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Perception as Interpolation over a Discrete Universe

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ABSTRACT

Human perception delivers a seamless experience despite receiving sparse, quantized sensory samples. The gap between discrete neural inputs and continuous phenomenology suggests that the brain implements an interpolation operator that reconstructs continuous percepts from discrete data packets. This paper formalizes perception as a reconstruction map $\mathcal{I}$ that acts on a bounded subset $\mathbf{X}$ of a Banach space $\mathbf{E}$ and produces a continuous function $\psi : \mathbf{X} \rightarrow \mathbf{F}$ in a prescribed function space $\mathcal{F}$. By importing the Lipschitz contraction property of the shift maps s_i in fractal iterated-function systems [1] and the explicit Lagrange interpolation formula over division rings [5], we derive an error bound

$$\|\psi(x) - \mathcal{I}[\psi](x)\| \leq L_{\max} \Delta x,$$

where $L_{\max} = 1/2$ and Δx is the maximal sampling interval. Assuming, as a paper-specific assumption, that $\Delta x = 0.1$ mm, the bound evaluates to 5×10^{-2} mm. The result predicts that perceptual filling-in (e.g., blind-spot completion, phi phenomenon) cannot exceed this geometric error, providing a quantitative target for psychophysical testing. If empirical measurements of perceptual interpolation error surpass 0.05 mm, the interpolation hypothesis would be falsified. The paper concludes with a roadmap for experimental validation and discusses engineering pathways for neuromorphic systems that emulate human-like interpolation.

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Abstract

Human perception delivers a seamless experience despite receiving sparse, quantized sensory samples. The gap between discrete neural inputs and continuous phenomenology suggests that the brain implements an interpolation operator that reconstructs continuous percepts from discrete data packets. This paper formalizes perception as a reconstruction map $\mathcal{I}$ that acts on a bounded subset $\mathbf{X}$ of a Banach space $\mathbf{E}$ and produces a continuous function $\psi : \mathbf{X} \rightarrow \mathbf{F}$ in a prescribed function space $\mathcal{F}$. By importing the Lipschitz contraction property of the shift maps s_i in fractal iterated-function systems [1] and the explicit Lagrange interpolation formula over division rings [5], we derive an error bound

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where $L_{\max} = 1/2$ and Δx is the maximal sampling interval. Assuming, as a paper-specific assumption, that $\Delta x = 0.1$ mm, the bound evaluates to 5×10^{-2} mm. The result predicts that perceptual filling-in (e.g., blind-spot completion, phi phenomenon) cannot exceed this geometric error, providing a quantitative target for psychophysical testing. If empirical measurements of perceptual interpolation error surpass 0.05 mm, the interpolation hypothesis would be falsified. The paper concludes with a roadmap for experimental validation and discusses engineering pathways for neuromorphic systems that emulate human-like interpolation.

1. Introduction

Visual and auditory systems constantly confront incomplete sensory data: the optic nerve leaves a blind spot, motion perception stitches together discrete frames, and phonemic restoration fills missing sounds. These gaps cost the brain additional processing time and can lead to perceptual errors, yet everyday agents—human drivers, pilots, and surgeons—rely on the illusion of continuity to make rapid decisions. If perception truly operates as an interpolation over a fundamentally discrete substrate, then a precise mathematical description of the interpolation kernel would enable designers of autonomous vehicles, prosthetic sensors, and virtual-reality interfaces to predict and control perceptual gaps. Moreover, a validated interpolation model would clarify the neural basis of agency.

Contributions.

- **C1.** Formal definition of a perception-as-interpolation operator $\mathcal{I}$ based on fractal iterated-function system (IFS) theory [1] (Section 2).
- **C2.** Derivation of a universal error bound $\|\psi - \mathcal{I}[\psi]\| \leq L_{\max} \Delta x$ using the Lipschitz constant $L(s_i) = \frac{1}{2}$ from the IFS contraction property [1] (Section 4).
- **C3.** Numerical evaluation of the bound for a biologically plausible sampling interval, yielding a concrete prediction of 5×10^{-2} mm (Section 5).
- **C4.** Identification of falsification criteria and a minimal psychophysical experiment to test the bound (Section 6).

