

Planck Fidelity: Fundamental Leakage Cost of Binary Truncation of Bosonic Ladders

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Abstract

The transmon is a bosonic oscillator whose spectrum is a discrete but unbounded ladder of energy eigenstates; encoding it as a qubit, i.e. restricting the computational space to $n \in \{0, 1\}$, is a radix choice rather than a physical necessity. We formalize the conjecture, originating in the QNFO "Rosetta" corpus, that any finite-time gate driven through the raising operator $a^\dagger$ on such a ladder incurs a strictly positive "Planck Loss" $L_P = 1 - \text{Tr}(P_{\text{qubit}} |\psi\rangle\langle\psi|)$, a leakage floor independent of materials and control engineering. We give two complementary derivations: an exact detuned-Rabi solution in the three-level truncation, and a first-order perturbative bound $L_2 = 8(\Omega/|\alpha|)^2 \sin^2(\alpha T_g/2)$. Under explicitly labeled illustrative parameters ($f_\Omega = 20$ MHz, $f_\alpha = 200$ MHz), a single π pulse carries coherent leakage $\approx 4.76 \times 10^{-4}$ and a 1000-gate sequence retains only $\approx 62\%$ subspace population absent leakage return. We quantify the truncation entropy $S_{\text{Base}} = \ln(d/2) = 1.791759$ nats for $d = 12$ resolvable levels, showing binary encoding uses $\approx 27.9\%$ of the ladder's Hilbert-space information content. We propose a Planck Fidelity benchmark, state falsification conditions, and argue the results motivate base- d qudit architectures.

1. Introduction

A qubit is, by definition, a two-dimensional quantum system. The transmon — a capacitively shunted Josephson junction behaving, to leading order, as a weakly anharmonic bosonic oscillator — is not such a system. Its spectrum is a discrete but unbounded ladder $\{|n\rangle\}_{n=0}^\infty$, and the computational subspace $\{|0\rangle, |1\rangle\}$ is carved out of it by fiat. The choice of radix 2 is an engineering convention inherited from the classical bit, not a consequence of the device physics. This observation, and the conjecture that the resulting leakage is a *fundamental* cost of the radix choice rather than an avoidable imperfection, is due to the QNFO "Rosetta" corpus [9], [10], which argues that the transmon is properly a qudit with $d \sim 12$ resolvable levels and that binary truncation of it constitutes a "radix error," not a physics error.

This paper takes the conjecture seriously as a quantitative research program. Our contributions are:

- 1. A formal statement of Planck Loss.** For a gate $U(T_g)$ generated by a Hamiltonian containing a raising operator $a^\dagger$, acting on an initial state in the qubit subspace, we define $L_P = 1 - \text{Tr}(P_{\text{qubit}} |\psi(T_g)\rangle\langle\psi(T_g)|)$ with $P_{\text{qubit}} = |0\rangle\langle 0| + |1\rangle\langle 1|$, and identify when $L_P > 0$ and in which singular limits $L_P \rightarrow 0$.
- 1. Two leakage derivations.** An exact solution for the $|2\rangle$ population in the three-level truncation under a resonant drive, and a first-order perturbative expression $L_2 = 8(\Omega/|\alpha|)^2 \sin^2(\alpha T_g/2)$, both evaluated numerically under clearly labeled illustrative parameters.
- 1. An entropy accounting of truncation.** Following the $S_{\text{Base}} = \ln(d/2)$ formula of [9], we compute the nats and bits discarded by binary truncation of a $d = 12$ ladder, and the fraction of the device's Hilbert-space information content lost to the qubit abstraction.
- 1. A benchmark proposal.** We define a "Planck Fidelity" metric $F_P = 1 - L_P$ and specify how it is to be compared against measured leakage.

A terminological hazard must be flagged at the outset. The name "Planck" in this program refers to Planck quantization of the ladder, in the sense of [9]. It has no physical connection to the European Space Agency's Planck cosmic microwave background (CMB) satellite, whose mission papers [1]–[8] dominate any literature search on the term. Section 2 discusses those papers explicitly, both to disambiguate the naming collision and because their methodological stance — extracting essentially all of the information present in a physical signal rather than a convenient subset — is a useful mirror for the information-theoretic argument of this paper.

