

Re-Deriving Quantum Entanglement from Classical Radon–Nikodým Hydrodynamics: A Formal Falsification Analysis of the Hydrodynamic Stability Hypothesis

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Abstract

The Hydrodynamic Stability Hypothesis (HSH) conjectures that standard quantum mechanics, including multi-particle entangled states such as the Greenberger–Horne–Zeilinger (GHZ) state, can be axiomatically re-grounded in classical measure theory: wavefunctions are interpreted as Radon–Nikodým densities that relax into basins of attraction of classical probability measures under a local hydrodynamic transport, restoring local realism without superdeterminism. We subject this conjecture to a formal test. We state the hypothesis as four explicit axioms (classical probability space, local state variables, local continuous transport, free measurement settings) and attempt an explicit derivation of the GHZ state and its Bell-type correlations purely from those axioms. We show with full arithmetic that the Mermin GHZ argument forces any model satisfying the axioms to obey a pointwise product constraint $P_1 P_2 P_3 P_4 = +1$, while the quantum predictions require the corresponding expectation -1 ; the Mermin expression reaches the quantum value $M_Q = 4$ against the classical bound $M_{LHV} = 2$, a violation factor of 2. A CHSH cross-check gives $2\sqrt{2} \approx 2.828427$ against the classical bound 2. The strong form of the hypothesis is therefore falsified: reproducing GHZ statistics forces either non-local variables or contradiction. As a clearly labeled projection, we compute the propagation timescales that the hypothesis's proposed sub-femtosecond particle-tracking simulations must resolve, and we discuss which weakened versions of the hypothesis survive.

1. Introduction

The relationship between quantum and classical mechanics remains an open problem more than three-quarters of a century after the term "entanglement" was coined for a peculiar feature inherent to quantum systems [3]. Semiclassical analyses of closed systems as Planck's constant becomes vanishingly small show that the fate of entanglement in that limit is subtle rather than trivially classical [3]. At the same time, a body of work constructs explicit bridges between classical and quantum descriptions: unified formulations providing a continuous passage between strictly classical and strictly quantized states [8], and entanglement-free regimes in which classical particle paradigms arise as limit cases of quantum states of interacting pairs [5].

The Hydrodynamic Stability Hypothesis (HSH), documented in the internal research record [9], takes the strongest possible position in this landscape: that quantum mechanics — including multi-particle entangled states such as the GHZ state — can be axiomatically re-derived from classical measure theory, via a local hydrodynamic picture in which wavefunctions relax into basins of attraction of classical Radon–Nikodým measures. The Radon–Nikodým theorem allows a measure absolutely continuous with respect to a base measure to be represented by a density; HSH proposes that the quantum wavefunction is, at the foundational level, such a density over a classical configuration space, and that preparation and measurement are stability phenomena: trajectories fall into attracting basins of these measures. Crucially, the hypothesis is local — the relaxation is supposed to proceed by continuous local wave propagation, so that apparent instantaneous state changes across spatial barriers are always accompanied by an underlying continuous local signal — and it disavows superdeterminism.

This position matters because, if correct, it would restore local realism without superdeterminism, a combination widely believed to be excluded by Bell-type analyses. It also matters practically: multipartite entanglement is the resource that empowers quantum networks beyond bipartite link-by-link distribution [2], enables hierarchical network protocols using hybrid entanglement between different degrees of freedom [6], and underlies tripartite coherence dynamics studied even when qubits couple to classical environments [4]. If entanglement were re-derivable classically, the resource picture underlying these network proposals would change fundamentally.

Rather than advocate, we test. Our method is deliberately adversarial to the hypothesis. We (i) write down the minimal axiom set that any classical Radon–Nikodým hydrodynamic re-grounding must satisfy, (ii) attempt an explicit derivation of the GHZ state and its measurement correlations from those axioms alone, and (iii) show that the derivation reaches a fork: either the model introduces variables that correlate spacelike-separated outcomes directly (abandoning locality), or it contradicts the quantum predictions it was supposed to re-derive. We execute this fork quantitatively with the Mermin GHZ game and a CHSH cross-check, all arithmetic shown. Finally, we give a projection analysis — clearly labeled as such, with stated assumptions — of the sub-femtosecond particle-tracking simulation program that the hypothesis proposes as a complementary test.

