dependent constant; that the bit is recoverable only as the derived, base-dependent quantity $S/\ln 2$ with conversion factor $1/\ln 2 = 1.442695$; and that every base- b representation of entropy is a fixed constant multiple of the same invariant, so that no observable at fixed theory content distinguishes the invariant formulation from the conventional one. Worked examples with full arithmetic are given: a horizon patch of assumed area $A = 10^{68}$ in Planck units carries $S = 2.5 \times 10^{67}$ nats, i.e. 3.60674×10^{67} bits, and permits $D \leq 10^{1.085736 \times 10^{67}}$. We identify the rounding of D to an integer as the one genuinely open conventional element and discuss limitations, failure modes, and falsification conditions.

1. Introduction

Physics claims universality, yet its units of information are inherited from engineering. The bit — one bit being the entropy of a two-outcome event, $S = \ln 2$ in natural units — exists because human communication systems favor two-state symbols. The choice of base 2 is a convention of human computation, not a feature of nature. The same criticism applies, though more weakly, to the practice of counting: Hilbert-space dimension is an integer, but "which integer" is meaningful only once a tensor-product factorization and a normalization convention have been chosen by an observer.

The holographic entropy bound offers an unusual opportunity. In the Bekenstein–Hawking framework, the entropy of a black hole horizon is proportional to its area, and the entropy of a finite spacetime region is bounded by a quantity of the form $\exp(A/4\ell_P^2)$. If one adopts Planck natural units, in which $c = \hbar = G = k_B = 1$, the Planck length ℓ_P becomes the unit of length, area becomes dimensionless, and the bound can be written as

$$D \leq e^{A/4},$$

where A is the dimensionless area and D is the dimension of the Hilbert space associated with the region. Expressed this way, the bound involves exactly two ingredients: a geometric invariant (area in Planck units) and the exponential function. The base e of the exponential is not chosen but forced by the requirement that entropy be additive under composition of independent systems: if two disjoint regions with areas A_1 and A_2 have independent Hilbert spaces of dimensions D_1 and D_2 , the composite dimension is $D_{12} = D_1 D_2$, and the entropy $S = \ln D$ defined by the Bekenstein–Hawking relation $S = A/4$ is fixed in nats by geometry. Choosing a different base for its expression introduces the constant factor $1/\ln b$ with no geometric origin.

The conjecture this paper develops, following a prior companion paper [12], is that the entire content of the holographic bound can be restated using only the dimensionless entropy $S = A/4$, with the bit recovered as a derived, base-dependent quantity $b = S/\ln 2$. If correct, information-theoretic bounds in gravitational physics are grounded in observer-independent geometry rather than in human conventions of notation and counting. Conversely, if the program fails — if some irreducibly conventional choice remains — the failure itself is informative, because it locates precisely where observer-dependence enters the foundations of entropy bounds.

Our contributions are: (i) a precise statement of the natural-unit formulation of the Bekenstein–Hawking entropy bound; (ii) explicit derivations showing which quantities are base-independent and which are not, with worked numerical examples computed in full; (iii) a structural result that all base- b representations are related by known constants and are therefore empirically equivalent, so the proposal’s significance is foundational rather than empirical; and (iv) an analysis of the integer-rounding problem for the Hilbert-space dimension D , which we argue is the one genuinely non-derivable convention left in the framework.

The paper is organized as follows. Section 2 reviews the supplied literature. Section 3 sets up the formal framework. Section 4 carries out the explicit derivations, including all arithmetic. Section 5 reports the computed results. Section 6 discusses limitations, failure modes, and falsification conditions. Section 7 concludes.

2. Background and Related Work

We review the twelve works supplied in the bibliography, in their given order. Several are only loosely connected to holography; we confine ourselves to what each entry’s own summary supports, and we note where a summary is thin or absent.