Throughout, "leakage" means population of states outside the computational subspace; "anharmonicity" means the detuning α of the $|1\rangle \rightarrow |2\rangle$ transition from the $|0\rangle \rightarrow |1\rangle$ transition; and "radix" means the number of distinguishable symbols per physical carrier.

2. Background and Related Work

The bibliography supplied for this paper is heterogeneous: eight entries concern the Planck CMB mission, and three concern the QNFO "Project Rosetta" corpus from which the present research idea originates. We discuss each strictly on the basis of its supplied summary, and we are explicit where a summary is thin.

The Planck CMB mission papers [1]–[8]. Reference [1], *The Scientific Programme of Planck*, states that for 40 years the CMB has been the most important source of information about the geometry and contents of the Universe, that only a small fraction of the available information had been extracted to date, and that Planck — the third space CMB mission after COBE and WMAP — is designed to extract essentially all of the information in the CMB temperature anisotropies. The design goal of exhaustive information extraction is the salient point for us: it is the exact inverse of the qubit abstraction, which deliberately discards most of the information a physical device carries. Reference [2], *Planck Early Results: The Planck mission*, records that the satellite was launched on 14 May 2009 and has been surveying the sky stably and continuously since 13 August 2009, with performance in line with expectations; a decade-scale, continuously validated survey is the kind of measurement discipline a leakage benchmark would ultimately require. Reference [3] describes the Planck Catalogue of Compact Sources (PCCS), nine single-frequency catalogues of compact Galactic and extragalactic sources over the entire sky, covering 30–857 GHz, with 90% completeness at 180 mJy in the best channel; the catalogue structure — many frequency channels resolving one sky — is a concrete instance of multi-channel (base- d -like) representation outperforming any single channel. Reference [4] describes the all-sky Sunyaev–Zeldovich cluster catalogue from the first 15.5 months of observations, containing 1227 entries — over six times the earlier ESZ sample — of which 861 are confirmed clusters; again, a census whose value lies in counting and classifying all resolvable objects rather than a privileged subset, the same move we advocate for transmon levels. Reference [5] exploits the Early Release Compact Source Catalogue to measure number counts dN/dS of extragalactic radio sources at 30, 44, 70, 100, 143 and 217 GHz, noting that the full-sky nature of the catalogue extends the measurement to the rarest and brightest sources; counting the population of each rung of a brightness ladder is structurally the exercise we propose for occupation numbers n of a bosonic ladder. Reference [6], on the HFI spectral response, states that the relative spectral response, including out-of-band signal rejection, of all HFI detectors was measured in ground-based cryogenic tests prior to launch. Out-of-band rejection is the direct observational analogue of computational-subspace leakage: signal arriving outside the designated band is indistinguishable, in its harmful effect, from population arriving outside $\{|0\rangle, |1\rangle\}$, and it must be characterized, not assumed away. Reference [7] presents the Second Planck Catalogue of Compact Sources, detected in single-frequency maps over the full mission duration and superseding previous versions; it illustrates that censuses are revised as instrument understanding improves, just as level counts d of a transmon are revised as coherence improves. Reference [8] describes the pipeline calibrating LFI timelines into thermodynamic temperatures over four years of operations, using the CMB dipole — spin-synchronous modulation as in 2013, now with the orbital component added — as the calibrator; calibration against an absolute reference is the analogue of the leakage benchmarking we propose in Section 3.5.

We emphasize for honesty: none of [1]–[8] says anything about transmons, qudits, or leakage. Their role here is (i) disambiguation of the term "Planck" and (ii) methodological analogy only.

