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Reexamining the Archimedean Axiom: A Non-Archimedean Substrate as a Unifying Coordinate Choice in Theoretical Physics

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ABSTRACT

The Archimedean axiom underlies the real-number continuum used throughout quantum field theory, general relativity, and high-energy accelerator design, yet it generates persistent “epicycles” such as ultraviolet divergences, set-theoretic paradoxes, the timeless Wheeler–DeWitt constraint, and software null-pointer failures. This work fills two gaps: (1) it supplies a computable adjudication rule—the Keplerian criterion—that ranks frameworks by the number of known quantities they derive per free parameter, and (2) it demonstrates concrete parameter reductions on both a fusion-engineering case and a non-Archimedean combinatorial case. Using published CFETR magnetohydrodynamic (MHD) design data we derive the geometric scaling $w_{cd}/R = 0.0124$, collapsing three continuum tunings into a single dimensionless number. On the non-Archimedean side we compute the exact ball cardinality of the Bruhat–Tits tree, obtaining a capacity ratio of 146.19 relative to a comparable Archimedean interval, and we evaluate lattice-counting benchmarks for voting-paradox probabilities. These results illustrate that a single non-Archimedean parameter can replace multiple continuum specifications and that the tree substrate offers orders-of-magnitude greater state capacity. If the non-Archimedean substrate proves operative, accelerator and fusion designers could streamline design freedom, reduce systematic budgets, and replace costly continuum-based parameter scans with discrete, parsimonious specifications.

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Abstract

The Archimedean axiom underlies the real-number continuum used throughout quantum field theory, general relativity, and high-energy accelerator design, yet it generates persistent “epicycles” such as ultraviolet divergences, set-theoretic paradoxes, the timeless Wheeler–DeWitt constraint, and software null-pointer failures. This work fills two gaps: (1) it supplies a computable adjudication rule—the Keplerian criterion—that ranks frameworks by the number of known quantities they derive per free parameter, and (2) it demonstrates concrete parameter reductions on both a fusion-engineering case and a non-Archimedean combinatorial case. Using published CFETR magnetohydrodynamic (MHD) design data we derive the geometric scaling $w_{cd}/R = 0.0124$, collapsing three continuum tunings into a single dimensionless number. On the non-Archimedean side we compute the exact ball cardinality of the Bruhat–Tits tree, obtaining a capacity ratio of 146.19 relative to a comparable Archimedean interval, and we evaluate lattice-counting benchmarks for voting-paradox probabilities. These results illustrate that a single non-Archimedean parameter can replace multiple continuum specifications and that the tree substrate offers orders-of-magnitude greater state capacity. If the non-Archimedean substrate proves operative, accelerator and fusion designers could streamline design freedom, reduce systematic budgets, and replace costly continuum-based parameter scans with discrete, parsimonious specifications.

1. Introduction

High-energy particle-physics programs and magnetic-confinement-fusion projects both face costly design bottlenecks that stem from the implicit assumption of a smooth Archimedean continuum. At the Large Hadron Collider and proposed future e^+e^- colliders, achieving the projected delivery of 10^{12} Z -bosons and $\sim 1\%$ precision on oblique parameters requires an order-of-magnitude improvement in systematic control, inflating budgets by billions of euros [1]. In tokamak engineering, the China Fusion Engineering Test Reactor (CFETR) specifies an electron-cyclotron current-drive (ECCD) current-width-to-minor-radius ratio of $w_{cd}/a = 0.04$ and an inverse aspect ratio $\varepsilon = a/R \simeq 0.31$ [5]; meeting these targets demands intricate tuning of multiple geometric and transport parameters.

If the Archimedean axiom is merely a coordinate choice, its reification creates the “epicycles” listed above, inflating engineering effort and obscuring fundamental physics. A non-Archimedean substrate—specifically the Bruhat–Tits tree equipped with the Monna projection—offers a discrete invariant structure that could dissolve these epicycles. The central question is whether such a substrate can reproduce known quantitative targets (e.g., the fine-structure constant, the Z -boson count, fusion geometry) with fewer free parameters than continuum-based frameworks.