[1] treats natural language processing as a branch of computer science combining artificial intelligence with linguistics, aimed at analyzing language elements such as writing or speech with software and converting them into information, and notes that each language’s own grammatical rules and vocabulary diversity make studies in this field complex. The connection to our topic is analogical only: "information" in applied contexts is defined relative to a symbol system with its own conventions, which is precisely the anthropocentric dependence we seek to remove from entropy bounds. The supplied summary is truncated and gives no further technical detail.

[2] argues that the holographic bound in physics constrains the complexity of life, that the finite storage capability of information in the observable universe requires "protein linguistics" in the evolution of life, and reports finding that the evolution of the genetic code determines the variance of amino acid frequencies and genomic GC content among species, with the linguistic mechanism confirmed by experimental observations (the summary is cut off mid-sentence). This is the closest supplied work to our theme: it takes the holographic bound as a physical constraint on information capacity and applies it outside gravitation. We use it only as evidence that holographic bounds are treated as substrate-independent capacity statements, which motivates asking in what units such a capacity is defined.

[3] develops computer-assisted proof methods involving non-classical inequalities for Shannon entropy and applies them to secret sharing schemes, in which a random secret value is transformed into shares distributed among participants so that only qualified groups recover it, and to hat guessing games. Its relevance is that Shannon entropy there is handled as an abstract quantity obeying inequalities, independent of any particular logarithm base; homogeneous inequalities are invariant under rescaling of the log base, exactly the invariance structure we formalize in Section 4.3.

[4] considers classical communication over a finite-dimensional quantum channel with memory using a separable-state input ensemble and local output measurements, and proposes algorithms for estimating and bounding the information rate of such setups, some based on so-called auxiliary channels, extending prior algorithms. The finite-

dimensional quantum channel is the same algebraic setting in which the Hilbert-space dimension D entering our definition of physical entropy lives; the supplied summary gives no further detail on the forms of the bounds.

[5] describes the Gamma Ray Large Area Space Telescope (GLAST) Large Area Telescope (LAT), a pair-production high-energy (> 20 MeV) gamma-ray telescope built by an international partnership for a satellite launch in 2006, and the collaboration's construction of a balloon flight engineering model. We cite it as an example of large-scale empirical astrophysics whose measurements are reported in human-defined units; the summary supplies no numerical effective area, so we draw no quantitative comparison.

[6] analyzes (mis)information operations: the massive diffusion of social media fosters disintermediation, changes how users are informed and engage in public debate, and the cognitive layer of users and related social dynamics define the nature and dimension of informational threats, with users tending to interact with information adhering to their preferred narrative. Its relevance is a sociological contrast: "information" as threat vector is defined relative to human cognition, the extreme opposite of the observer-independent entropy we define below.

[7] investigates a multi-terminal source coding problem under logarithmic loss fidelity, which does not necessarily lead to an additive distortion measure, motivated by extending the Information Bottleneck method to a multi-source scenario where several encoders cooperatively build rate-limited descriptions to maximize information. Logarithmic loss is a fidelity measure under which the relevant quantities are entropies in some base; the supplied summary does not state which base, which itself illustrates the base-convention arbitrariness we study.

[8] develops a theoretical framework for defining and identifying flows of information in computational systems, modeling a computational system as a directed graph with "clocked" nodes that send transmissions along edges at discrete times, seeking a definition capturing dynamic flow of information about a specific message. The summary is truncated; we use it only as evidence that information-flow definitions are convention-laden even in classical computation.

[9] (QNFO: Ten-Fingered Trap) has an empty supplied summary; it gives no content we can use, and we cite it only to acknowledge the corpus from which this project arises.

[10] (QNFO: The Threshold of, DOI 10.5281/zenodo.19202950) likewise has an empty supplied summary; no statement about its content can be made here.

[11] (QNFO: Super-Universe, DOI 10.5281/zenodo.19347807) likewise has an empty supplied summary; no statement about its content can be made here.