The QNFO corpus [9]–[11]. Reference [9], *Project Rosetta: The Approximation Entropy & The Fractal Limits of Digital Physics* — v2.0 (<https://papers.qnfo.org/papers/rosetta-fractal-math/>), is the direct source of the central conjecture. Its summary states that v2.0 corrects v1.0: the transmon spectrum is discrete (Planck quantization), so binary truncation is a radix error, not a physics error; the transmon is a qudit with $d \sim 12$, not a qubit; and the base-truncation entropy is $S_{\text{Base}} = \ln(d/2) \sim 1.79$ nats. We adopt the quantity S_{Base} and recompute it exactly in Section 4.1, finding agreement. Reference [10], *The Two-Level Lie: The Transmon Is Not a Qubit — And the Entire Field Knows It* (<https://papers.qnfo.org/papers/the-two-level-lie/>), supplies the framing: its v2.0 adds "Part II: The Approximation Entropy," described as a meta-mathematical framework quantifying the irreducible cost of translating continuous bosonic physics to discrete digital computation, and introduces a "Rosetta Constant," a "Trotter Wall," and a "Fractal Wall theorem" together with a falsifiable calorimetry experimental proposal. The supplied summary gives no further detail on these objects, so we do not use them quantitatively; we note only that the falsifiability orientation of [10] matches the falsification criteria we state in Section 6. Reference [11], *Thermodynamic Scaling of 4-Kelvin*

Topological Processors (<https://papers.qnfo.org/papers/thermodynamic-scaling-of-4-kelvin-topological-processors/>), has an empty summary in the supplied bibliography; we cannot state what it contains, and relate it to our argument only through its title, which suggests a thermodynamic-scaling perspective on alternative hardware that would be the natural venue for an entropy-cost accounting such as S_{Base} . This is recorded as a limitation in Section 6.

Gap. Strictly within the supplied corpus, no work derives a quantitative leakage bound on transmon gates from the radix-truncation viewpoint. Neither [9] nor [10] supplies the derivations given below; they are original to this paper, built on the conjecture's own premises.

3. Methods

3.1 Device model

We model the transmon as a weakly anharmonic bosonic oscillator with Hamiltonian ($\hbar = 1$)

$$H_0 = \omega a^\dagger a + \frac{\alpha}{2} a^\dagger a^\dagger a a,$$

where $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$, ω is the $|0\rangle \rightarrow |1\rangle$ transition frequency, and α is the anharmonicity. The energy of level n is $E_n = \omega n + \frac{\alpha}{2}n(n-1)$, so the $|1\rangle \rightarrow |2\rangle$ transition is detuned from the qubit transition by exactly α . A classical drive near the qubit frequency adds, in the rotating frame and within the rotating-wave approximation (RWA),

$$H_d(t) = \Omega \epsilon(t) (a^\dagger e^{i\alpha t} + a e^{-i\alpha t}),$$

with Rabi amplitude Ω and dimensionless envelope $\epsilon(t)$, $|\epsilon(t)| \leq 1$.

3.2 Definitions

- **Qubit projector:** $P_{\text{qubit}} = |0\rangle\langle 0| + |1\rangle\langle 1|$.
- **Planck Loss:** for an initial state in the qubit subspace,

$$L_P = 1 - \text{Tr}(P_{\text{qubit}} |\psi(T_g)\rangle\langle\psi(T_g)|) = \sum_{n \geq 2} |c_n(T_g)|^2.$$

- **Planck Fidelity:** $F_P = 1 - L_P$.
- **Base-truncation entropy** (from [9]): $S_{\text{Base}} = \ln(d/2)$ nats.

3.3 Perturbative leakage derivation

Work in the interaction picture with respect to H_0 . To first order in Ω , the amplitude to transition from $|1\rangle$ to $|2\rangle$ is, using $\langle 2|a^\dagger|1\rangle = \sqrt{2}$,

$$c_2^{(1)}(T_g) = -i\Omega\sqrt{2} \int_0^{T_g} \epsilon(t) e^{i\alpha t} dt.$$

For a square envelope $\epsilon(t) = 1$ (an idealization giving the cleanest closed form):

$$c_2^{(1)}(T_g) = -i\Omega\sqrt{2} \frac{e^{i\alpha T_g} - 1}{i\alpha} = \Omega\sqrt{2} e^{i\alpha T_g/2} \frac{2 \sin(\alpha T_g/2)}{\alpha}.$$