Our conclusion is that the strong form of HSH is falsified by the standard GHZ argument, executed entirely within the hypothesis's own classical measure-theoretic language. We are explicit in Section 6 about what this does and does not rule out.

2. Background and Related Work

We review the supplied literature in four strands, restricting every statement about a work to what its supplied entry states.

Entanglement as a dynamical and statistical resource. The entry for arXiv:2111.08018v2 [1] articulates a philosophy drawn from statistical mechanics and random-matrix theory: individual instances of complex quantum circuits are essentially intractable to simulate, but the statistical properties of random ensembles obey simple universal laws, and this philosophy is applied to the dynamics of quantum information in ideal and noisy quantum circuits. This is directly relevant to HSH: a Radon–Nikodým measure is an ensemble object, not a single-trajectory object, so any classical hydrodynamic re-grounding must live at exactly the ensemble-statistical level of description that [1] emphasizes. Our axiom set in Section 3 is therefore phrased at the ensemble level.

Multipartite entanglement in networks. The entry for arXiv:2103.14759v3 [2] states that a quantum internet aims at distributing bipartite entanglement between distant nodes, but that multipartite entanglement between nodes may empower the quantum internet for additional or better applications in communications, sensing, and computation, and presents an algorithm for generating multipartite entanglement between distributed nodes. The entry for arXiv:2004.13176v3 [6] introduces hybrid entangled states — entanglement between different degrees of freedom — as enabling various quantum computational and communication tasks with fewer resources, and aims to exploit this advantage in communication over quantum networks, noting that entanglement shared over networks faces limitations; the supplied summary is truncated before specifying the nature of those limitations. Together, [2] and [6] establish that multipartite entanglement of exactly the GHZ type we analyze is an operational, technologically targeted resource, not a mathematical curiosity; any re-grounding of quantum mechanics must reproduce it.

The classical–quantum boundary. The entry for arXiv:1110.0157v1 [3] states that 75 years after the term "entanglement" was coined, the connection between quantum and classical mechanics remains an open problem, and discusses, drawing on results in semiclassical systems, the fate of entanglement in a closed system as Planck's constant becomes vanishingly small; the summary is truncated at this point. The entry for arXiv:1206.5715v2 [5] identifies three classical, pre-quantum paradigms for describing particles — the material point, the test-particle, and the diluted particle (droplet model) — as limit cases of a quantum regime in which pairs of particles interact without getting entangled, an entanglement-free regime that the (truncated) summary connects to decoherence-related descriptions. These two works

frame the exact question HSH answers in the opposite direction: [3] and [5] study how classicality emerges from quantum mechanics, whereas HSH claims the reverse reduction. Our falsification result is consistent with the framing of [3], in which the quantum–classical connection is treated as an open problem rather than a solved reduction.

GHZ dynamics under classical environments. The entry for arXiv:2107.11259v1 [4] addresses the dynamics of entanglement and coherence for three non-interacting qubits initially prepared as a maximally entangled GHZ-like state coupled with independent classical environments, considering two different Gaussian environment models; the summary truncates before giving results. This is the closest supplied antecedent to HSH's program, since it couples a GHZ state to explicitly classical environments. Because the supplied summary gives no further detail on the results, we use it only to establish that GHZ-plus-classical-environment models are an active modeling class, and attribute no specific findings from it.

Classical–quantum unification and interpolation programs. The entry for arXiv:2010.03984v2 [8] states the goal of overcoming the two separate languages of classical and quantum systems and creating a smooth, common procedure providing a continuous passage between the strictly classical and the strictly quantized state, with path integration among the (truncated) tools mentioned. The entry for arXiv:2207.14810v3 [7] shows that the scaling of the α -QPE family of phase-estimation algorithms — for which Wang, Higgott and Brierley [2019, Phys. Rev. Lett. 122 140504] showed a continuous trade-off between quantum speedup and circuit depth — can be naturally and succinctly derived within a classical–quantum framework using quantum singular value transformations; the summary truncates before further details. Both [7] and [8] represent interpolation strategies between fixed classical and fixed quantum formalisms; HSH is more radical, claiming the quantum formalism is derivable from the classical one. Our result marks the precise point where such a derivation, if local, must fail.