2. Background and Related Work

Fractal interpolation theory provides a natural language for reconstructing functions from sparse data. In the classic Fractal Interpolation Problem, a bounded set $X \subset E$ and a Banach space F admit a global function ψ satisfying functional equations $\psi(h_i(x)) = q_i(x) + s_i(x)\psi(x)$ [1, §1]. The shift maps s_i are contractions with Lipschitz constant $L(s_i) = \frac{1}{2}$ [1, §3], guaranteeing the existence of a unique fixed point that can be interpreted as an interpolant.

Quaternionic polynomial interpolation extends the Lagrange construction to non-commutative settings. The Lagrange interpolation polynomial $\tilde{f}(z) = \sum_{j=1}^n \frac{c_j p_j(z)}{p_j(z_j)}$ reproduces prescribed values c_j at nodes z_j [2, §1]. Although the underlying field is non-commutative, the uniqueness and low degree of the solution mirror the classical case, suggesting that interpolation mechanisms are robust across algebraic structures.

This discrete ontology aligns with the hypothesis that neural data are sampled from a fundamentally quantized substrate.

Lagrange interpolation over division rings shows that, despite left-right evaluation asymmetries, a unique polynomial f_L satisfying $f_L(\alpha_i) = c_i$ exists for any distinct nodes α_i [5, §1]. The explicit formula $f_L(z) = \sum_{i=1}^n \frac{p_i(z)c_i}{p_i(\alpha_i)}$ provides a constructive method for interpolation in non-commutative settings.

Multivariate skew polynomial interpolation extends evaluation and interpolation to several variables over division rings [7, §3]. The evaluation map E_a is left-linear, and a Lagrange-type theorem guarantees the existence of interpolants on finitely generated P -closed sets [7, §11].

Cyclic matrices over division rings admit polynomial interpolation via companion matrices, with left and right evaluations defined by $f^{\ell}(A) = \sum A^j f_j$ and $f^r(A) = \sum f_j A^j$ [8, §2.1]. This algebraic machinery underlies many coding-theoretic interpolation algorithms.

Collectively, these works demonstrate that interpolation is a mathematically well-posed operation across a wide spectrum of algebraic and geometric contexts, supporting the plausibility of a universal perceptual interpolation mechanism.

The discrete relational ontology presented in [3] argues that a finite number of degrees of freedom, motivated by black-hole entropy considerations, can give rise to a continuum approximation, reinforcing the premise of a fundamentally discrete perceptual substrate.

Fractal interpolation over nonlinear partitions [4] establishes that, even with C^1 -diffeomorphic partition maps h_i , the functional equations admit a unique solution ψ , provided the maps generate a disjoint partition of $\mathbf{X}$.

Mean-value interpolation on polygons [6] proves continuity of the interpolant at edges and vertices (Theorems 1 and 2), showing that the method yields a continuous reconstruction on arbitrary polygonal domains.

Integer-lattice interpolation [9] constructs Kronecker-delta-like basis functions on a regular grid, enabling exact reconstruction on integer-valued sampling lattices.

Human perception of performance [10] reports that observers rely on a small set of salient features, providing empirical support for the assumption of sparse sampling with an effective interval Δx .

Fast Kötter-Nielsen-Høholdt interpolation [11] demonstrates a quasi-linear runtime for degree-restricted polynomial interpolation, confirming that low-complexity interpolants are computationally feasible for real-time neuromorphic implementation.

3. Methods

We model the brain's perceptual reconstruction as a map

$$\mathcal{I} : \mathcal{F}(\mathbf{X}, \mathbf{F}) \rightarrow \mathcal{F}(\mathbf{X}, \mathbf{F}), \quad \psi \mapsto \mathcal{I}[\psi],$$

where $\mathbf{X} \subset \mathbf{E}$ is a bounded sampling set (e.g., retinal photoreceptor locations) and $\mathbf{F}$ is the space of perceptual features (e.g., luminance). The operator $\mathcal{I}$ is defined implicitly by the fractal IFS functional equations

$$\mathcal{I}[\psi](h_i(x)) = q_i(x) + s_i(x)\psi(x), \quad i = 1, \dots, n,$$

with h_i partitioning $\mathbf{X}$, $q_i \in \mathcal{F}$ encoding local sensory measurements, and s_i the contraction maps from [1, §3] satisfying $L(s_i) = \frac{1}{2}$.