Hence

$$L_2 = |c_2^{(1)}|^2 = \left(\Omega\sqrt{2}\right)^2 \left(\frac{2 \sin(\alpha T_g/2)}{\alpha}\right)^2 = 8 \left(\frac{\Omega}{|\alpha|}\right)^2 \sin^2\left(\frac{\alpha T_g}{2}\right),$$

valid in the perturbative regime $8(\Omega/|\alpha|)^2 \ll 1$. Higher levels $n \geq 3$ contribute at higher order and are neglected (see Section 6).

3.4 Exact three-level solution

Within the truncation $\{|0\rangle, |1\rangle, |2\rangle\}$, the $|1\rangle \leftrightarrow |2\rangle$ pair is a driven two-level system with coupling $g = \Omega\sqrt{2}/2 = \Omega/\sqrt{2}$ and detuning $\delta = \alpha$. For initial condition $c_1(0) = 1$, the detuned-Rabi solution gives

$$P_2(t) = \frac{2\Omega^2}{\alpha^2 + 2\Omega^2} \sin^2\left(\frac{\alpha}{2}\sqrt{1 + 2r^2} t\right), \quad r = \frac{\Omega}{\alpha}.$$

Two exact consequences: the peak leakage is $P_{2,\max} = \frac{2r^2}{1 + 2r^2}$, and the time-averaged leakage (using $\langle \sin^2 \rangle = 1/2$ over a Rabi period) is $\overline{P}_2 = \frac{r^2}{1 + 2r^2}$.

3.5 Strict-positivity argument

Writing $c_2^{(1)}(T_g) = -i\Omega\sqrt{2}\tilde{\epsilon}(\alpha)$ with $\tilde{\epsilon}(\alpha) = \int_0^{T_g} \epsilon(t)e^{i\alpha t} dt$, we have $L_2 = 2\Omega^2|\tilde{\epsilon}(\alpha)|^2$. For $L_2 = 0$ exactly, the finite-time drive spectrum must vanish precisely at the anharmonicity. For a compactly supported envelope, $\tilde{\epsilon}$ is an entire function and can vanish at isolated parameter values, so specific combinations of (α, T_g, ϵ) can null the first-order leakage. However: (i) the null condition is a measure-zero constraint, not generically satisfiable jointly with the gate constraint on the qubit subspace; (ii) second-order pathways remain, so the total $L_P = \sum_{n \geq 2} |c_n|^2$ is strictly positive for any finite-time, finite-amplitude drive of the stated form; and (iii) $L_P \rightarrow 0$ only in the singular limits $T_g \rightarrow \infty$ or $\Omega \rightarrow 0$. The robust statement of the conjecture is therefore: $L_P > 0$ for generic finite-time gates, with pointwise zeros confined to a measure-zero set — consistent with the exact solution of Section 3.4, where $P_2(t)$ vanishes at isolated revival times $t_k = k\pi / (\frac{\alpha}{2}\sqrt{1 + 2r^2})$. This is a model-class result, not a theorem over all conceivable Hamiltonians (Section 6).

3.6 Worst-case uniform-population scenario

As a deliberately pessimistic, separately labeled scenario (not a physical prediction), assume population uniformly distributed over the d experimentally resolvable levels ($d \approx 12$ per [9]). The qubit-subspace population is then $2/d$ and the leakage is $L_P^{(\text{wc})} = (d - 2)/d$. This quantifies the cost of total radix ignorance, not the coherent leakage of a driven gate.

3.7 Benchmark protocol

The Planck Fidelity benchmark compares (i) the predictions of Sections 3.3–3.4, evaluated with the device's measured α and calibrated Ω, T_g , against (ii) measured leakage from standard leakage-characterization sequences on a device with $d \approx 12$ resolvable levels as posited in [9]. Agreement within the perturbative regime supports the model; systematic excess attributes to non-RWA, higher-level, or materials effects — which, per the conjecture, add to rather than excuse the radix floor.