Internal program context. The QNFO corpus entry [10] describes an archive of a research record investigating the Post-Quantum Synthesis (PQS) framework, a neutral investigation of PQS's four sub-pillars mapped against external literature, including an analysis of AI gate-check convergence between independent evaluators. The entries [9], [11], and [12] are supplied with empty summaries: [9] is the Hydrodynamic Stability Hypothesis itself, taken here as the conjecture under test as restated in the grounding material; [11] (a category-theoretic foundation for finite measurement, "Valuation Without R") and [12] (POST QUANTUM SYNTHESIS) supply no further detail, so we make no claims about their content beyond their titles, while noting that [11]'s title suggests a finite, real-number-free measurement formalism that would be a natural axiomatic substrate for a measure-theoretic re-derivation — a possibility we cannot verify from the supplied material.

3. Methods

3.1 Axiomatization of the hypothesis

We formalize the strong form of HSH as four axioms, chosen to be generous: each is the weakest assumption the hypothesis's own description requires. Let $(\Lambda, \mathcal{F}, \mu)$ be a classical probability space, with Λ a configuration space of hidden classical states $\lambda \in \Lambda$.

- **(A1) Classical measure grounding.** Every quantum state corresponds to a measure $\mu_\psi \ll \mu$ with Radon–Nikodým derivative $\rho_\psi = \frac{d\mu_\psi}{d\mu}$, such that all quantum predictions are recoverable as functionals of ρ_ψ .
- **(A2) Local state variables.** For N spatially separated parties, the configuration decomposes as $\lambda = (\lambda_1, \dots, \lambda_N)$ with $\lambda_i \in \Lambda_i$, and each party's outcome is a local function $A_i = f_i(\lambda_i, s_i)$ of its own variable and its own setting s_i .
- **(A3) Local continuous transport.** The time evolution $\mu_t = (\Phi_t)_* \mu_0$ is generated by a local flow Φ_t on Λ propagating information at finite speed $v_{\max}$, so that no influence connects spacelike-separated events; preparation and measurement correspond to relaxation into basins of attraction $\mathcal{B}_k \subset \Lambda$ of stable measures, the observed outcome indexing the basin.

- **(A4) Free settings.** The distribution of measurement settings is statistically independent of λ (no superdeterminism).

The strong claim of HSH is that A1–A4 suffice to derive the quantum formalism, including entangled states.

3.2 Falsification protocol

Given A1–A4, we ask whether the model can reproduce the quantum joint statistics of the GHZ state

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

under local Pauli measurements X and Y on each of the three qubits. The protocol is: (i) derive the classical constraint that A1–A4 impose on the four relevant triple correlations, (ii) compute the quantum values of those same correlations from the GHZ state, and (iii) compare. Any mismatch falsifies the strong form, because the hypothesis claims to re-derive the quantum statistics from the classical axioms. This is a derivation-and-contradiction analysis, not a numerical simulation; the complementary simulation target is specified as a projection in Section 4.5.

3.3 Simulation projection protocol

The complementary test proposed by the hypothesis is sub-femtosecond particle tracking of the hydrodynamic transport Φ_t , to check whether apparent instantaneous state changes across spatial barriers are accompanied by continuous local wave propagation. We compute, as labeled projections with stated assumptions, the propagation timescales such a simulation must resolve for the hypothesis's locality claim to be checked.

4. Analysis

4.1 Quantum correlations of the GHZ state (inputs and derivation)

Input 1 (explicit assumption, not sourced). We take the GHZ state to be $|\text{GHZ}\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$. The supplied entry for [4] mentions only a maximally entangled GHZ-like state of three qubits and does not state the explicit superposition; the formula above is therefore adopted here as a working assumption of the analysis, not as a claim derived from the bibliography.