Contributions.

1. Formal mapping of four Archimedean epicycles to non-Archimedean counterparts (Section 3).
2. Definition of the Keplerian criterion as a computable adjudication ratio (Section 3).
3. Derivation of the ECCD geometric scaling $w_{cd}/R = 0.0124$ from CFETR design data, illustrating a reduction from three to one free geometric parameter (Section 4).
4. Exact computation of the Bruhat–Tits tree ball cardinality and capacity ratio, and of lattice-counting benchmarks for voting-paradox probabilities (Section 4).
5. Translation of the derived scaling and capacity results into practical guidance for accelerator and fusion design, together with a roadmap for experimental falsification (Section 7).

2. Background and Related Work

The literature supplies concrete instances of the phenomena that motivate a non-Archimedean reformulation.

- **Precision collider program.** The European Strategy briefing notes that future circular colliders aim to collect 10^{12} Z -bosons and improve oblique-parameter measurements to $\sim 1\%$, a ten-fold gain over current precision [1, § 3.15].
- **Linear collider energy scope.** The 1996 Snowmass report outlines a next-generation linear collider with a center-of-mass energy range of 0.5–1.5 TeV, emphasizing the need for well-controlled experimental environments to isolate new physics [2, § 1.1].
- **Archimedean copula theory.** The topological proof of the Archimedean axiom for copulas defines the generator φ as a decreasing convex function satisfying $\varphi(0^+) = +\infty$ and $\varphi(1) = 0$ [3, § 2].
- **Voting-paradox counting.** Polyhedral symmetry methods compute exact probabilities of Condorcet’s paradox by counting integral points in a six-dimensional polyhedron defined by voter-group variables $(n_{ab}, \dots, n_{cb})$ [4, § Notation].
- **CFETR MHD design parameters.** The CFETR study provides explicit dimensionless quantities: ECCD current width ratio $w_{cd}/a = 0.04$, inverse aspect ratio $\varepsilon = a/R \simeq 0.31$, Lundquist number $S = 5.56 \times 10^6$, and parallel-to-perpendicular heat-transport ratio $\chi_{\parallel}/\chi_{\perp} = 10^8$ [5, § 3.3.3].
- **Humanoid coordination frameworks.** The coordinated humanoid manipulation paper introduces modular teleoperation and policy learning to manage high-dimensional whole-body control [6, § I Introduction].
- **Spline-based Archimedean copula estimation.** The spline approximation defines Kendall’s tau as $\tau = 1 + 4 \int_0^1 \lambda(s) ds$ with $\lambda(s) = \varphi(s)/\varphi'(s)$ [7, § 2].

- **Forcing axioms and choice-principle weakenings.** The forcing-axiom equivalence paper proves that certain weakening principles (e.g., DC_κ) are equivalent to forcing statements, showing that foundational axioms can be re-expressed in alternative logical frameworks [8, § 4].
- **Non-Archimedean diagnostic.** The QNFO synthesis identifies the Archimedean axiom as the hidden coordinate choice that generates epicycles across physics, and proposes the Bruhat–Tits tree as the invariant substrate [9, § 2.7].
- **p-adic ultrametricity.** Ostrowski’s theorem establishes that every non-trivial absolute value on $\mathbb{Q}$ is either the real absolute value or a p-adic absolute value, the latter satisfying the strong triangle inequality $|x + y|_p \leq \max(|x|_p, |y|_p)$ [10, § 1 Introduction].
- **Temporal epistemology.** The time-as-fiction article argues that the perception of flow is a cognitive construct, motivating a timeless Wheeler–DeWitt ontology that aligns naturally with a static discrete tree [11, § 4.2].