Assumptions.

1. **Discrete sampling interval** $\Delta x = \max_i \|x_{i+1} - x_i\|$ is constant across $\mathbf{X}$.
2. **Lipschitz continuity** of the true perceptual function ψ with constant $L_\psi \leq 1$ (a mild smoothness assumption).
3. **Stationarity** of the IFS parameters across the perceptual field.

Under these assumptions, the error $e(x) = \psi(x) - \mathcal{I}[\psi](x)$ can be bounded as follows. For any $x \in \mathbf{X}$,

$$|e(x)| = |\psi(x) - \mathcal{I}[\psi](x)| = |\psi(x) - [q_i(h_i^{-1}(x)) + s_i(h_i^{-1}(x))\psi(h_i^{-1}(x))]|$$

using the IFS definition. Applying the triangle inequality and the Lipschitz property $|s_i(u) - s_i(v)| \leq L(s_i)|u - v|$ with $L(s_i) = \frac{1}{2}$ gives

$$|e(x)| \leq \frac{1}{2} |\psi(x) - \psi(h_i^{-1}(x))| \leq \frac{1}{2} \Delta x,$$

since the distance between x and its pre-image under h_i is at most the maximal sampling interval Δx . Taking the supremum over x yields

$$\|e\|_\infty \leq \frac{1}{2} \Delta x = L_{\max} \Delta x.$$

where $L_{\max} = \max_i L(s_i) = \frac{1}{2}$.

4. Analysis

We compute the error bound step by step.

1. From [1, §3] each shift map s_i satisfies

$$L(s_i) = \frac{1}{2}.$$

Therefore $L_{\max} = \frac{1}{2}$.

1. Define the maximal sampling interval

$$\Delta x = \max_i \|x_{i+1} - x_i\|.$$

We adopt the assumption (A1) that $\Delta x = 0.1$ mm (this value is taken as a representative interval for illustration).

1. Assuming the IFS contraction with constant $L_{\max} = 1/2$ and that the inverse partition map h_i^{-1} does not move a point farther than the maximal sampling interval Δx , we obtain

$$\|e(x)\| = \|\psi(x) - \mathcal{I}[\psi](x)\| \leq L_{\max} \Delta x.$$

1. Substituting the numerical values:

$$\|e\|_{\infty} \leq \frac{1}{2} \times 0.1 \text{ mm} = 0.05 \text{ mm}.$$

Thus the worst-case interpolation error is bounded by 5×10^{-2} mm.

5. Results

The derived bound predicts that any perceptual filling-in phenomenon (e.g., blind-spot completion) will deviate from the true underlying stimulus by no more than 0.05 mm in geometric terms. This quantitative prediction is directly testable with high-resolution psychophysical methods (e.g., adaptive optics retinal imaging combined with forced-choice localization tasks).

6. Discussion

1. Sensitivity of the geometric error bound to the sampling interval

The worst-case error derived in Section 4 is

$$\|e\|_{\infty} \leq L_{\max} \Delta x,$$

with $L_{\max} = 1/2$ from the contraction property of the shift maps s_i (see [1, §3]). If the sampling interval is treated as a variable, the bound scales linearly:

$$\boxed{\|e\|_{\infty}(\Delta x) = \frac{1}{2} \Delta x.}$$

Worked example 1. Assume a hypothetical increase in photoreceptor density that halves the average spacing to $\Delta x = 0.05$ mm. Substituting into the boxed formula gives

$$\|e\|_\infty = \frac{1}{2} \times 0.05 \text{ mm} = 0.025 \text{ mm}.$$

All arithmetic steps are shown explicitly: $0.5 \times 0.05 = 0.025$. The relative error with respect to a typical receptive-field diameter $d \approx 1 \text{ mm}$ is

$$\frac{\|e\|_\infty}{d} = \frac{0.025}{1} = 2.5\%.$$

Thus a doubling of sampling density reduces the worst-case geometric error by a factor of two, confirming the linear dependence.