Input 2 (explicit assumption, not sourced). We assume the Pauli operators X and Y each have eigenvalues ± 1 . No supplied bibliography entry states this; it is taken as a standard assumption of the quantum formalism under test, not as a sourced fact.

Step 1: action of Pauli strings on GHZ. Using $X|0\rangle = |1\rangle$, $X|1\rangle = |0\rangle$, $Y|0\rangle = i|1\rangle$, $Y|1\rangle = -i|0\rangle$:

$$XXX|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|111\rangle + |000\rangle) = +|\text{GHZ}\rangle,$$

$$XYY|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}((i)(i)|011\rangle + (-i)(-i)|100\rangle) = \frac{1}{\sqrt{2}}(-|011\rangle - |100\rangle) = -|\text{GHZ}\rangle,$$

and identically for YXY and YYX by symmetry of the GHZ state under qubit exchange.

Step 2: quantum triple products. Since $XXX|\text{GHZ}\rangle = |\text{GHZ}\rangle$, the product of the three outcomes for settings (X, X, X) is deterministically

$$\langle XXX \rangle_Q = +1.$$

Since XYY , YXY , YYX each act as -1 on $|\text{GHZ}\rangle$,

$$\langle XYY \rangle_Q = \langle YXY \rangle_Q = \langle YYX \rangle_Q = -1.$$

These four stabilizer eigenvalues are the quantum predictions any successful re-derivation must reproduce. (We deliberately do not use the (Y, Y, Y) setting: its expectation in $|\text{GHZ}\rangle$ requires phase bookkeeping that adds nothing to the argument, since the four settings above already suffice for the contradiction.)

4.2 The classical constraint from axioms A1–A4

Under A1–A4, each party $i \in \{1, 2, 3\}$ carries a local variable λ_i and predetermined outcomes $A_i(\lambda_i, s_i) \in \{-1, +1\}$ for settings $s_i \in \{X, Y\}$. Define the four triple products:

$$P_1 = A_1^X A_2^X A_3^X, \quad P_2 = A_1^X A_2^Y A_3^Y, \quad P_3 = A_1^Y A_2^X A_3^Y, \quad P_4 = A_1^Y A_2^Y A_3^X.$$

Each $P_k \in \{-1, +1\}$ pointwise. Their product, computed pointwise for every fixed λ :

$$P_1 P_2 P_3 P_4 = (A_1^X)^2 (A_1^Y)^2 (A_2^X)^2 (A_2^Y)^2 (A_3^X)^2 (A_3^Y)^2 = (+1)^6 = +1,$$

since each local outcome appears exactly twice and squares to 1. Taking expectations and using that the pointwise identity holds almost everywhere with respect to μ (A1):

$$\langle P_1 P_2 P_3 P_4 \rangle_\mu = +1.$$

But the quantum values require $\langle P_1 \rangle_Q = +1$ and $\langle P_2 \rangle_Q = \langle P_3 \rangle_Q = \langle P_4 \rangle_Q = -1$, so the quantum product of the four correlations is

$$(+1) \times (-1) \times (-1) \times (-1) = -1 \neq +1.$$

The contradiction is already complete here: the classical axioms force the pointwise product $+1$; the quantum statistics require -1 .

4.3 Mermin bound and violation factor

Equivalently, in Mermin-inequality form: for any model satisfying A1–A4,

$$|\langle P_1 \rangle - \langle P_2 \rangle - \langle P_3 \rangle - \langle P_4 \rangle| \leq 2,$$

because the pointwise expression $|P_1 - P_2 - P_3 - P_4| = 2$ whenever $P_1 P_2 P_3 P_4 = +1$. Check of the cases: if all four $P_k = +1$, then $|1 - 1 - 1 - 1| = 2$; if exactly two are -1 (e.g. $P_2 = P_3 = -1$), then $|1 + 1 + 1 - 1| = 2$; the all-four-negative case also has product $+1$ (four negatives is an even count) and gives $|-1 - (-1) - (-1) - (-1)| = |-1 + 1 + 1 + 1| = 2$. Hence every assignment with $P_1 P_2 P_3 P_4 = +1$ — zero, two, or four negative terms — yields exactly $|P_1 - P_2 - P_3 - P_4| = 2$, and the classical bound is $M_{\text{LHV}} = 2$.

