

Number Systems as Stratified Distinction Operations: An Enclosure Hierarchy over the Mark

Number Systems as Stratified Distinction Operations: An Enclosure Hierarchy over the Mark

Abstract

We develop a formal sketch of the conjecture that the major number systems arise from successive primitive acts of distinction applied to a Laws-of-Form-like mark. We model each historical-cognitive stage — tallying ($\mathbb{N}$), ratio comparison ($\mathbb{Q}$), convergent sequences (computable reals), lossy projection of a tree-like limit (the continuum), phase enclosure ($\mathbb{C}$), divisibility enclosure ($\mathbb{Q}_p$), and simultaneous valuations (the adeles) — as the closure of the previous stage under a single enclosure operator E_i . For each stratum we state the minimal algebraic structure the operator generates, define the inter-stratum morphisms, and introduce a quantitative information-loss functional Λ measuring the distinguishable-state deficit of a projection between adjacent strata, with the Monna map from a p -adic tree to the real line as the canonical example. We carry out explicit arithmetic for the state counts and loss budgets of the first strata: a binary p -adic enclosure at depth $n = 10$ carries $2^{10} = 1024$ distinguishable states, while a real enclosure at precision $\epsilon = 2^{-3}$ carries $2^3 = 8$, giving a Monna-projection loss of $\Lambda = 7$ bits; the loss is linear in depth, $\Lambda(n) = n - 3$, and vanishes at matched depth $n = 3$. We situate the program against the QFNO stratigraphy literature and adjacent work on noncanonical number systems, stratified β -numbers, and p -adic congruence phenomena, and state falsifiability conditions.

1. Introduction

The received picture of the number systems is a tower of completions: $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$, with the p -adic fields $\mathbb{Q}_p$ and the adèle ring $\mathbb{A}$ attached to $\mathbb{Q}$ by a parallel, seemingly independent branch. This paper takes seriously the conjecture — articulated in the QFNO measurement-stratigraphy program [10] and in the Adelic Physics Program's claim that the physically accessible base field is $\mathbb{Q}$ rather than $\mathbb{R}$ [9] — that this progression is not an accident of mathematical history but the trace of an increasingly refined sequence of *distinction operations*: primitive acts of drawing, enclosing, and comparing boundaries on an undifferentiated mark.

The claim has a precise form. Following the idempotence-first critique of arithmetic in [12], in which $1 + 1 = 2$ is read as presupposing two distinguishable marks and quantity itself as "created distinction," we posit a primitive mark M and a single type of act, the *distinction operation* D , which when applied to a structure S produces its enclosure-closure $E(S)$. Each number system is then the minimal algebraic structure generated by one application of one operator to the previous stratum. The sequence of operators is:

$$\{M\} \xrightarrow{E_1: \text{tally}} \mathbb{N} \xrightarrow{E_2: \text{ratio-compare}} \mathbb{Q} \xrightarrow{E_3: \text{convergent sequence}} \mathbb{R}_{\text{comp}} \xrightarrow{E_4: \text{lossy tree projection}} \mathbb{R} \xrightarrow{E_5: \text{phase enclosure}} \mathbb{C} \xrightarrow{E_6: \text{divisibility enclosure}} \mathbb{Q}_p \xrightarrow{E_7: \text{simultaneous}}$$

(We index the operators by their *target* stratum; the arrow labels name the operation.)

Three questions drive the paper. (i) *Recovery*: does each operator, applied minimally, actually generate the intended structure? (ii) *Morphism*: what is the canonical map between adjacent strata, and what does it destroy? (iii) *Loss*: can the destruction be measured? For (iii) we propose a distinguishable-state count per stratum and a loss functional Λ , and we compute it explicitly for the tree-to-continuum projection — the "Monna map" of [10] — in Section 4.

Our contribution is deliberately a *sketch with proofs of small pieces*, not a completed formalization. We prove (in the elementary but fully explicit sense of Section 4) the state-count formulas for the tally, enclosure, and p -adic strata, derive the loss budget for the Monna projection at finite depth, and state the recovery claims for the higher strata as conjectures with falsifiability conditions. No simulation or external measurement is reported; every number in Section 5 is computed by hand in Section 4.

2. Background and Related Work

We review the twelve supplied sources, citing only what their own summaries support.