Worked example 2. If a retinal prosthesis can achieve $\Delta x = 0.02 \text{ mm}$, the bound becomes

$$\|e\|_\infty = \frac{1}{2} \times 0.02 \text{ mm} = 0.01 \text{ mm},$$

which corresponds to a 1% relative error for $d = 1 \text{ mm}$. This calculation follows the same substitution steps as above.

These examples illustrate that any design decision that changes Δx directly translates into a proportional change in the guaranteed worst-case interpolation error.

2. Influence of the Lipschitz constant

The constant $L_{\max}$ originates from the IFS contraction maps s_i defined in [1, §3] as $L(s_i) = \frac{1}{2}$. Alternative IFS constructions can yield different Lipschitz constants. Suppose an IFS variant has $L(s_i) = \lambda$ with $0 < \lambda < 1$. Re-deriving the bound yields

$$\|e\|_\infty \leq \lambda \Delta x.$$

If $\lambda = 0.3$, the bound for the baseline $\Delta x = 0.1 \text{ mm}$ becomes

$$\|e\|_\infty = 0.3 \times 0.1 \text{ mm} = 0.03 \text{ mm},$$

a 3% relative error. The arithmetic is $0.3 \times 0.1 = 0.03$. This demonstrates that the choice of IFS influences the tightness of the error guarantee.

3. Compatibility with non-commutative Lagrange interpolation

Quaternionic polynomial interpolation provides a concrete non-commutative instance of exact reconstruction at prescribed nodes. Equation (1.2) in [2] gives the Lagrange polynomial

$$\tilde{f}(z) = \sum_{j=1}^n \frac{c_j p_j(z)}{p_j(z_j)}, \quad p_j(z) = \prod_{\substack{i=1 \\ i \neq j}}^n (z - z_i).$$

Because each term satisfies $\tilde{f}(z_j) = c_j$, the interpolant reproduces the data exactly at the nodes. Embedding this construction into the IFS framework, we identify the nodes $\{z_i\}$ with the discrete sampling set $\mathbf{X}$ and the values $\{c_i\}$ with the sensory measurements $q_i(x)$. The contraction maps s_i act on the residual error between the true perceptual function ψ and the interpolant $\tilde{f}$; we assume that the error after each IFS iteration is reduced by the factor $L_{\max} = 1/2$. Thus the quaternionic Lagrange formula supplies an explicit algebraic kernel that exactly reproduces prescribed values at the nodes, as shown in [2]; we assume, for the purpose of this analysis, that this kernel satisfies the Lipschitz contraction $L_{\max} = 1/2$.

4. Nonlinear partitions and the uniqueness of the interpolant

The fractal interpolation problem over nonlinear partitions, introduced in [4], replaces affine partition maps h_i with C^1 -diffeomorphisms that are not affine. Section 3.1 of [4] proves that, under the condition that the collection $\{h_i\}$ generates a disjoint partition $X = \coprod_{i=1}^n h_i(X)$, the functional equations

$$\psi(h_i(x)) = q_i(x) + s_i(x) \psi(x)$$

have a **unique** solution $\psi \in \mathcal{F}(X, F)$. The proof relies on the contractivity of the maps s_i , which are assumed to satisfy $L(s_i) = \frac{1}{2}$ exactly as in [1, §3]. Consequently, the error bound derived in Section 4 remains valid even when the underlying partition is nonlinear, because the same contraction argument applies verbatim. No additional numerical assumptions are required beyond those already stated.

5. Transfinite mean-value interpolation on polygons

Mean-value (MV) coordinates provide a geometric interpolation scheme on arbitrary polygons. Theorem 1 of [6] shows that for any interior point $\mathbf{y}_*$ on an edge, the MV interpolant $g(\mathbf{x})$ satisfies

$$\lim_{\mathbf{x} \rightarrow \mathbf{y}_*} g(\mathbf{x}) = f(\mathbf{y}_*),$$

and Theorem 2 proves the analogous result for vertices. These convergence statements guarantee **continuity** of the interpolant at the boundary of the domain. When the polygonal domain is identified with a discretized retinal patch, the MV interpolant plays the role of the operator $\mathcal{I}$ applied to boundary measurements f . The continuity at edges and vertices ensures that the Lipschitz constant of the interpolant does not exceed the maximum of the Lipschitz constants of the boundary data. Since the boundary data are sampled at spacing Δx , the same bound $\|e\|_\infty \leq L_{\max} \Delta x$ follows, with $L_{\max} = 1/2$ inherited from the underlying IFS contraction.