The quantum values give

$$|\langle P_1 \rangle_Q - \langle P_2 \rangle_Q - \langle P_3 \rangle_Q - \langle P_4 \rangle_Q| = |(+1) - (-1) - (-1) - (-1)| = |1 + 1 + 1 + 1| = 4 = M_Q.$$

Step by step: $1 + 1 = 2$; $2 + 1 = 3$; $3 + 1 = 4$. The violation gap and factor are

$$\Delta M = M_Q - M_{\text{LHV}} = 4 - 2 = 2, \quad \frac{M_Q}{M_{\text{LHV}}} = \frac{4}{2} = 2.$$

4.4 CHSH cross-check

For two parties, the CHSH combination is

$$S = E(a, b) + E(a, b') + E(a', b) - E(a', b'),$$

with each $E \in [-1, 1]$ classically, giving the classical bound $|S| \leq 2$: pointwise, $|ab + ab' + a'b - a'b'| \leq 2$ since $b, b' \in \{\pm 1\}$ and one of the two pairs cancels for every assignment. As a clearly labeled projection with stated assumptions — namely, the assumption that standard quantum mechanics predicts the Tsirelson-type maximum $S_{\max} = 2\sqrt{2}$ for singlet-type correlations, a value not supplied by any bibliography entry — we take

$$S_{\max} = 2\sqrt{2}.$$

Arithmetic: $\sqrt{2} \approx 1.414214$ (check: $1.414214^2 = 2.0000006 \approx 2$), so $2\sqrt{2} \approx 2 \times 1.414214 = 2.828427$. The violation factor over the classical bound is

$$\frac{2\sqrt{2}}{2} = \sqrt{2} \approx 1.414214.$$

Thus any local classical Radon–Nikodým model reproducing two-party entangled statistics is confined to $|S| \leq 2$, while quantum mechanics reaches $2\sqrt{2} \approx 2.828427$; the gap is a factor of $\sqrt{2} \approx 1.414214$, confirming the three-party result at the two-party level.

4.5 The fork and the falsification verdict

The derivation forces exactly two options:

- **Option (i):** the model abandons A2, introducing non-local variables so that A_i depends on all settings. Then the pointwise factorization $P_1 P_2 P_3 P_4 = +1$ fails and the quantum values become reproducible — but the hypothesis's locality claim (A2, A3) is gone.
- **Option (ii):** the model keeps A2 and simply fails to reproduce the quantum stabilizer values, i.e., it contradicts the quantum statistics it was supposed to re-derive.

Either way, the strong form of HSH — local classical measure theory reproducing GHZ statistics — is falsified. This is the GHZ argument executed entirely within the hypothesis's own axiomatic language; no experiment is needed, only the requirement of internal consistency with the quantum predictions.

4.6 Simulation timescale projection (explicitly labeled projection)

Assumptions (stated). (P1) The hydrodynamic transport propagates influence at most at $v_{\max} = c = 3.0 \times 10^8 \text{ m s}^{-1}$ — the most generous choice for the hypothesis, since any slower speed makes the required continuous local signal more detectable. (P2) The spatial barrier separating two parties has width $L = 1.0 \times 10^{-3} \text{ m}$ (millimeter scale, typical of lab-scale separated systems). (P3) The simulation timestep must be at most one-tenth of the propagation time to resolve the signal (Nyquist-type sampling requirement).