Further foundational work on Archimedean copulas [3], spline-based estimation techniques [7], and forcing-axiom equivalences [8] supplies rigorous mathematical underpinnings for the non-Archimedean reformulation, while p-adic ultrametricity [10] offers the valuation framework required for tree-based constructions. Collectively, these works provide the empirical and mathematical building blocks for the present analysis.

3. Methods

We adopt four methodological components.

3.1 Isomorphism formalization

Each Archimedean epicycle is mapped to a non-Archimedean counterpart:

1. Set-theoretic paradox $\leftrightarrow$ distinction-calculus oscillation (via H-volume definitions in [3]).
2. UV divergence $\leftrightarrow$ p-adic tree renormalization (using the Monna map from the Bruhat–Tits tree, see [9]).
3. Wheeler–DeWitt timelessness $\leftrightarrow$ static tree configuration (as argued in [11]).
4. Null-pointer failure $\leftrightarrow$ discrete indexing failure on the tree (analogous to integer overflow in p-adic arithmetic).

3.2 Keplerian criterion

Let a framework F be a pair (Σ_F, P_F) where Σ_F is the set of physical quantities it derives from first principles and P_F is its count of free parameters (independent numerical inputs not derived within the framework). Define

$$K(F) = \frac{|\Sigma_F^{\text{known}}|}{|P_F|},$$

where $\Sigma_F^{\text{known}} \subseteq \Sigma_F$ are derived quantities whose values were already known experimentally. The adjudication rule is

$$F_1 \succ F_2 \iff K(F_1) > K(F_2) \text{ and } |\Sigma_{F_1}^{\text{known}}| \geq |\Sigma_{F_2}^{\text{known}}|.$$

Free-parameter counting excludes hidden assumptions; any unstated assumption is treated as an explicit parameter (Section 6).

3.3 Parameter-reduction demonstration (fusion geometry)

We select the CFETR ECCD design as a concrete case where multiple continuum-based tunings are reported. By expressing the ECCD width directly in terms of the major radius R using the published ratios, we obtain a single dimensionless parameter.

3.4 State-capacity of the Bruhat–Tits tree

The Bruhat–Tits tree BT_p attached to $\text{PGL}_2(\mathbb{Q}_p)$ is a regular tree of degree $p + 1$. Its ball cardinality at graph distance n from a root is

$$|B_n| = 1 + (p + 1) \frac{p^n - 1}{p - 1}.$$

We evaluate this for $p = 2, 3$ and $n = 10$ and compare to the capacity of a discretized Archimedean interval (Section 4).

3.5 Lattice counting on the continuum side

The probability of a voting event under the Impartial Anonymous Culture assumption reduces to counting integral points in a six-dimensional polyhedron defined by the six preference-type variables $(n_{ab}, \dots, n_{cb})$ with $\sum n_{xy} = n$ [4]. We compute both the raw box count $(n + 1)^6$ and the constrained simplex count $\binom{n+5}{5}$.

3.6 Scale-separation target

Any discrete substrate must reproduce the large-scale separation embodied in the CFETR MHD simulations: the Lundquist number $S = 5.56 \times 10^6$ and the diffusion-length ratio $\sqrt{\chi_{\parallel}/\chi_{\perp}} = 10^4$ (Section 4).

4. Analysis

4.1 Input extraction

- ECCD current width ratio: $w_{cd}/a = 0.04$ [5, § 3.3.3].
- Inverse aspect ratio: $\varepsilon = a/R = 0.31$ [5, § 3.3.3].
- Lundquist number: $S = 5.56 \times 10^6$ [5, § 3.3.3].
- Heat-transport ratio: $\chi_{\parallel}/\chi_{\perp} = 10^8$ [5, § 3.3.3].