4. Analysis

Every input number is stated with its source; every arithmetic step is shown.

4.1 Base-truncation entropy

Inputs. $d = 12$ resolvable levels — stated in [9] ("transmon is a qudit, $d \sim 12$ "). Formula $S_{\text{Base}} = \ln(d/2)$ — stated in [9].

$$S_{\text{Base}} = \ln\left(\frac{12}{2}\right) = \ln 6 = \ln 2 + \ln 3 = 0.693147 + 1.098612 = 1.791759 \text{ nats}.$$

In bits, dividing by $\ln 2 = 0.693147$:

$$S_{\text{Base}} = \frac{1.791759}{0.693147} = 2.584963 \text{ bits}.$$

This reproduces the " ~ 1.79 nats" quoted in [9] to the precision given there.

4.2 Fraction of Hilbert-space information discarded

Inputs. $\ln 12$ with $d = 12$ (from [9]); $S_{\text{Base}} = 1.791759$ nats (Section 4.1).

$$\ln 12 = 2 \ln 2 + \ln 3 = 1.386294 + 1.098612 = 2.484906 \text{ nats.}$$

$$f_{\text{disc}} = \frac{S_{\text{Base}}}{\ln d} = \frac{1.791759}{2.484906} = 0.721125.$$

Binary truncation discards 72.1% of the ladder's Hilbert-space information content, retaining $1 - 0.721125 = 0.278875$, i.e. 27.9%. Equivalently, the radix efficiency of qubit encoding is $\eta = \log_2 2 / \log_2 12 = 1/3.584963 = 0.278942 \approx 27.9\%$, and one $d = 12$ oscillator faithfully requires $\lceil \log_2 12 \rceil = 4$ qubits merely to represent the radix.

4.3 Qudit–qubit capacity ratio

$$\frac{\ln 12}{\ln 2} = \frac{2.484906}{0.693147} = 3.584963 = \log_2 12.$$

A single $d = 12$ qudit carries the information of ≈ 3.58 qubits.

4.4 Numerical evaluation of the leakage (illustrative parameters, Set B)

Inputs. All parameters in this subsection are *illustrative assumptions*, not measurements: $f_\Omega = 20$ MHz, $f_\alpha = 200$ MHz, so

$$r = \frac{f_\Omega}{f_\alpha} = \frac{20}{200} = 0.1, \quad r^2 = 0.01.$$

Peak and time-averaged leakage (Section 3.4):

$$P_{2,\text{max}} = \frac{2 \times 0.01}{1 + 2 \times 0.01} = \frac{0.02}{1.02} = 0.019608 \approx 1.96 \times 10^{-2}; \quad \bar{P}_2 = \frac{0.01}{1.02} = 0.009804 \approx 9.80 \times 10^{-3}.$$

Leakage at the end of a π pulse. The resonant π -pulse duration is

$$t_\pi = \frac{1}{2f_\Omega} = \frac{1}{2 \times 20 \times 10^6 \text{ s}^{-1}} = 2.5 \times 10^{-8} \text{ s} = 25 \text{ ns.}$$

The Rabi phase argument at $t = t_\pi$ is $\theta = \frac{\alpha}{2} \sqrt{1 + 2r^2} t_\pi = \pi f_\alpha \sqrt{1.02} t_\pi$. Factor by factor: $\pi f_\alpha t_\pi = \pi \times 200 \times 10^6 \times 2.5 \times 10^{-8} = \pi \times 5 = 15.70796 \text{ rad}$; $\sqrt{1.02} = 1.0099505$; hence $\theta = 15.70796 \times 1.0099505 = 15.86445 \text{ rad}$. Reduce modulo 2π : $15.86445 - 4\pi = 3.29808 \text{ rad}$; $3.29808 - \pi = 0.15649 \text{ rad}$, so $\sin \theta = -\sin(0.15649)$. With $\sin x \approx x - x^3/6$ at $x = 0.15649$: $x^3 = 3.829 \times 10^{-3}$, $x^3/6 = 6.38 \times 10^{-4}$, so $\sin(0.15649) \approx 0.15585$ and $\sin^2 \theta \approx 0.024288$. Therefore

$$P_2(t_\pi) = P_{2,\text{max}} \times \sin^2 \theta = 0.019608 \times 0.024288 = 4.76 \times 10^{-4}.$$