Derivation. Minimum propagation time:

$$t_{\min} = \frac{L}{v_{\max}} = \frac{1.0 \times 10^{-3} \text{ m}}{3.0 \times 10^8 \text{ m s}^{-1}} = 3.333333 \times 10^{-12} \text{ s} \approx 3.33 \text{ ps}.$$

Required timestep: $\Delta t \leq t_{\min}/10 = 3.333333 \times 10^{-13} \text{ s} \approx 333 \text{ fs}$, which lies within the sub-femtosecond regime the hypothesis's simulation program targets. Number of timesteps to cover one propagation across the barrier:

$$N_{\text{steps}} = \frac{t_{\min}}{\Delta t} = 10.$$

For a macroscopic barrier $L = 1.0 \text{ m}$:

$$t_{\min} = \frac{1.0}{3.0 \times 10^8} = 3.333333 \times 10^{-9} \text{ s} \approx 3.33 \text{ ns}, \quad N_{\text{steps}} = \frac{3.33 \times 10^{-9}}{3.33 \times 10^{-13}} = 1.0 \times 10^4.$$

These are projections conditional on P1–P3. They establish the temporal resolution the tracking program must achieve, but, per Section 4.5, they cannot rescue the strong form of the hypothesis: the GHZ contradiction is kinematic, not dynamical — no continuous local propagation, however fast, restores the pointwise factorization $P_1 P_2 P_3 P_4 = +1$ that A2 imposes.

5. Results

All numbers below are computed in Section 4 with shown arithmetic; none are empirical measurements.

1. **GHZ quantum correlations** (derived from the stabilizer relations of $|\text{GHZ}\rangle$, Section 4.1): $\langle XXX \rangle_Q = +1$; $\langle XYY \rangle_Q = \langle YXY \rangle_Q = \langle YYX \rangle_Q = -1$.
2. **Classical pointwise constraint** (from axioms A1–A4, Section 4.2): $P_1 P_2 P_3 P_4 = (+1)^6 = +1$ for every λ , hence $\langle P_1 P_2 P_3 P_4 \rangle_\mu = +1$, which conflicts with the quantum product $(+1)(-1)(-1)(-1) = -1$.
3. **Mermin bound and violation** (Section 4.3): classical $|\langle P_1 \rangle - \langle P_2 \rangle - \langle P_3 \rangle - \langle P_4 \rangle| \leq 2$; quantum value $M_Q = 4$; gap $\Delta M = 2$; violation factor $4/2 = 2$.
4. **CHSH cross-check** (Section 4.4): classical bound 2; quantum maximum $2\sqrt{2} \approx 2.828427$; violation factor $\sqrt{2} \approx 1.414214$.
5. **Falsification verdict** (Section 4.5): the strong form of HSH is falsified; any classical measure-theoretic model reproducing GHZ statistics must violate axiom A2 (locality of outcome functions) or contradict the quantum predictions.
6. **Simulation timescale projections** (Section 4.6, conditional on assumptions P1–P3): for $L = 1.0 \times 10^{-3} \text{ m}$, $t_{\min} = 3.333333 \times 10^{-12} \text{ s}$ and required $\Delta t \leq 3.333333 \times 10^{-13} \text{ s}$ ($\approx 333 \text{ fs}$), with $N_{\text{steps}} = 10$; for $L = 1.0 \text{ m}$, $t_{\min} = 3.333333 \times 10^{-9} \text{ s}$ and $N_{\text{steps}} = 1.0 \times 10^4$. These are projections with stated assumptions, not measurements.

6. Discussion

What the falsification does and does not show. The derivation in Sections 4.2–4.5 falsifies the *strong* claim: that GHZ-type Bell-violating correlations follow from classical measure theory plus local hydrodynamic transport plus freedom of settings. It does not falsify weaker versions. In particular, a version of HSH that drops A2 — allowing the Radon–Nikodým measure to be non-local on Λ — or that reinterprets "locality" at the level of the flow rather than the response functions, escapes the argument. We flag this as the hypothesis's most likely retreat position, and it is exactly the position that mainstream non-local interpretations occupy; the value of the derivation is that it shows HSH cannot simultaneously keep locality, measurement freedom, and the GHZ correlations.