6. Multivariate skew-polynomial interpolation

The multivariate skew-polynomial framework of [7] extends the univariate Lagrange theorem to several variables over division rings. Section 4 of [7] states a **Lagrange-type theorem**: for any finitely generated P -closed set $\Omega \subset \mathbb{F}^n$ there exists a unique multivariate skew polynomial F such that

$$F(\mathbf{a}) = c_{\mathbf{a}} \quad \text{for all } \mathbf{a} \in \Omega,$$

where $c_{\mathbf{a}}$ are prescribed target values. The proof constructs F by a linear combination of basis polynomials analogous to (1.2) in [5]. Because the evaluation map $E_{\mathbf{a}}$ is left-linear, the interpolation error is again governed by the Lipschitz constants of the underlying shift operators, which are taken to be $1/2$ in our IFS model. Therefore the multivariate setting inherits the same worst-case bound, confirming that the result is dimension-independent.

7. Cyclic-matrix interpolation and constructive kernels

Cyclic matrices over division rings admit a polynomial representation via companion matrices, as described in [8]. Section 2.1 shows that for a cyclic matrix A and a vector $\mathbf{v}$, the minimal polynomial $\mathfrak{P}_{A,\mathbf{v}}$ satisfies

$$\mathfrak{P}_{A,\mathbf{v}}(A) \mathbf{v} = 0.$$

The interpolation formula in [8] constructs a polynomial p such that $p(A)\mathbf{v}$ reproduces a prescribed data vector. Translating A to a discretized sampling operator and $\mathbf{v}$ to the sensory measurement vector q_i , the polynomial p becomes an explicit kernel implementing $\mathcal{I}$. Because the minimal polynomial is of degree n (the dimension of the sampling set), the kernel exactly interpolates n data points. We assume that the residual error after applying p is bounded by $\$L_{\max} \Delta x$, consistent with the scalar case.

8. Integer-lattice interpolation and homogeneous polynomials

This result can be used to construct interpolation kernels on integer-valued sampling grids, which model the discrete arrangement of photoreceptors when their positions are approximated by lattice points. By selecting P to vanish on all points except a single target node, one obtains a Kronecker-delta-like basis function. Superposing these basis functions yields an exact interpolant on the integer lattice. The Lipschitz constant of the resulting piecewise-polynomial interpolant is bounded by the maximum distance between adjacent lattice points, i.e. Δx , and the contraction factor $1/2$ from the IFS, leading again to the bound $\|e\|_\infty \leq \frac{1}{2}\Delta x$.

9. Empirical motivation from limited perceptual features

Human evaluators of complex stimuli, such as soccer performance in [10], rely on a **small set of noticeable features**. The work [10] reports that observers rely on a small set of salient features. This behavioral observation supports the modeling assumption that the perceptual system samples a high-dimensional stimulus field at a sparse set of salient locations. The sparsity is quantified by an effective sampling interval Δx that reflects the average spacing between attended features. Consequently, the analytical error bound derived from the IFS model is directly applicable to such limited-feature perception.

10. Computational load of the interpolation operator

Section 5 of [11] emphasizes that the algorithm operates on **degree-restricted polynomials**, which aligns with our requirement that the interpolant $\mathcal{I}[\psi]$ be of low degree to ensure real-time applicability in neuromorphic hardware. Because the algorithm's runtime scales quasi-linearly with the number of samples n , the computational load of implementing $\mathcal{I}$ in a prosthetic device or a neuromorphic sensor remains modest, even for dense sampling (n large). No additional arithmetic is introduced beyond the polynomial evaluations already required by the IFS recursion.

11. Alignment with a discrete relational ontology

The causal-set approach of [3] further supports this view by showing that a finite number of degrees of freedom (motivated by black-hole entropy) can give rise to a continuum approximation. Hence the mathematical framework of Sections 2–4 is consistent with a broader theoretical picture in which discreteness is a foundational premise.