Sequence-level survival. If each of N independent gates leaks probability $L = 4.76 \times 10^{-4}$ with no leakage recovery, the subspace survival probability is $S_N = (1 - L)^N \approx e^{-NL}$. For $N = 1000$: $NL = 0.47624$, so

$$S_{1000} \approx e^{-0.47624} = 0.6212.$$

Under these assumptions a 1000-gate qubit-encoded circuit retains subspace population only $\approx 62\%$ of the time absent active leakage return.

4.5 Numerical evaluation of the perturbative bound (illustrative parameters, Set C)

Inputs. Again illustrative assumptions: $\Omega/|\alpha| = 0.05$ (perturbative check: $8 \times 0.05^2 = 0.02 \ll 1$, satisfied); gate timing $\alpha T_g/2 = \pi/2$ (worst case within the first half-period of the anharmonicity beat).

$$L_2 = 8 \times (0.05)^2 \times \sin^2\left(\frac{\pi}{2}\right) = 8 \times 0.0025 \times 1 = 0.02.$$

The corresponding Planck Fidelity is $F_P = 1 - 0.02 = 0.98$. Gate-averaging $\sin^2$ over uniformly distributed gate times (mean $1/2$):

$$\overline{L_2} = 8 \times 0.0025 \times \frac{1}{2} = 0.01, \quad \overline{F_P} = 0.99.$$

Scaling check. Doubling $\Omega/|\alpha|$ from 0.05 to 0.10 gives $\overline{L_2} = 4 \times 0.01 = 0.04$; the ratio $0.04/0.01 = 4 = 2^2$ confirms the quadratic scaling numerically.

4.6 Worst-case uniform-population scenario

Input. $d = 12$ (from [9]); uniform-population assumption (Section 3.6).

$$L_P^{(\text{wc})} = \frac{d-2}{d} = \frac{10}{12} = \frac{5}{6} \approx 0.8333.$$

This is a scenario value under a deliberately pessimistic assumption, not a prediction for any driven gate.

5. Results

All numbers below are computed in Section 4 with shown arithmetic, or are labeled projections under stated assumptions.

1. **Base-truncation entropy** (computed, Section 4.1): for $d = 12$, $S_{\text{Base}} = \ln 6 = 1.791759$ nats = 2.584963 bits, matching the ~ 1.79 -nat value stated in [9].
2. **Discarded information fraction** (computed, Section 4.2): $f_{\text{disc}} = 0.721125$, i.e. 72.1% of the $\ln 12 = 2.484906$ -nat Hilbert-space information content; radix efficiency $\eta \approx 27.9\%$; one $d = 12$ oscillator requires $\lceil \log_2 12 \rceil = 4$ qubits.
3. **Qudit capacity ratio** (computed, Section 4.3): $\log_2 12 = 3.584963$.
4. **First-order leakage bound** (derived, Section 3.3): $L_2 = 8(\Omega/|\alpha|)^2 \sin^2(\alpha T_g/2)$.
5. **Exact three-level leakage** (derived, Section 3.4): $P_{2,\text{max}} = 2r^2/(1 + 2r^2)$, $\overline{P}_2 = r^2/(1 + 2r^2)$.
6. **Illustrative operating point, Set B** (computed from stated assumptions, Section 4.4): at $r = 0.1$, $P_{2,\text{max}} = 1.96 \times 10^{-2}$, $\overline{P}_2 = 9.80 \times 10^{-3}$, per- π -pulse leakage $P_2(t_\pi) = 4.76 \times 10^{-4}$, and $S_{1000} \approx 0.6212$. Model projections, not measurements.
7. **Illustrative operating point, Set C** (computed from stated assumptions, Section 4.5): at $\Omega/|\alpha| = 0.05$, $\alpha T_g/2 = \pi/2$: $L_2 = 0.02$ ($F_P = 0.98$); gate-averaged $\overline{L_2} = 0.01$ ($\overline{F_P} = 0.99$); doubling the drive ratio quadruples the averaged floor ($0.01 \rightarrow 0.04$).
8. **Worst-case scenario value** (computed under the uniform-population assumption, Section 4.6): $L_P^{(\text{wc})} = 5/6 \approx 0.8333$ for $d = 12$.
9. **Strict positivity** (proved at model-class level, Section 3.5): $L_P > 0$ for generic finite-time, finite-amplitude $a^\dagger$ -driven gates; $L_P = 0$ only in the singular limits $T_g \rightarrow \infty$ or $\Omega \rightarrow 0$.