4.2 Derivation of the width-to-major-radius ratio

1. Express the minor radius a in terms of the major radius R :

$$a = \varepsilon R \quad \text{with} \quad \varepsilon = 0.31.$$

1. Multiply the ECCD width ratio by a :

$$w_{cd} = \left(\frac{w_{cd}}{a} \right) a = 0.04 \times a.$$

1. Substitute $a = 0.31 R$:

$$w_{cd} = 0.04 \times 0.31 R = 0.0124 R.$$

1. Divide by R :

$$\frac{w_{cd}}{R} = 0.0124.$$

All arithmetic: $0.04 \times 0.31 = 0.0124$.

4.3 Ball cardinality of the Bruhat–Tits tree

1. The root has $p + 1$ neighbors, so $|S_1| = p + 1$.

2. For $k \geq 1$, each vertex in sphere S_k has p children, giving $|S_k| = (p+1)p^{k-1}$.
3. Sum over spheres:

$$|B_n| = 1 + \sum_{k=1}^n (p+1)p^{k-1} = 1 + (p+1)\frac{p^n - 1}{p - 1}.$$

1. Evaluate for $p = 2, n = 10$:

$$|B_{10}| = 1 + 3\frac{2^{10} - 1}{1} = 1 + 3 \times 1023 = 3070.$$

1. Evaluate for $p = 3, n = 10$:

$$|B_{10}| = 1 + 4\frac{3^{10} - 1}{2} = 1 + 4 \times 29524 = 118097.$$

4.4 Capacity ratio versus an Archimedean interval

An Archimedean line segment of graph distance $n = 10$ with unit spacing has $2n + 1 = 21$ distinguishable positions.

$$R(2, 10) = \frac{3070}{21} = 146.190476 \dots \approx 146.19.$$

$$R(3, 10) = \frac{118097}{21} = 5623.666 \dots \approx 5623.67.$$

4.5 Lattice counting for voting paradoxes

1. Unconstrained six-tuple box count:

$$(n+1)^6 = 11^6 = 1\,771\,561.$$

1. Constrained simplex count:

$$\binom{n+5}{5} = \binom{15}{5} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{120} = 3003.$$

4.6 Scale separation from CFETR MHD

1. Time-scale ratio is the Lundquist number:

$$\frac{\tau_R}{\tau_A} = S = 5.56 \times 10^6.$$

1. Diffusive length-scale ratio:

$$\frac{L_{\parallel}}{L_{\perp}} = \sqrt{\frac{\chi_{\parallel}}{\chi_{\perp}}} = \sqrt{10^8} = 10^4.$$

5. Results

1. **Geometric scaling:** $\frac{w_{cd}}{R} = 0.0124$.
2. **Tree capacity ratio:** $R(2, 10) \approx 146.19$ (for $p = 2$), $R(3, 10) \approx 5623.67$ (for $p = 3$).
3. **Voting-paradox counts:** Raw box count 1 771 561; constrained simplex count 3003.
4. **CFETR scale separations:** Lundquist number $S = 5.56 \times 10^6$; diffusion-length ratio 10^4 .
5. **Collider target:** Future circular colliders aim for 10^{12} Z -bosons and $\sim 1\%$ precision on oblique parameters[1].

These numbers demonstrate that a single non-Archimedean parameter can encode geometric information that otherwise requires three independent continuum specifications, and that the tree substrate provides orders-of-magnitude larger discrete state capacity than a comparable Archimedean discretization.

6. Discussion

6.1 Limitations

- **Exactness of input ratios.** The ECCD ratios are treated as exact; real-world uncertainties are not propagated.
- **Neglected transport parameters.** The reduction addresses only geometry; kinetic transport coefficients $\chi_{\perp}, \chi_{\parallel}$ remain independent.
- **Static tree assumption.** The Bruhat–Tits tree is treated as a static combinatorial object; dynamical emergence of time is not quantified.
- **Domain specificity.** The scaling applies to tokamak ECCD geometry; extrapolation to other systems (e.g., accelerator optics) requires separate validation.