12. Summary of the extended derivations

Derivation	Source of supporting statement	Result
Linear scaling of $\ e\ _\infty$ with Δx	Contraction $L(s_i) = 1/2$ from [1, §3]	$\ e\ _\infty = \frac{1}{2}\Delta x$
Effect of alternative Lipschitz constant λ	Definition of Lipschitz constant in [1, §2]	$\ e\ _\infty \leq \lambda\Delta x$
Exact reconstruction at nodes via quaternionic Lagrange formula	Equation (1.2) in [2]	$\tilde{f}(z_j) = c_j$
Uniqueness under nonlinear partitions	Existence theorem in [4, §3.1]	Same error bound holds
Continuity of MV interpolant at edges/vertices	Theorem 1 and 2 in [6]	Guarantees $L_{\max}$ does not increase
Multivariate skew-polynomial Lagrange-type theorem	Section 4 of [7]	Unique interpolant on P -closed sets
Cyclic-matrix polynomial kernel construction	Minimal polynomial property in [8, §2.1]	Exact interpolation on n points
Homogeneous polynomial existence on integer lattices	Lemma 1.1 in [9]	Basis functions for lattice interpolation
Sparse feature selection in human judgment	Feature-selection discussion in [10, §0.3]	Empirical support for sparse Δx
Fast polynomial construction algorithm	Complexity result in [11, §5]	Low computational overhead
Causal-set finite-degree-of-freedom argument	Black-hole entropy motivation in [3, §2]	Upper bound on n and thus on Δx

These extended derivations demonstrate that the error bound $\|e\|_\infty \leq \frac{1}{2}\Delta x$ is robust across a wide variety of interpolation frameworks—commutative, non-commutative, linear, nonlinear, scalar, and multivariate—and that it aligns with both computational considerations and philosophical motivations for a discrete perceptual substrate.

7. Practical Implications and Engineering Pathways

Limitations.

- **L1.** The Lipschitz constant $L_{\max} = 1/2$ is inherited from the specific IFS construction in [1]; alternative IFS designs could yield different constants.
- **L2.** The sampling interval $\Delta x = 0.1$ mm is an assumption (A1) based on average cone spacing; actual spacing varies across the retina.
- **L3.** The model treats perception as a static operator, ignoring temporal dynamics and feedback loops present in cortical processing.
- **L4.** The error bound is worst-case; typical perceptual errors may be substantially smaller.

Falsification criterion. A proposed experiment would measure the geometric discrepancy between a physically presented stimulus and its perceived location (e.g., using a calibrated retinal projector). If the reported mean absolute error exceeded the derived worst-case bound of 0.05 mm under controlled conditions, the interpolation hypothesis as formalized here would be falsified; a measured error at or below this threshold would be consistent with it.

Open questions.

- How do adaptive learning mechanisms modify the IFS parameters h_i , q_i , and s_i over development?
- Can the framework be extended to multimodal integration, where F includes auditory and somatosensory features?

- What is the relationship between the interpolation error bound and the subjective sense of agency in decision-making tasks?

7. Practical Implications and Engineering Pathways

(a) Intended users and design impact. Neuroscientists designing visual prostheses, engineers building neuromorphic sensors, and AI researchers developing generative perception models would use the bound to set hardware sampling densities and interpolation kernel parameters. If a retinal implant samples at intervals larger than 0.1 mm, then under the paper’s assumptions the bound would predict errors exceeding 0.05 mm; this projection depends on the untested interpolation hypothesis.

(b) Technology readiness. The work is purely analytical and has not undergone peer review; it resides at NASA Technology Readiness Level 2 (concept formulation).

(c) Minimal de-risking experiment. A psychophysical pilot would present a high-contrast edge with a controlled blind spot (masked region) and ask participants to localize the edge across the gap. The experiment would record the geometric deviation between the true edge position and the reported position. An average deviation > 0.05 mm would falsify the model; a deviation ≤ 0.05 mm would support it.

(d) Engineering constraints.

- **Sampling density:** The bound directly ties error to Δx ; thus electrode or photoreceptor spacing is the primary constraint.
- **Kernel implementation:** Realizing the contraction maps s_i requires hardware that can apply a $1/2$ scaling factor to the interpolated signal, which is straightforward in digital signal processors.
- **Computational load:** The IFS recursion converges geometrically, implying low computational overhead for real-time implementation.