The direct ancestor of this paper is the QFNO measurement-stratigraphy framework [10], which proposes a "Stratigraphy of Measurement" reinterpreting the history of number systems as a sequence of expanding distinction operations, each era adding a new enclosure: marking (giving $\mathbb{N}$), comparing ratios (giving $\mathbb{Q}$), and convergent sequences (giving a computable real line $\mathbb{R}_{\text{comp}}$). The present paper adopts exactly this stratum list, extends it through $\mathbb{C}$, $\mathbb{Q}_p$, and $\mathbb{A}$, and adds the quantitative loss functional that [10] does not supply; the summary of [10] is truncated mid-list, so we cannot attribute to it any strata beyond $\mathbb{R}_{\text{comp}}$ and treat our extension as our own conjecture.

The epistemological motivation comes from the Adelic Physics Program [9], which proposes that the physically accessible base field of physics is $\mathbb{Q}$, not $\mathbb{R}$, and that Ostrowski's theorem demands all p -adic completions of $\mathbb{Q}$ be physically meaningful; that paper supplies the epistemological and pedagogical infrastructure for taking the p -adic branch as primary rather than peripheral. If the base field is $\mathbb{Q}$, then the stratification question — how each completion of $\mathbb{Q}$ arises from a distinct act of distinction — becomes foundational rather than merely classificatory.

The cognitive-primitive reading is supported by [12], a critique-first paper arguing that arithmetic's hidden premise is that $1 + 1 = 2$ presupposes two distinguishable marks, that idempotence ($AA = A$) is the primitive law, and that quantity is created distinction rather than repetition; it labels its reconstruction program (building $\mathbb{Z}$ from a "!" modality) a conjecture with falsifiability conditions. We adopt the same discipline: our recovery claims below are labeled conjecture or theorem accordingly. The anti-Platonist, embodied-cognition orientation is reinforced by [11], a review of Lakoff & Núñez's *Where Mathematics Comes From* identifying convergence on anti-Platonism, the symbol-concept distinction, and a critique of formalism, with extensions including an ultrametric formalization of the four grounding metaphors; the ultrametric extension is directly relevant to our divisibility-enclosure stratum, though the truncated summary gives no further detail on its content.

Among the arXiv sources, the most structurally relevant is [6] on noncanonical number systems in the integers: it investigates whether every integer can have a finite expansion on a given integer base b when the digit set does not contain 0, proves that such digit sets exist, and provides infinitely many examples for every base b with $|b| \geq 4$. This bears directly on our tally stratum: it shows that the *canonical* digit system (digits including 0) is one choice among many, i.e., that the enclosure operator E_1 admits noncanonical realizations — evidence that the operator, not the particular digit set, is the invariant.

The word "stratified" in our title has a precedent in [3], which introduces modified versions of P. Jones's β -numbers for Carnot groups, called stratified β -numbers, and proves an analogue of Jones's traveling salesman theorem on 1-rectifiability of sets for any Carnot group when previous notions of β -numbers are replaced by the stratified ones, generalizing both directions of the theorem. The analogy is methodological: in both settings, a stratified structure supplies the correct scale-sensitive measurement of how well a set fits a smooth object — precisely the role we want a stratified distinction measure to play for number systems, though [3] operates in Carnot groups and makes no claim about number-system foundations.

The p -adic stratum connects to a cluster of congruence results. [2] proves, for any prime $p \geq 7$, the congruence $\sum_{k=1}^{p-1} H_k^2/k^2 \equiv \frac{4}{5}pB_{p-5} \pmod{p^2}$ for harmonic numbers $H_n = \sum_{k=1}^n 1/k$, confirming a conjecture of Z.-W. Sun, along with two similar congruences modulo p^2 . [4] poses many challenging conjectures on congruences involving binomial coefficients and Apéry-like numbers, and [7] poses further such conjectures modulo p^3 for odd primes p . These works are evidence that mod- p^k structure — the arithmetic of the divisibility enclosure E_6 — is a live and deep layer of number theory; their summaries state conjectures and proofs of congruences and give no interpretive framework, so we use them only as witnesses that the p -adic stratum carries nontrivial, still-open structure.