[12] (QNFO: Non-Anthropocentric Natural Units: From the Bekenstein Bound to Ostrowski's Theorem, DOI 10.5281/zenodo.21480756) states that it strips away all anthropocentric constructions from the Bekenstein–Hawking entropy bound, treating area as a dimensionless geometric invariant, defining the exponential bound without logarithmic base, and taking the integer Hilbert-space dimension as a universal invariant; it further connects to Ostrowski's theorem, noting that the Archimedean completion is (summary truncated). This is the direct predecessor of the present paper; our contribution is to make the base-independence claim precise, to exhibit the bit as a derived quantity with explicit conversion factor, to prove the empirical-equivalence result of Section 4.3, and to analyze the integer-rounding problem, which [12]'s summary does not address.

Notably absent from the supplied bibliography are the primary sources for the Bekenstein bound, Bekenstein–Hawking thermodynamics, and the covariant entropy bound; since we may cite only the supplied list, our statements about those bounds rest on the project's framing and on [2] and [12], which explicitly invoke the holographic and Bekenstein–Hawking bounds. This is a genuine weakness of the corpus, acknowledged rather than hidden.

3. Methods

3.1 Natural-unit setting

We adopt Planck natural units throughout:

$$c = \hbar = G = k_B = 1.$$

In these units the Planck length ℓ_P is the unit of length, the Planck area ℓ_P^2 is the unit of area, and any physical area A_{phys} is represented by the dimensionless number

$$A = \frac{A_{\text{phys}}}{\ell_P^2}.$$

The Boltzmann constant k_B , which converts temperature to energy, is set to 1, so entropy is measured in nats: one nat is the entropy of an event with Hilbert-space dimension e .

3.2 The base-free entropy bound

The Bekenstein–Hawking entropy of a horizon of area A_{phys} is

$$S_{\text{BH}} = \frac{k_B A_{\text{phys}}}{4 \ell_P^2},$$

which in natural units becomes

$$S_{\text{BH}} = \frac{A}{4}.$$

We take this as the definition of the entropy associated with a region's boundary. The corresponding bound on the Hilbert-space dimension D of the region is obtained by inverting the thermodynamic relation $S = \ln D$ (valid for a maximally mixed state on a D -dimensional Hilbert space):

$$D \leq e^S = e^{A/4}.$$

This is the central object of the paper. Its ingredients are the dimensionless area A , the integer 4 (a geometric factor from the Bekenstein–Hawking relation), and the exponential function. The nat is thus distinguished among logarithm bases by the additivity of $S = \ln D$ under composition and by the geometric fixing of $S = A/4$; any other base introduces the constant $1/\ln b$ with no geometric origin.

3.3 Recovery of the bit as a derived quantity

The bit is defined by

$$b = \frac{S}{\ln 2} = \frac{A}{4 \ln 2}.$$

The constant $\ln 2 \approx 0.693147$ is a pure mathematical constant, so the bit is well-defined — but its presence in any formula marks exactly where a human convention (binary representation) has been imported. Our method is to carry S (nats) through all derivations and convert to bits only at the output stage, so that every intermediate statement is base-free.

3.4 Method of analysis

We proceed by: (a) stating each quantity in the conventional formulation; (b) rewriting it in natural units; (c) classifying it as base-free (expressible via $S = A/4$ alone), derived (expressible only with an explicit base constant such as $\ln 2$ or $\ln 10$), or conventional (requiring a choice not fixed by mathematics or geometry, such as integer rounding of D); and (d) computing worked numerical examples with full arithmetic. We use no simulation and no empirical input beyond the structural assumptions stated above.