Against ourselves: possible escapes we cannot exclude. First, our formalization A1–A4 is our own; the source record [9] supplies no summary text, so the hypothesis's author might formalize "stability" or "locality" differently in ways that evade our axioms. The falsification is conditional on our formalization being faithful. Second, we assumed deterministic response functions $A_i \in \{\pm 1\}$; stochastic local response functions change the bookkeeping but not the underlying pointwise product constraint, though a fully rigorous treatment of the stochastic case is not given here. Third, the connection between "basins of attraction" (A3) and Born-rule statistics is nowhere derived in this paper; even if the correlation structure were classically reproducible, the *probabilities* would need a separate derivation, and [11]'s category-theoretic finite-measurement program (whose supplied entry gives no detail beyond the title) might be the intended substrate — we cannot verify this from the supplied material.

Limitations of the quantitative work. The timescale projections of Section 4.6 depend entirely on the assumed barrier width L and the generous choice $v_{\max} = c$; the dimensional requirement of the hydrodynamic model is itself an unspecified parameter of HSH [9], so the projections bound only the temporal resolution, not the full computational cost,

which grows with phase-space dimensionality and was not computed here. The Mermin and CHSH bounds are standard results re-derived within our axiom system, not novel theorems; their novelty here lies only in their application to the HSH axiom set.

Failure modes. If the hypothesis's defenders respond by weakening A2 while retaining the word "local" for the transport Φ_t , the falsification loses its target; the paper's contribution then becomes a clarification of which notion of locality is incompatible with GHZ statistics. If a regularised (non-singular) family of classical measures could be shown to approximate the GHZ correlations while keeping A2, our use of deterministic pointwise assignments would be the vulnerable point — but the product constraint $P_1 P_2 P_3 P_4 = +1$ holds for any local assignment, deterministic or not, so this escape route is closed in the deterministic case and, we argue, in the stochastic case by the same algebra.

What would falsify our falsification. A single explicit construction of local response functions $A_i(\lambda_i, s_i)$ satisfying A1–A4 that reproduces all four stabilizer values $\langle P_1 \rangle = +1$, $\langle P_2 \rangle = \langle P_3 \rangle = \langle P_4 \rangle = -1$ would falsify our falsification, since it would contradict the pointwise product constraint $P_1 P_2 P_3 P_4 = +1$ derived in Section 4.2; by the algebra of Section 4.2, no such construction exists within the axioms, but an explicit demonstration outside our axiomatization — for example, under a different formalization of "stability" or "locality" than ours — would overturn the verdict. Similarly, an experimental or theoretical demonstration that the GHZ stabilizer values themselves admit a local stochastic measure-theoretic model with $\langle P_1 P_2 P_3 P_4 \rangle_\mu = -1$ would break the argument at its pointwise step.

7. Conclusion

We formalized the Hydrodynamic Stability Hypothesis as four axioms (A1–A4): classical probability grounding via Radon–Nikodým densities, local state variables, local continuous transport, and free measurement settings. Within these axioms we derived, with fully shown arithmetic, that the pointwise product of the four GHZ triple correlations satisfies $P_1 P_2 P_3 P_4 = (+1)^6 = +1$, while the quantum stabilizer values of $|\text{GHZ}\rangle$ require the product $(+1)(-1)(-1)(-1) = -1$. In Mermin form the classical bound is $M_{\text{LHV}} = 2$ against the quantum value $M_Q = 4$, a violation factor of 2; a CHSH cross-check gives $2\sqrt{2} \approx 2.828427$ against the classical bound 2, a factor of $\sqrt{2} \approx 1.414214$. The strong form of HSH — local classical measure theory reproducing GHZ statistics without superdeterminism — is therefore falsified: reproducing the quantum correlations forces abandonment of axiom A2 (locality of the response functions) or contradiction with the quantum predictions. As labeled projections conditional on assumptions P1–P3, we computed the timescales ($t_{\min} = 3.333333 \times 10^{-12}$ s for $L = 1.0 \times 10^{-3}$ m; 3.333333×10^{-9} s for $L = 1.0$ m) that the hypothesis's sub-femtosecond particle-tracking program must resolve; these bound the temporal resolution of the proposed simulation but cannot rescue the strong form, since the GHZ contradiction is kinematic, not dynamical. Weaker versions of the hypothesis that relinquish locality of the response functions remain outside the reach of this argument and are the natural retreat position, as discussed in Section 6.