Two further arXiv entries bear on the base-field and generation themes. [1] is a mostly expository note explaining a proof of Tate's two conjectures for algebraic cycles of arbitrary codimension on certain products of elliptic curves and abelian surfaces over number fields; its relevance is that the objects live over number fields — the field $\mathbb{Q}$ and its finite extensions that our stratification takes as primitive — though the summary supplies no further connection. [5] is an expository survey of the Caccetta–Hággkvist conjecture (that a finite directed graph with at least n/k edges out of each vertex contains a directed cycle of length at most k), including Hamidoune's proofs of the conjecture for Cayley graphs and vertex-transitive graphs using additive number theory; it illustrates how generation acts on discrete structures — the combinatorial shadow of our tally operator — but the summary gives no direct number-systems content. Finally, [8] proves an equivalence concerning the number of distinct prime factors of sums of super-powers $s_n = \sum_{i=1}^k \prod_{j=0}^{\ell} x_{i,j}^{n^j}$, relating growth in n to prime-factor abundance; we cite it as a witness that multiplicative (valuation-theoretic) structure of sequences is quantitatively accessible, while noting the summary is truncated and supports no stronger use.

3. Methods

3.1 The mark and the distinction operation

Following [12], we begin with an undifferentiated mark M satisfying the idempotent law $MM = M$: re-marking does not double. A *distinction operation* D is any act that partitions a structure into distinguishable classes. An *enclosure operator* E is a closure-type operator on structures: $S \subseteq E(S)$, $E(E(S)) = E(S)$, and E is monotone. Each stratum S_i is defined as $S_i = E_i(S_{i-1})$.

3.2 The strata

Stratum 0 (the mark). $S_0 = \{M\}$, a one-point structure with $MM = M$. Distinguishable states: $N_0 = 1$.

Stratum 1 (tallying, $\mathbb{N}$). E_1 is the *tally enclosure*: iterated juxtaposition of marks, each new mark distinguishable from the pile by position. The structure generated is the free monoid on one generator modulo commutativity of the pile: $\mathbb{N}$ with addition. Distinguishable states at tally depth n : the counts $0, 1, \dots, n$, so $N_1(n) = n + 1$.

Stratum 2 (ratio comparison, $\mathbb{Q}$). E_2 encloses pairs of tallies under an equivalence: $(a, b) \sim (c, d)$ iff $ad = bc$. The minimal field of fractions of $\mathbb{Z}$ is $\mathbb{Q}$.

Stratum 3 (convergent sequences, $\mathbb{R}_{\text{comp}}$). E_3 encloses Cauchy sequences of rationals under the equivalence $x_n \sim y_n$ iff $|x_n - y_n| \rightarrow 0$. Applied to all Cauchy sequences this gives $\mathbb{R}$; restricted to computable ones it gives $\mathbb{R}_{\text{comp}}$, the stratum named in [10].

Stratum 4 (lossy tree projection, the continuum). E_4 is the *Monna-type projection* π_M : a tree-like limit structure (a rooted tree of nested enclosures, isomorphic at the state-count level to a p -adic ball, cf. Section 4) is projected lossily onto an ordered line, discarding the branching metric and keeping only linear order and a real precision ε . The recovery claim — that π_M applied to the tree limit yields $\mathbb{R}$ — is **Conjecture C4** of this paper.

Stratum 5 (phase enclosure, $\mathbb{C}$). E_5 encloses the real line with a phase circle: pairs (r, θ) with $r \in \mathbb{R}$, $\theta \in \mathbb{R}/2\pi\mathbb{Z}$, giving $\mathbb{C} \cong \mathbb{R} \times S^1$ as a real structure with multiplication $(r, \theta) \cdot (r', \theta') = (rr', \theta + \theta')$.

Stratum 6 (divisibility enclosure, $\mathbb{Q}_p$). E_6 encloses $\mathbb{Q}$ under a single prime's divisibility: the completion with respect to the p -adic absolute value $|x|_p = p^{-v_p(x)}$. The state structure at depth n is the residue ring $\mathbb{Z}/p^n\mathbb{Z}$, with $N_6(n) = p^n$ distinguishable states.

Stratum 7 (simultaneous valuations, $\mathbb{A}$). E_7 performs all divisibility enclosures at once: $\mathbb{A} = \mathbb{R} \times \prod_p \mathbb{Q}_p$ (restricted product). This is the stratum demanded by the Ostrowski argument of [9]: if the base field is $\mathbb{Q}$, all completions are physically meaningful, and the adèles are their simultaneous enclosure.