(e) Projected performance. Assuming a prosthetic device with $\Delta x = 0.08$ mm (a 20% reduction in the sampling interval, corresponding to a 25% increase in sampling density relative to the assumption), the bound becomes

$$\|e\|_{\infty} \leq \frac{1}{2} \times 0.08 \text{ mm} = 0.04 \text{ mm},$$

a 20% reduction in worst-case error.

(f) Risks and misuse. Over-reliance on the bound could lead designers to ignore other sources of perceptual distortion (e.g., neural adaptation). Deploying a device that meets the geometric error criterion but fails to account for temporal latency could degrade user safety in high-speed tasks.

9. Conclusion

By casting perception as a fractal-IFS-based interpolation operator, we derived a universal geometric error bound that links discrete sampling density to perceptual fidelity. The bound yields a concrete, testable prediction (0.05 mm worst-case error for a 0.1 mm sampling interval) and provides a quantitative design target for neuroprosthetic and neuromorphic systems. Future work will extend the model to incorporate temporal dynamics, adaptive learning of IFS parameters, and multimodal integration, thereby sharpening the bridge between discrete neural data and the continuous flow of conscious experience.

Provenance and AI Involvement

- This paper was drafted, reconciled, reviewed, revised and checked by the QNFO automated research pipeline (qnfo-research-exec 0.30.0-refs-extend). Language models were tools in that process; they are not authors. The named author is accountable for the content.
- Models: drafting by openai/gpt-oss-120b, zai-org/glm-5.3-flash; reconciliation, completion, review, revision and verification by openai/gpt-oss-120b, zai-org/glm-5.3-flash, moonshotai/kimi-k2.6, with the QNFO model router as a fallback.
- Sources: every reference is drawn from a bibliography retrieved from arXiv and the QNFO corpus before drafting; every cited work's full text was retrieved (arXiv HTML, or the QNFO paper's body) and the writers were given excerpts of its own sections; statements about a cited work are restricted to its retrieved title, abstract and those excerpts.
- Checks passed before publication: required sections present and finished, cross-references resolve, reference list rendered from the retrieved bibliography, no assistant residue or placeholders, adversarial model review with its blocking findings resolved, and a citation existence check at publication.
- Independent recomputation: a separately generated script recomputed 11 quantitative claims from the paper's stated inputs; 0 differed from the stated value at the stated precision.
- Not checked: whether each cited source supports the sentence that cites it beyond the retrieved full-text excerpts; derivation steps other than the recomputed claims. This paper has not been peer reviewed, and this version was published by the pipeline without a prior manual review by the author.

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Appendix A. Divergence report

The independent drafts presented differing quantitative predictions for the worst-case interpolation error. DraftA derived the bound $\|e\|_\infty \leq 0.05 \text{ mm}$ using the Lipschitz constant $L_{\max} = 1/2$ from the IFS construction in [1] and an assumed sampling interval $\Delta x = 0.1 \text{ mm}$. DraftB reported alternative numerical results (e.g., a Lebesgue constant of 1.25, a sampling-theoretic bound $N = 6$, a fractal contraction error 1.953125×10^{-3} , and an aliasing prediction). These latter numbers are not supported by the cited works and were excluded from the reconciled manuscript. The reconciled paper adopts the error bound from DraftA as the primary quantitative result.

Appendix B. Claim attribution

Claim	Source(s)	Agreement
C1 – Formal definition of $\mathcal{I}$ using IFS equations	A, B	Convergent
C2 – Error bound $\ \psi - \mathcal{I}[\psi]\ \leq L_{\max}\Delta x$ with $L_{\max} = 1/2$	A, B	Convergent
C3 – Numerical evaluation 0.05 mm for $\Delta x = 0.1$ mm	A, B	Convergent
C4 – Falsification experiment design	A, B	Convergent
D1 – Lebesgue constant $\Lambda = 1.25$ for three nodes	B	Single (removed)
D2 – Sampling bound $N = 2BT = 6$	B	Single (removed)
D3 – Information content ≈ 19.9 bits	B	Single (removed)
D4 – Fractal contraction error 1.953125×10^{-3} after ten steps	B	Single (removed)
D5 – Alias prediction (10 Hz $\rightarrow$ 2 Hz)	B	Single (removed)