6. Discussion

Limitations of the derivations. The perturbative bound $L_2 = 8(\Omega/|\alpha|)^2 \sin^2(\alpha T_g/2)$ is first order in Ω and neglects levels $n \geq 3$; the exact three-level solution of Section 3.4 is itself a truncation. The strict-positivity claim of Section 3.5 is a model-class result: it holds for finite-time, finite-amplitude drives of the stated $a^\dagger$ -form under the rotating-wave approximation, not for all conceivable Hamiltonians, and shaped envelopes can null the first-order leakage at isolated, measure-zero parameter points. Second-order pathways, non-RWA effects, and materials-level mechanisms all add to the coherent leakage computed here and are not modeled.

Status of the numerical values. Every leakage number in Sections 4.4 and 4.5 is computed from explicitly labeled illustrative parameters ($f_\Omega = 20$ MHz, $f_\alpha = 200$ MHz; $\Omega/|\alpha| = 0.05$), not from a measured device. The sequence-level survival $S_{1000} \approx 0.6212$ assumes independent gates and no leakage return or reset; any leakage-reduction unitary or post-selection invalidates it. The worst-case value $L_P^{(\text{wc})} = 5/6$ is a scenario under a uniform-population assumption, not a physical prediction.

What would falsify the claims. The central conjecture would be falsified by a finite-time, finite-amplitude gate on a transmon-like ladder that reaches unit qubit-subspace fidelity with generic gate parameters, i.e. a robust (not measure-zero) family of drives with $L_P = 0$ exactly. The entropy accounting would be falsified if the resolvable level count d were shown not to be a well-defined device property, since $S_{\text{Base}} = \ln(d/2)$ depends directly on d : the value $d \sim 12$ is asserted in [9] and not independently established here.

Open questions. Whether the S_{Base} quantity of [9] corresponds to an operationally measurable thermodynamic or channel capacity cost remains open; the supplied summary of [11] is empty, so its relation to thermodynamic scaling of alternative hardware could not be assessed here. A full treatment of optimal control (DRAG-like pulse shaping) against the leakage floor, and extension of the benchmark protocol of Section 3.7 to measured devices, are left to future work.

7. Conclusion

We formalized the radix-truncation conjecture of the QNFO "Rosetta" corpus [9], [10] as a quantitative leakage program for transmon-encoded qubits. We defined Planck Loss L_P and Planck Fidelity $F_P = 1 - L_P$, derived a first-order bound $L_2 = 8(\Omega/|\alpha|)^2 \sin^2(\alpha T_g/2)$ and an exact three-level expression $P_{2,\text{max}} = 2r^2/(1 + 2r^2)$, and proved strict positivity of L_P at the model-class level for generic finite-time gates. Under labeled illustrative parameters, a single π pulse leaks $\approx 4.76 \times 10^{-4}$ and a 1000-gate sequence retains $\approx 62\%$ subspace population absent leakage return. The binary truncation of a $d = 12$ ladder discards $S_{\text{Base}} = \ln 6 = 1.791759$ nats, i.e. 72.1% of the ladder's Hilbert-space information content, retaining $\approx 27.9\%$. These results, together with the proposed Planck Fidelity benchmark and stated falsification conditions, motivate treating the transmon as a base- d qudit and treating the qubit radix as a measurable approximation cost rather than a free abstraction.

References

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