3.3 Morphisms and the loss functional

Between adjacent strata $S_i \rightarrow S_{i+1}$ we take the canonical map induced by the operator. The *information-loss functional* is

$$\Lambda_{i \rightarrow i+1}(\text{params}) = \log_2 N_i(\text{params}) - \log_2 N_{i+1}(\text{params}),$$

the base-2 logarithm of the ratio of distinguishable-state counts, i.e., the number of bits destroyed by the projection. For the Monna projection π_M from a depth- n , base- p tree to a real line at precision ε , we compute Λ explicitly in Section 4.

3.4 Falsifiability conditions

Following the discipline of [12], each recovery claim is a conjecture with a stated falsifier. **F1** (tally): if some number system in the sequence cannot be generated by any closure operator over the mark, the stratification fails. **F2** (loss): if Λ_{Monna} computed from state counts is not monotone in tree depth at fixed real precision, the loss functional does not measure projection loss. **F3** (adeles): if a completion of $\mathbb{Q}$ exists that is not captured by any single enclosure operator in our list, the operator list is incomplete.

4. Analysis

All inputs below are either definitions of Section 3 or stated here; every arithmetic step is shown.

Input 1. Tally depth $n = 9$ (a hand-tally of nine marks). Source: chosen parameter.

Input 2. Binary prime $p = 2$, tree depth $n = 10$. Source: chosen parameter.

Input 3. Real precision $\varepsilon = 2^{-3}$, i.e., the unit interval partitioned into cells of width 2^{-3} . Source: chosen parameter.

Input 4. Small primes $2, 3, 5$ for the finite adèle-type product. Source: chosen parameter.

Derivation 1 (tally states). With $n = 9$ marks, the distinguishable counts are $\{0, 1, \dots, 9\}$, so

$$N_1(9) = 9 + 1 = 10.$$

Derivation 2 (enclosure states at precision ε). The unit interval $[0, 1]$ partitioned into cells of width $\varepsilon = 2^{-3}$ has

$$N_{\mathbb{R}}(\varepsilon) = \frac{1}{\varepsilon} = \frac{1}{2^{-3}} = 2^3 = 8$$

distinguishable cells.

Derivation 3 (*p*-adic tree states). The residue ring $\mathbb{Z}/p^n\mathbb{Z}$ with $p = 2$, $n = 10$ has exactly p^n elements:

$$N_6(10) = 2^{10} = 1024.$$

Equivalently, the rooted binary tree of depth 10 has $2^{10} = 1024$ leaves, one per residue class modulo 2^{10} .

Derivation 4 (Monna loss). The Monna projection π_M maps the depth-10 binary tree onto the real line at precision $\varepsilon = 2^{-3}$. The loss functional is

$$\Lambda_{\text{Monna}} = \log_2 N_6(10) - \log_2 N_{\mathbb{R}}(\varepsilon) = \log_2(1024) - \log_2(8) = 10 - 3 = 7.$$

So the projection destroys $\Lambda_{\text{Monna}} = 7$ bits of distinguishable-state information at these parameters. As a check, the ratio form gives the same answer:

$$\Lambda_{\text{Monna}} = \log_2 \left(\frac{N_6(10)}{N_{\mathbb{R}}(\varepsilon)} \right) = \log_2 \left(\frac{1024}{8} \right) = \log_2(128) = 7,$$

and $2^7 = 128$ confirms the ratio.

Derivation 5 (monotonicity of the loss in depth, F2 test). Fix $\varepsilon = 2^{-3}$ and vary depth n . For general n ,

$$\Lambda_{\text{Monna}}(n) = \log_2(2^n) - \log_2(2^3) = n - 3,$$

which is strictly increasing in n ; hence the loss functional passes falsifiability test **F2** at these parameters (monotone in tree depth at fixed real precision). At $n = 3$ the loss is $\Lambda = 3 - 3 = 0$: a depth-3 binary tree projects onto a precision- 2^{-3} real line with no state-count deficit — the two enclosures are state-count isomorphic at matched parameters.

Derivation 6 (finite adele-type product). With primes 2, 3, 5, the product of residue counts at depth $n_p = 1$ for each prime is

$$N_{\mathbb{A}, \text{fin}} = \prod_{p \in \{2, 3, 5\}} p = 2 \times 3 \times 5 = 30.$$

At depth $n_p = 2$ for each prime it is $2^2 \times 3^2 \times 5^2 = 4 \times 9 \times 25 = 900$.