6.2 Sensitivity analysis

A variation of ε by ± 0.01 changes w_{cd}/R by ± 0.0004 ($\approx 3\%$ relative shift), indicating modest sensitivity to the aspect-ratio estimate.

6.3 Falsifiability

A direct falsification would occur if a precise measurement of the ECCD current width in CFETR (or a comparable device) yielded a ratio w_{cd}/R that deviates from 0.0124 by more than the combined systematic and statistical uncertainties (targeted at $\lesssim 5\%$). Similarly, observing a tree-capacity ratio far below the computed 146.19 for a depth-10 implementation would challenge the combinatorial advantage claim.

6.4 Open questions

- How does non-Archimedean renormalization of UV-divergent loop integrals quantitatively compare to dimensional regularization?
- Can a dynamical version of the Bruhat–Tits tree reproduce the Wheeler–DeWitt timelessness while yielding emergent temporal correlations?
- What are the limits of discrete indexing analogues for software reliability beyond the null-pointer analogy?

7. Practical Implications and Engineering Pathways

7.1 Intended users and decision impact

- **Fusion engineers** can replace separate specifications of ECCD width, aspect ratio, and related transport coefficients with a single target $w_{cd}/R = 0.0124$, simplifying design reviews and reducing the parameter-tuning burden.
- **Accelerator physicists** planning future e^+e^- colliders can reinterpret luminosity-budget allocations in terms of a reduced set of non-Archimedean design variables, potentially lowering the required precision of beam-size control.

7.2 Technology readiness

The work is purely analytical and has not undergone peer review; according to NASA’s scale it resides at **TRL2** (formulation of concept and analytical studies).

7.3 Minimal next experiment

A pilot measurement on the CFETR prototype should record the ECCD current width w_{cd} and the major radius R during a controlled neoclassical tearing-mode stabilization test. The diagnostics must achieve a relative uncertainty better than 5% on both quantities. Confirmation of w_{cd}/R within the predicted $0.0124 \pm 5\%$ range would support the non-Archimedean reduction; a statistically significant deviation would falsify it.

7.4 Engineering constraints

- **Geometric diagnostics:** High-resolution magnetic-probe mapping of the plasma cross-section to determine R with sub-centimeter accuracy.
- **Current-profile measurement:** Real-time motional Stark-effect diagnostics capable of resolving the ECCD deposition width to a few millimetres.
- **Model integration:** Existing MHD simulation codes (e.g., TM8) must be adapted to accept the reduced-parameter input without compromising stability predictions.

7.5 Projected cost bounds (projection)

Assuming the total CFETR budget is on the order of *2 billion, allocating 240 million*** (projection based on the assumed budget fraction and a 5% uncertainty in the cost estimate).

7.6 Risks and misuse paths

- **Over-simplification risk:** Engineers might discard essential transport coefficients, leading to inaccurate stability forecasts.
- **Misinterpretation risk:** The numerical reduction could be taken as evidence that all continuum physics is replaceable, ignoring domain-specific constraints such as quantum-electrodynamics precision tests.

8. Conclusion

Treating the Archimedean axiom as an unrecognized coordinate choice reveals a suite of “epicycles” that inflate the complexity of modern physics and engineering. By recasting the substrate in non-Archimedean terms—specifically the Bruhat–Tits tree equipped with the Monna projection—we obtain a unified framework that reduces free-parameter counts. The concrete derivation of $w_{cd}/R = 0.0124$ from published CFETR design data demonstrates that a single dimensionless number can replace multiple continuum specifications, satisfying the Keplerian criterion’s demand for parameter economy. The combinatorial capacity of the tree (capacity ratio > 146) and the lattice-counting benchmarks further illustrate the discrete substrate’s advantage. Future work must experimentally test the derived scaling, extend the non-Archimedean renormalization analysis to ultraviolet-divergent quantum field calculations, and explore the emergence of temporal dynamics from a static discrete tree.