References

- [1] Entanglement dynamics in hybrid quantum circuits. arXiv:2111.08018v2. <https://arxiv.org/abs/2111.08018v2> [2] Distributing Multipartite Entanglement over Noisy Quantum Networks. arXiv:2103.14759v3. <https://arxiv.org/abs/2103.14759v3> [3] Entanglement, the quantum formalism and the classical world. arXiv:1110.0157v1. <https://arxiv.org/abs/1110.0157v1> [4] Probing tripartite entanglement and coherence dynamics in pure and mixed independent classical environments. arXiv:2107.11259v1. <https://arxiv.org/abs/2107.11259v1> [5] Coherent states and the classical-quantum limit considered from the point of view of entanglement. arXiv:1206.5715v2. <https://arxiv.org/abs/1206.5715v2> [6] Hierarchical Quantum Network using Hybrid Entanglement. arXiv:2004.13176v3. <https://arxiv.org/abs/2004.13176v3> [7] Simplifying a classical-quantum algorithm interpolation with quantum singular value transformations. arXiv:2207.14810v3. <https://arxiv.org/abs/2207.14810v3> [8] A Unified Combination of Classical and Quantum Systems. arXiv:2010.03984v2. <https://arxiv.org/abs/2010.03984v2> [9] QNFO: Hydrodynamic Stability

Hypothesis [10] QNFO: PQS AI-Evaluation Audit: Post-Quantum Synthesis Investigation and AI Gate-Check Analysis
 [11] QNFO: Valuation Without R: A Category-Theoretic Foundation for Finite Measurement [12] QNFO: POST
 QUANTUM SYNTHESIS

Appendix A. Divergence report

No unresolved substantive divergences remain in the reconciled text. Two convention choices that independent drafts could reasonably make differently are documented here explicitly. (i) **Deterministic vs. stochastic response functions:** the main text adopts deterministic response functions $A_i \in \{\pm 1\}$ as the primary bookkeeping convention; the pointwise product constraint $P_1 P_2 P_3 P_4 = +1$ is stated in Section 6 to carry over to stochastic local response functions by the same algebra, and this convention choice does not affect the verdict. (ii) **Propagation speed for the projection:** the projection of Section 4.6 adopts $v_{\max} = c = 3.0 \times 10^8 \text{ m s}^{-1}$ as the most generous choice for the hypothesis; any smaller assumed speed only shortens $t_{\min} = L/v_{\max}$ and tightens the required timestep $\Delta t \leq t_{\min}/10$, so the projection is an upper bound on the required timescales under this convention.

Appendix B. Claim attribution

Claim	Substance	Source drafts	Status
C1	GHZ stabilizer values $\langle XXX \rangle_Q = +1$, $\langle XYY \rangle_Q = \langle YXY \rangle_Q = \langle YYX \rangle_Q = -1$	A, B, C	CONVERGENT
C2	Pointwise classical constraint $P_1 P_2 P_3 P_4 = +1$ from local response functions	A, B, C	CONVERGENT
C3	Quantum product of the four correlations equals -1 , contradicting $+1$	A, B, C	CONVERGENT
C4	Mermin bound $M_{\text{LHV}} = 2$, quantum value $M_Q = 4$, violation factor 2	A, B, C	CONVERGENT
C5	CHSH cross-check: classical bound 2, quantum maximum $2\sqrt{2} \approx 2.828427$, factor $\sqrt{2} \approx 1.414214$	A, B	CONVERGENT
C6	Falsification verdict against the strong form of HSH; fork between abandoning A2 and contradicting quantum predictions	A, B, C	CONVERGENT
C7	Simulation timescale projections $(t_{\min}, \Delta t, N_{\text{steps}})$ under assumptions P1–P3	A, B	CONVERGENT
C8	Choice of $v_{\max} = c$ as the generous upper bound for the projection	A	SINGLE
C9	Stochastic response functions preserve the pointwise product constraint (argued, not fully proven)	B	SINGLE
C10	Retreat position: weakening A2 escapes the argument and matches mainstream non-local interpretations	A, B	CONVERGENT