4. Analysis

4.1 Classification of quantities

Quantity	Expression	Status
Area	$A = A_{\text{phys}}/\ell_P^2$	base-free (geometric)
Entropy	$S = A/4$	base-free (nats)
Hilbert dimension bound	$D \leq e^{A/4}$	base-free
Bits	$b = S/\ln 2$	derived (base 2)
Decimal digits	$d = S/\ln 10$	derived (base 10)
Integer dimension	$D \in \mathbb{Z}$	conventional (rounding)

The only conventional entry is the last: the bound $D \leq e^{A/4}$ constrains a real number, but physical Hilbert spaces have integer dimension, so the operative bound is

$$D \leq \lfloor e^{A/4} \rfloor,$$

and the floor function is a counting convention imported from human mathematics. We quantify this in Section 4.4.

4.2 Input numbers and their sources

The only numerical inputs are mathematical constants of the standard real functions, not empirical measurements:

- $\ln 2 = 0.693147 \dots$ (definition of the natural logarithm).
- $\ln 10 = 2.302585 \dots$ (definition of the natural logarithm).
- The structural constants 4 (from the bound $S \leq A/4$) and the illustrative areas chosen for the worked examples.

All arithmetic below is shown step by step.

4.3 Derivation 1: conversion constants and the bit as a derived quantity

The horizon entropy in invariant form is $S = A/4$ nats. The bit count is

$$b = \frac{S}{\ln 2} = \frac{A}{4 \ln 2}.$$

Compute the conversion constant:

$$\frac{1}{\ln 2} = \frac{1}{0.693147} = 1.442695 \dots$$

Check: $1.442695 \times 0.693147 = 1.000000$ to six decimals. Therefore

$$b = 1.442695 \times \frac{A}{4} = 0.360674 A,$$

since $1.442695/4 = 0.360674$ (check: $0.360674 \times 4 = 1.442696$, consistent to the displayed precision). So each Planck area unit ($A = 1$) carries $S = 1/4 = 0.25$ nats = 0.360674 bits. The factor 0.360674 is a pure unit conversion, exactly as $1 \text{ meter} = 100 \text{ cm}$; it encodes no physics.

For the three common bases:

$$\frac{1}{\ln 2} = 1.442695, \quad \frac{1}{\ln e} = 1, \quad \frac{1}{\ln 10} = \frac{1}{2.302585} = 0.4342945.$$

Check the last: $0.4342945 \times 2.302585 = 1.000000 \dots$ (the standard identity $\log_{10} e = 0.4342945 \dots$). The ratio between representations is

$$\frac{b}{d} = \frac{\ln 10}{\ln 2} = \frac{2.302585}{0.693147} = 3.321928,$$

which is $\log_2 10$ (check: $2^{3.321928} = 10$ by construction of the logarithm). Every member of the family $\{S/\ln b\}$ is a fixed constant multiple of S ; the family has one degree of freedom, and that degree of freedom is the convention.

Empirical-equivalence argument. Any physical prediction of the theory is a statement of the form "the entropy of region $\mathcal{R}$ is at most $A/4$ ". Suppose an observable O depended on the representation; then O would be a function $f(S/\ln b)$ for some b , i.e. a function of S alone with a b -dependent rescaling absorbed. Two observers using bases b_1 and b_2 compute observables related by

$$g_{b_2}(S) = g_{b_1}\left(\frac{\ln b_1}{\ln b_2} S\right),$$

and $\ln b_1 / \ln b_2$ is a known constant. Hence the representations are empirically equivalent at fixed theory content; the invariant formulation differs from the conventional one only in which representation is taken as primitive. This is the paper's central structural result, and it is a consistency theorem, not an experimental prediction.

4.4 Derivation 2: worked example, a macroscopic horizon patch

Input number. We assume, as a labeled illustrative choice, a horizon patch with dimensionless area

$$A = 10^{68} \quad (\text{assumption: patch area in Planck units}).$$

This magnitude is chosen for illustration only; no empirical claim is made about any specific physical system.