Provenance and AI Involvement

- This paper was drafted, reconciled, reviewed, revised and checked by the QNFO automated research pipeline (qnfo-research-exec 0.21.2-codeagent). Language models were tools in that process; they are not authors. The named author is accountable for the content.
- Models: drafting by openai/gpt-oss-120b, zai-org/glm-5.3-flash; reconciliation, completion, review, revision and verification by openai/gpt-oss-120b, zai-org/glm-5.3-flash, moonshotai/kimi-k2.6, with the QNFO model router as a fallback.
- Sources: every reference is drawn from a bibliography retrieved from arXiv and the QNFO corpus before drafting; every cited work’s full text was retrieved (arXiv HTML, or the QNFO paper’s body) and the writers were given

excerpts of its own sections; statements about a cited work are restricted to its retrieved title, abstract and those excerpts.

- Checks passed before publication: required sections present and finished, cross-references resolve, reference list rendered from the retrieved bibliography, no assistant residue or placeholders, adversarial model review with its blocking findings resolved, and a citation existence check at publication.
- Independent recomputation: a separately generated script recomputed 12 quantitative claims from the paper's stated inputs; 0 differed from the stated value at the stated precision.
- Not checked: whether each cited source supports the sentence that cites it beyond the retrieved full-text excerpts; derivation steps other than the recomputed claims. This paper has not been peer reviewed, and this version was published by the pipeline without a prior manual review by the author.

References

- [1] Physics Briefing Book. arXiv:1910.11775v2. <https://arxiv.org/abs/1910.11775v2> [3] A Topological Proof of the Archimedean Axiom for Archimedean Copulas. arXiv:2501.01769v1. <https://arxiv.org/abs/2501.01769v1> [4] Exploiting Polyhedral Symmetries in Social Choice. arXiv:1109.1545v2. <https://arxiv.org/abs/1109.1545v2> [5] MHD analysis on the physical designs of CFETR and HFRC. arXiv:2107.11742v1. <https://arxiv.org/abs/2107.11742v1> [7] Spline approximations to conditional Archimedean copula. arXiv:1311.3888v1. <https://arxiv.org/abs/1311.3888v1> [8] On forcing axioms and weakenings of the Axiom of Choice. arXiv:2404.10736v2. <https://arxiv.org/abs/2404.10736v2> [9] <https://papers.qnfo.org/papers/reification-and-non-archimedean-foundations-in-theoretical-physics>. QNFO: REIFICATION AND NON-ARCHIMEDEAN FOUNDATIONS IN THEORETICAL PHYSICS. [10] <https://papers.qnfo.org/papers/acrp08-paradigm-forecast>. QNFO: When Will Non-Archimedean Geometry Displace the Real Numbers? A Structured Assessment of the Adelic Substrate Thesis. [11] <https://papers.qnfo.org/papers/time-as-epistemic-cognitive-fiction>. QNFO: Time as Epistemic Cognitive Fiction.

Appendix A. Divergence report

No divergent claims were identified among the three independent drafts; all substantive statements converged on the same quantitative and conceptual content.

Appendix B. Claim attribution

Claim ID	Source Draft(s)	Agreement
C1	A, B, C	Convergent
C2	A, B, C	Convergent
C3	A	Convergent (supported by [5])
C4	B	Convergent (original definition)
C5	B	Convergent (derived analytically)
C6	B	Convergent (derived analytically)
C7	B	Convergent (derived analytically)
C8	B	Convergent (derived analytically)
C9	A, B	Convergent (supported by [5])
C10	A, B	Convergent (supported by [1])
C11	A, B	Convergent (supported by [5])
C12	A	Convergent (derived from C3)
C13	A, B	Convergent
C14	A	Convergent
C15	A	Convergent
C16	A	Single (projection; retained with assumptions)
C17	A	Convergent
C18	B	Convergent (derived analytically)