Derivation 7 (stratum count). The operator list of Section 3.2 contains seven operators $E_1, \dots, E_7$ acting on the mark stratum S_0 , producing strata $S_1, \dots, S_7$; total strata including the mark: $7 + 1 = 8$.

5. Results

Every number below is computed in Section 4; no simulation or external measurement is reported.

Quantity	Value	Source derivation
Tally states at depth $n = 9$	$N_1(9) = 10$	Derivation 1
Real cells at $\varepsilon = 2^{-3}$	$N_{\mathbb{R}} = 8$	Derivation 2
Binary tree states at depth $n = 10$	$N_6(10) = 1024$	Derivation 3
Monna loss at ($p=2, n=10, \varepsilon=2^{-3}$)	$\Lambda = 7$ bits	Derivation 4
General Monna loss at depth n , $\varepsilon = 2^{-3}$	$\Lambda(n) = n - 3$	Derivation 5
Loss-free matching depth	$n = 3$ ($\Lambda = 0$)	Derivation 5
Finite adele-type product, depth 1, primes $\{2, 3, 5\}$	30	Derivation 6
Finite adele-type product, depth 2	900	Derivation 6
Total strata (mark + 7 operators)	8	Derivation 7

Result R1. The Monna projection loss is exactly linear in tree depth at fixed real precision: $\Lambda(n) = n - 3$ bits for $\varepsilon = 2^{-3}$, $p = 2$.

Result R2. There exists a matched-parameter pair (depth $n = 3$, precision $\varepsilon = 2^{-3}$) at which the p -adic tree and the real enclosure have identical distinguishable-state counts ($2^3 = 8$), i.e., zero loss. This is the precise sense in which the tree-like limit structure and the continuum are "the same information, differently metricized" at matched resolution — and the sense in which deeper trees carry strictly more information than any fixed-precision continuum.

Result R3. The stratum sequence has eight levels including the mark, and the finite simultaneous-valuation product over $\{2, 3, 5\}$ at depth 1 is 30 states.

Projection (labeled, not computed). If the linear law $\Lambda(n) = n - \log_2(1/\varepsilon)$ holds for general p and ε (assumption: state counts remain p^n and $1/\varepsilon$), then for $p = 3$, $n = 5$, $\varepsilon = 10^{-2}$ the projected loss is $\log_2(3^5) - \log_2(100) \approx 7.925 - 6.644 \approx 1.28$ bits, with uncertainty bounded by the unproven generality of the state-count formulas; we flag this as a projection under stated assumptions, not a result.

6. Discussion

Limitations. The recovery claims for strata 4, 5, and 7 (Conjectures C4, C5, C7) are unproven here; we have only verified state-count arithmetic for strata 0, 1, 3, and 6 at finite depth. The loss functional Λ counts *states*, not *structure*: two projections can destroy the same number of bits while destroying very different algebra (e.g., order vs. branching). The Monna map analysis assumes the tree is exactly a p -adic ball in state count; a general tree-like limit structure might have different branching profiles, changing Λ . The finite adele-type product of Derivation 6 is a toy: the true adele ring uses a restricted product with the real factor, and our count of 30 says nothing about its topology.

Failure modes. If the tally operator E_1 can generate structures other than $\mathbb{N}$ (as the noncanonical digit systems of [6] suggest — digit sets without 0 that still give finite expansions for every integer, for infinitely many examples in every base $|b| \geq 4$), then "the operator determines the stratum" is false: the operator determines only an equivalence class of digit systems, and a choice point remains inside each stratum. This is the strongest internal threat to the thesis, and we flag it rather than resolve it.

Similarly, the congruence literature [2], [4], [7] shows mod- p^k structure of great depth (congruences modulo p^2 and p^3 for harmonic, binomial, and Apéry-like sums); if that depth requires operators beyond E_6 — e.g., operators on Bernoulli or Apéry structures — the seven-operator list is incomplete (falsifier **F3**).

What would falsify the claims. **F1:** exhibit a number system in the historical sequence not generated by any closure operator over the mark. **F2:** exhibit a projection whose state-count loss decreases with refinement — we showed monotonicity only for the linear family $\Lambda(n) = n - 3$. **F3:** exhibit a completion of $\mathbb{Q}$ outside $\{\mathbb{R}, \mathbb{Q}_p : p \text{ prime}\}$ — by the Ostrowski-based argument of [9] this should be impossible, which is why the adele stratum is the safest of our conjectures. A fourth falsifier targets the loss functional itself: if two intuitively different projections (say, Monna and a phase-discard) have identical Λ but intuitively different information content, Λ is not an adequate measure and must be refined, perhaps by the stratified β -number methodology of [3], which measures fit-to-smooth-object at multiple scales in a stratified group setting.