Step 1: entropy in nats.

$$S = \frac{A}{4} = \frac{10^{68}}{4} = 2.5 \times 10^{67} \text{ nats.}$$

Step 2: bits. Using $1/\ln 2 = 1.442695$:

$$b = \frac{S}{\ln 2} = 2.5 \times 10^{67} \times 1.442695 = 3.60674 \times 10^{67} \text{ bits.}$$

Arithmetic check: $2.5 \times 1.442695 = 3.6067375$, so $b = 3.60674 \times 10^{67}$ (5 significant figures).

Step 3: Hilbert-space dimension bound.

$$D \leq e^S = e^{2.5 \times 10^{67}}.$$

This number is too large to write directly; we express it via its base-10 logarithm. Using $\ln 10 = 2.302585$:

$$\log_{10} D \leq \frac{S}{\ln 10} = \frac{2.5 \times 10^{67}}{2.302585} = 1.085736 \times 10^{67}.$$

Arithmetic check: $2.5/2.302585 = 1.085736 \dots$ (check: $1.085736 \times 2.302585 = 2.500000$), so $D \leq 10^{1.085736 \times 10^{67}}$.

Step 4: consistency relation.
 $b = \log_2 10 \times \log_{10} D = 3.321928 \times 1.085736 \times 10^{67} = 3.6067 \times 10^{67}$, agreeing with the direct computation of Step 2.

Step 5: base-dependence audit. The statements $S = 2.5 \times 10^{67}$ nats and $D \leq e^{2.5 \times 10^{67}}$ contain no chosen base. The statements $b = 3.60674 \times 10^{67}$ bits and $\log_{10} D \leq 1.08574 \times 10^{67}$ each contain an explicit base constant ($\ln 2, \ln 10$). The audit confirms the classification of Section 4.1.

4.5 Derivation 3: the integer-rounding deficit

The floor function introduces a loss:

$$\Delta = e^{A/4} - \lfloor e^{A/4} \rfloor \in [0, 1).$$

For small areas this matters proportionally. Take the minimal nontrivial case $A = 4$ (one Planck area times the geometric factor 4):

$$e^{A/4} = e^1 = 2.71828 \dots,$$

so

$$D \leq \lfloor e \rfloor = 2, \quad \Delta = 2.71828 - 2 = 0.71828.$$

The relative deficit is

$$\frac{\Delta}{e} = \frac{0.71828}{2.71828} = 0.26424,$$

i.e. the integer bound discards about 26.4% of the allowed dimension at $A = 4$. Arithmetic check: $0.71828/2.71828 = 0.264241 \dots$

For the macroscopic example of Section 4.4, the absolute deficit remains below 1 while $D \sim 10^{1.085736 \times 10^{67}}$, so the relative deficit is astronomically small. The rounding convention is therefore consequential exactly in the regime of Planck-scale areas — the regime where quantum gravity, not information theory, dominates — and inconsequential macroscopically. This locates the anthropocentric residue precisely: it survives only where the theory is least controlled.

4.6 Derivation 4: comparison with the conventional formulation

The conventional formulation writes the bound as $S \leq A_{\text{phys}}/(4\ell_P^2)$ with S in bits:

$$S_{\text{bits}} \leq \frac{A_{\text{phys}}}{4\ell_P^2 \ln 2}.$$

The natural-unit formulation writes $S \leq A/4$ with S in nats. The two differ by the constant factor $\ln 2 \approx 0.693147$ and by the unit of area. Since both $\ln 2$ and ℓ_P are fixed once and for all, the two formulations are related by a fixed change of units and make identical predictions for every observable. We state this as a theorem-level claim: **no observable consequence distinguishes the invariant formulation from the conventional one**. The significance of the program is therefore foundational (which quantities are primitive) rather than empirical (which numbers differ).

5. Results

All numbers below are computed in Section 4 with shown arithmetic; the input areas are labeled illustrative assumptions, not empirical measurements.

R1 (Base-free core). The holographic entropy bound admits the formulation

$$D \leq e^{A/4}, \quad S = \frac{A}{4},$$

containing no logarithm-base constant. The bit appears only in the derived expression $b = S / \ln 2$.

R2 (Conversion constants). The bit-to-nat conversion is $1 / \ln 2 = 1.442695$; the digit-to-nat conversion is $1 / \ln 10 = 0.4342945$; the bit-to-digit ratio is $\log_2 10 = 3.321928$. One Planck area ($A = 1$) corresponds to a maximum entropy of $S = 0.25$ nats = 0.360674 bits = 0.1085736 decimal digits (computed as $0.25 \times 0.4342945 = 0.1085736$; check: $0.1085736 / 0.4342945 = 0.250000$).

R3 (Worked example, labeled illustrative). For the assumed $A = 10^{68}$: $S = 2.5 \times 10^{67}$ nats; $b = 3.60674 \times 10^{67}$ bits; $D \leq 10^{1.085736 \times 10^{67}}$, i.e. $\log_{10} D \leq 1.085736 \times 10^{67}$. This is a mathematical illustration of the formalism, not a claim about any physical region.

R4 (Rounding deficit at minimal scale). At $A = 4$: $e^{A/4} = e = 2.71828 \dots$, so the integer bound is $D \leq 2$, with relative deficit $\Delta/e = 0.26424$ (about 26.4%).

R5 (Invariance and empirical equivalence). Any homogeneous entropy inequality is invariant under change of entropy base, since base change multiplies all entropy terms by the same constant $1 / \ln b$. Consequently, the natural-unit and conventional formulations are related by a fixed change of units ($\ln 2$ and ℓ_P) and are empirically indistinguishable; no experiment can prefer one over the other. The choice of primitive representation is methodological.

R6 (Projection, with assumptions). If a physical region has boundary area A_{phys} , its bit capacity under the bound is projected as $b = 0.360674 \times A_{\text{phys}} / \ell_P^2$, with uncertainty dominated entirely by the unknown physical applicability of the bound in sub-Planckian or non-saturating regimes; the arithmetic conversion itself is exact. No specific A_{phys} is asserted.

6. Discussion

Limitations. First, the reformulation is representational. By R5, it makes no new predictions; a skeptic can correctly say that writing $S \leq A/4$ instead of $b \leq A/(4 \ln 2)$ changes nothing operational. Our defense is that foundational clarity has value independent of new predictions, but we concede the result is weaker than "area and bit are universal invariants" might suggest: the bit is *not* an invariant under our proposal — it is explicitly demoted to a derived unit — so the paper's real claim is that only $\{A, S, D\}$ are candidate invariants, and the bit is not among them. Second, the illustrative areas inherit their status as assumptions; all Results that depend on them do too. Third, the derivation assumes the Bekenstein–Hawking relation $S = A/4$ as a definition; a reader who

regards it as a derived result requiring quantum-gravity input will find our "base-free" claim conditional on that relation. Fourth, the additivity argument for the nat (Section 3.2) shows the nat is *distinguished* among logarithm bases, but it does not show that entropy must be logarithmic at all; that assumption is inherited from the Bekenstein–Hawking framework.

Failure modes. The program fails if (i) the constant 4 in $S = A/4$ turns out to encode a convention (e.g., a choice of horizon slicing) rather than geometry — a derivation from first principles is outside this paper's scope and outside the supplied literature; (ii) the Hilbert-space dimension D is not well-defined for a sub-region, because Hilbert-space factorization is itself observer-dependent — in that case the "integer universal invariant" of [12] is not invariant, and our rounding analysis (R4) quantifies the wrong thing; (iii) entropy at Planck scale is not additive, breaking the uniqueness argument for the natural logarithm; or (iv) the microscopic states of a holographic cell are not countable by a finite integer D — e.g., if the relevant Hilbert space is infinite-dimensional — in which case $S = \ln D$ is undefined and the premise collapses.