Open questions. (i) Does the loss-free matching of Result R2 generalize: for every p and every dyadic precision $\varepsilon = 2^{-m}$, is there a depth n with $\Lambda = 0$? Under our formulas the answer is yes iff $\log_2 p$ divides appropriately — for $p = 3$ and $\varepsilon = 2^{-3}$, $\log_2(3^n) = 3$ has no integer solution, so no: irrational log-ratios mean generic pairs have nonzero loss. (ii) Can the simulation program — measuring distinguishable states, precision, and loss profiles under each operator — be run at scale, and does it reproduce the linear law of R1? (iii) Does the ultrametric formalization of grounding metaphors mentioned in [11] supply the cognitive counterpart of E_6 ? (iv) What is the loss functional for the adele stratum, where the product is infinite and restricted?

7. Conclusion

We have sketched a stratified-hierarchy formalization of the conjecture that the number systems $\mathbb{N}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, \mathbb{Q}_p, \mathbb{A}$ arise from seven successive distinction operations over a mark, aligned with the QFNO stratigraphy [10], the adelic base-field thesis [9], and the idempotence-first critique [12]. We proved in full arithmetic the finite-depth state counts for the tally, real-enclosure, and p -adic strata, established the linearity and vanishing of the Monna-projection loss at matched parameters, and stated the remaining recovery claims as conjectures with explicit falsifiability conditions. The framework is offered as a research program, not a completed theory: its value lies in making the “one stratum, one operator, one measurable loss” discipline precise enough to be attacked.

References

[1] A note on Tate’s conjectures for abelian varieties. arXiv:2112.15164v2. <https://arxiv.org/abs/2112.15164v2> [2] Proof of a congruence for harmonic numbers conjectured by Z.-W. Sun. arXiv:1108.1171v2. <https://arxiv.org/abs/1108.1171v2> [3] Stratified β -numbers and traveling salesman in Carnot groups. arXiv:1902.03268v2. <https://arxiv.org/abs/1902.03268v2> [4] Conjectures on congruences involving binomial coefficients and Apéry-like numbers. arXiv:2007.12042v4. <https://arxiv.org/abs/2007.12042v4> [5] The Caccetta-Haggkvist conjecture and additive number theory. arXiv:math/0603469v1. <https://arxiv.org/abs/math/0603469v1> [6] Noncanonical number systems in the integers. arXiv:0804.2190v1. <https://arxiv.org/abs/0804.2190v1> [7] New conjectures involving binomial coefficients and Apéry-like numbers. arXiv:2111.04538v2. <https://arxiv.org/abs/2111.04538v2> [8] On the number of distinct prime factors of a sum of super-powers. arXiv:1511.08784v3. <https://arxiv.org/abs/1511.08784v3> [9] DOI 10.5281/zenodo.21686727. QNFO: The Adelic Physics Program: Epistemological Foundations and Communications Framework. [10] DOI 10.5281/zenodo.21705220. QNFO: The History and Future of Measurement Stratigraphy, Number Theory, and Valuation Theory. [11] DOI 10.5281/zenodo.21440894. QNFO: Embodied Mathematics After Lakoff & Nunez: A QNFO Perspective. [12] DOI 10.5281/zenodo.21916939. QNFO: The Idempotent Core: Quantity as Broken Distinction and the Hidden Assumptions of Arithmetic and Algebra.

Appendix A. Divergence report

The three drafts agreed on the qualitative framework (stratified enclosure operators over a mark; strata $\mathbb{N}, \mathbb{Q}, \mathbb{R}_{\text{comp}}, \mathbb{C}, \mathbb{Q}_p, \mathbb{A}$; Monna map as canonical lossy projection; falsifiability discipline from [12]) but diverged on the quantitative model:

- **D1 (state-count convention).** Draft A used a doubling model: each of six strata doubles distinguishable states, giving $s_6 = 2^6 = 64$ and a simplified 1-bit Monna loss. Drafts B and C used budget/depth-parameterized models. *Conflict:* A’s model fixes the number of strata as the exponent; B/C treat depth and precision as free parameters. *Resolution:* the main text adopts B’s parameterized convention (Derivations 1–7), because it keeps depth and precision as explicit inputs and yields the falsifiable linear law $\Lambda(n) = n - 3$; A’s doubling model is a special case of it (six binary distinctions at unit budget) and is retained only as that special case, not as a headline result.
- **D2 (Monna loss value).** Draft A reported a loss of 1 bit (from a binary-phase simplification); Draft B reported $\Lambda = 7$ bits at $(p=2, n=10, \varepsilon=2^{-3})$; Draft C reported a retention fraction $\rho_3(10) = 11/1024 \approx 0.0107$ for a leftmost-path projection. *Conflict:* the three models measure different projections (phase collapse vs. tree-to-line state-count deficit vs. path retention). *Resolution:* the main text adopts B’s state-count deficit as the definition of Λ and reports $\Lambda = 7$ bits; A’s 1-bit figure and C’s retention fraction are recorded here as alternative conventions under different projection definitions, not as conflicting measurements of the same quantity.
- **D3 (budget model).** Draft C additionally computed a budget-constrained comparison: at $b = 10$ binary distinctions, tallying yields $b + 1 = 11$ states while positional enclosure yields $2^{10} = 1024$, ratio $\rho_1(10) = 1024/11 \approx 93.09$; and for $p = 3$ at budget 10, maximal depth $k = 6$ with $3^6 = 729$ classes. Drafts A and B did not include this model. *Status:* SINGLE (Draft C). It is arithmetically self-contained and consistent with the adopted convention, but it is not integrated into the main-text derivations to avoid mixing conventions; it is available as an alternative budget-based reading of the same strata.
- **D4 (operator indexing).** Draft A indexed six operators E_1 – E_6 with the continuum reached by Cauchy completion and no separate tree-projection stratum; Drafts B and C used seven operators with an explicit lossy tree-projection stratum before phase enclosure. *Resolution:* the main text adopts the seven-operator indexing of Draft B, which separates the completion act (giving $\mathbb{R}_{\text{comp}}$) from the lossy projection act (giving the full continuum), since the loss functional is defined on exactly that projection.

Appendix B. Claim attribution

Claim ID	Substance	Source drafts	Agreement
C1	Number systems arise from successive distinction operations on a mark	A, B, C	CONVERGENT
C2	Stratum list $\mathbb{N}, \mathbb{Q}, \mathbb{R}_{\text{comp}}, \mathbb{C}, \mathbb{Q}_p, \mathbb{A}$	A, B, C	CONVERGENT
C3	Idempotence-first reading of arithmetic from [12] as motivation	A, B, C	CONVERGENT
C4	Adelic thesis (base field $\mathbb{Q}$, Ostrowski) from [9] motivates p -adic/adele strata	A, B, C	CONVERGENT
C5	Stratigraphy framework [10] as direct ancestor; summary truncated after $\mathbb{R}_{\text{comp}}$	B, C	CONVERGENT
C6	Noncanonical digit systems [6] show operator, not digit set, is the invariant	B, C	CONVERGENT
C7	Loss functional Λ defined as $\log_2$ ratio of distinguishable-state counts	B, C	CONVERGENT
C8	Monna loss value $\Lambda = 7$ bits at $(p=2, n=10, \varepsilon=2^{-3})$	B	SINGLE (adopted per D2)

Claim ID	Substance	Source drafts	Agreement
C9	Linear law $\Lambda(n) = n - 3$ at fixed $\varepsilon = 2^{-3}$, with loss-free depth $n = 3$	B, C	CONVERGENT
C10	Finite adele-type product counts 30 (depth 1) and 900 (depth 2) over primes $\{2, 3, 5\}$	B	SINGLE
C11	Falsifiability conditions F1–F3 following the discipline of [12]	A, B, C	CONVERGENT
C12	Seven-operator indexing with explicit lossy tree-projection stratum	B, C	CONVERGENT (adopted per D4)
C13	Doubling model $s_6 = 2^6 = 64$ with 1-bit Monna loss	A	SINGLE (retained only as special case per D1)
C14	Budget-constrained comparison ($\rho_1(10) = 1024/11 \approx 93.09$; $p = 3$, $k = 6$, $3^6 = 729$)	C	SINGLE (recorded in D3, not integrated)
C15	Leftmost-path retention fraction $\rho_3(10) = 11/1024 \approx 0.0107$	C	SINGLE (alternative convention per D2)