What would falsify the claims. R5 is falsified by exhibiting a well-defined observable whose value depends on the logarithm base used in the bookkeeping while all invariant quantities (A , S , D) are held fixed; we know of no candidate, but the burden is on us to state the condition. R1 is falsified if the bound requires a base-dependent correction term (e.g., logarithmic corrections to $S = A/4$ with coefficients involving $\ln 2$ or π from state counting); such corrections would reintroduce base constants into the core formula. The foundational claim — that the nat/area formulation is *preferable* — is not empirically falsifiable at all; it is a methodological thesis, and could only be undermined by showing that some deep structure (e.g., information-theoretic inequalities of the kind studied in [3] and [7]) singles out base 2 intrinsically. The supplied summaries of [3] and [7] do not state any such base preference, so no support for base-2 primacy exists in the cited corpus.

Arguing against ourselves. The strongest objection is that "anthropocentric" is a red herring: the natural logarithm is no more observer-independent than base 2; both are human mathematical constructs, and the true invariant is the *pair* (S , base) up to rescaling. Our reply is that invariance here means invariance under the specific group of base rescalings exhibited in Section 4.3, and the nat is the canonical representative of the orbit only because the map $D \mapsto e^S$ is the identity-structured choice; we do not claim e is metaphysically privileged, only that fixing one representative and deriving the rest makes the convention explicit. A second objection: the bibliography contains no primary holography sources, so the paper's grounding in Bekenstein–Hawking physics rests on the project framing and on [2] and [12]; this is a genuine weakness of the supplied corpus, acknowledged rather than hidden.

Open questions. (1) Is the factor 4 conventional, and can it be derived rather than inserted? (2) Does a sub-region of a holographic system have a well-defined Hilbert-space dimension at all, given that factorization is ambiguous? (3) Can the floor function $\lfloor e^{A/4} \rfloor$ be replaced by a geometrically motivated integer, e.g. via a counting of Planck-cell states, which would remove the last conventional element? (4) Do the non-classical entropy inequalities studied in [3] admit base-free analogues beyond the homogeneous case — i.e., do any of them mix entropy with non-entropy quantities in a way that breaks rescaling invariance? The summaries of [3] and [4] describe inequality and rate bounds for Shannon and channel information, and a systematic base-audit of such results is a natural follow-up. (5) The truncated summary of [12] invokes Ostrowski's theorem concerning Archimedean completion; integrating that number-theoretic thread with the rounding problem of Section 4.5 is left open, since the supplied summary does not state the connection in enough detail to assess it. (6) Whether the finite-storage premise used in [2] — that the observable universe's information capacity is finite — can be stated without any base convention in cosmological settings. (7) Whether quantum channel information rates as bounded in [4] for finite-dimensional memory channels admit the same nat-only restatement; the summary of [4] does not state the form of its bounds, so this is unresolved from the supplied material.

7. Conclusion

Within the Bekenstein–Hawking framework, the entropy bound can be written as $D \leq e^{A/4}$ with A a dimensionless area in Planck units and $S = \ln D$ the physical entropy of a finite Hilbert space of integer dimension D . In this form the bound contains no logarithm base and no counting convention; the bit is recovered as the derived quantity $S/\ln 2$ with conversion factor 1.442695, and every other base-dependent representation is a fixed constant multiple of the same invariant. We showed that no observable at fixed theory content distinguishes the invariant formulation from the conventional bit-based one, so the contribution is foundational and methodological: it identifies exactly which quantities in holographic bounds are candidate universal invariants (A , S , D) and which are conventions (the base, hence the bit), and it locates the remaining conventional residue in the integer rounding of D , consequential only at Planck-scale areas where the relative deficit reaches 26.4% at $A = 4$. Future work must derive the coefficient $1/4$ internally, resolve the factorization ambiguity of sub-region Hilbert spaces, and integrate the Ostrowski-theoretic thread of [12].

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