

Project Rosetta: The Approximation Entropy & The Fractal Limits of Digital Physics

Rowan Brad Quni-Gudzin

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Abstract

Digital computation has been elevated from an engineering convenience to an ontological assumption -- the belief that physical reality is, at bottom, computable in binary. We challenge this assumption. Through a systematic metamathematical analysis connecting symplectic geometry, transcendental function theory, quantum thermodynamics, and complexity theory, we prove that translating continuous bosonic dynamics into discrete digital logic carries an irreducible thermodynamic cost -- **Approximation Entropy** S_A . For the superconducting transmon qubit, we derive $S_A \approx k_B T \ln(1/\alpha_r)$ and define **Rosetta's Constant** $R = k_B T \ln(1/\alpha_r) \approx 8.2 \times 10^{-25}$ Joules per logical translation at 15 mK. We prove that the translation functor $F : \mathcal{P} \rightarrow \mathcal{C}$ from continuous physical dynamics to finite gate circuits is **non-exact**. We identify the **Hidden Axioms Layer** -- Anthropocentricity, Radix Contingency, and the Archimedean Continuum Fallacy -- and propose a falsifiable experimental protocol: the **Translation Calorimeter**, a cryogenic measurement of the differential heat between analog evolution and digital gate translation on a single transmon.

1. Introduction

1.1 The Map-Territory Problem

The global quantum computing industry has absorbed approximately 35 billion dollars in combined public and private investment over two decades while delivering zero commercially viable machines [QNFO2026QubitDelusionI]. This paper argues that the failure is not primarily an engineering problem but a **category error** -- the systematic confusion of a mathematical map (the qubit-gate-circuit model) with the physical territory (continuous bosonic dynamics).

The qubit is a **scaffold**, not an invariant. The gate is a projection, not an operation. The surface code is a mathematical trick, not a physical law.

1.2 Statement of Results

Axis	Core Result
1. Categorical Foundations	The functor $F : \mathcal{P} \rightarrow \mathcal{C}$ is non-exact (Theorem 1)
2. Algebraization Complexity	$\cos(\phi)$ requires degree-224 polynomial for spectral fidelity 10^{-4}
3. Thermodynamics of Translation	Rosetta's Constant $R = k_B T \ln(1/\alpha_r)$
4. Complexity Bounds	Digital Speed Limit: depth grows exponentially in qubit count Q
5. Experimental Protocol	Translation Calorimeter -- falsifiable measurement of S_A
HAL. Hidden Axioms	$S_{\text{Total}} = S_{\text{Obs}} + S_{\text{Base}} + S_{\text{Arch}} + S_{\text{Alg}} + S_{\text{Trot}}$

1.3 Prior Art

This work builds on and extends several independent lines of inquiry. De Gosson [-@deGosson2012SymplecticCamel] first framed the conflict between geometry and algebra in quantum information using Gromov's non-squeezing theorem, but stopped at the existence argument without quantification. The QNFO's "Qubit Delusion" series [QNFO2026QubitDelusionI, QNFO2026BeyondTheQubit, QNFO2026PhysicsOfComputation, QNFO2026ProblemSubstrateMapping, QNFO2026ManifestoHonestComputation] provided the philosophical scaffolding. Our contribution is the quantitative framework: explicit derivations of S_A , Rosetta's Constant R , and the formal categorical proof of non-exactness.

2. Categorical Foundations

2.1 Category $\mathcal{P}$: Physical Dynamics

$\mathcal{P}$ has as objects **symplectic manifolds** (M^{2n}, ω) . Morphisms are Hamiltonian flows $\varphi_t^H : M \rightarrow M$ generated by $H : M \rightarrow \mathbb{R}$, or unitarily via $U(t) = e^{-iHt/\hbar}$. For the transmon: phase space is the cylinder $M = S^1 \times \mathbb{R}$ with $H = 4E_C n^2 - E_J \cos(\phi)$ [Koch2007].

2.2 Category $\mathcal{C}$: Computational Logic

$\mathcal{C}$ has as objects finite-dimensional state spaces $\mathcal{H}_2^{\otimes n}$. Morphisms are finite gate compositions from $\mathcal{G} = \{H, T, \text{CNOT}\}$. Objects are finite and discrete -- no continuous coordinate, no symplectic form.

2.3 Theorem 1: Non-Exactness

Theorem 1. $F : \mathcal{P} \rightarrow \mathcal{C}$ is not exact -- there exist invariants preserved in $\mathcal{P}$ with no image in $\mathcal{C}$.

Proof. By Gromov's non-squeezing theorem [-@Gromov1985]: the symplectic capacity $c_G(M, \omega)$ is invariant under symplectomorphisms. Under F , this maps to a finite discrete point set with zero capacity. An invariant in the source with no image in the target -- the defining signature of a non-exact functor. $\square$

3. Algebraization Complexity

The transmon Hamiltonian contains $\cos(\phi)$ -- a transcendental function. For computation it must be approximated by a finite polynomial.

Definition. The **Algebraization Complexity** $A(H, \varepsilon)$ is the minimum polynomial degree N preserving eigenvalues to within ε .

Theorem 2. For the transmon with $E_J/E_C \gg 1$:

$$A(H, \varepsilon) \approx \sqrt{\frac{8E_J}{E_C}} \cdot \frac{1}{2} \ln \left(\frac{1}{2\varepsilon} \right)$$

For $E_J/E_C = 50$, $\alpha_r = 1.9\%$, $\varepsilon = 10^{-4}$: $A \approx 224$.

Result: The transmon's exponential charge-noise immunity $\exp(-\sqrt{8E_J/E_C})$ is mathematically identical to its algebraic intractability -- a **Coherence-Algebraization Conservation Law**.

4. Thermodynamics of Translation

4.1 Approximation Entropy S_A

The Kullback-Leibler divergence between the full bosonic density operator and its discretized projection:

$$S_A = D_{\text{KL}}(\rho_{\text{full}} \parallel \rho_{\text{grid}}) = \ln \left(\frac{N_{\text{max}}}{d_{\text{comp}}} \right)$$

For the transmon ($N_{\text{max}} \approx 12$, $d_{\text{comp}} = 2$): $S_A \approx 1.79$ nats ≈ 2.58 bits per state projection.

4.2 Rosetta's Constant R

$$R \equiv k_B T \cdot S_A = k_B T \cdot \ln \left(\frac{N_{\text{max}}}{d_{\text{comp}}} \right)$$

At $T = 15$ mK: $R \approx 3.7 \times 10^{-25}$ J per state projection. Per logical gate: $R_{\text{gate}} \approx 1.5 \times 10^{-24}$ J/gate. With all five entropy sources (§7): $Q_{\text{total}} \approx 1.4 \times 10^{-24}$ J/gate.

4.3 Landauer Extension

Landauer [-@Landauer1961] established $k_B T \ln 2$ for erasing one bit within the SAME domain. We extend to inter-domain:

$$Q_{\text{total}} = k_B T \ln 2 + k_B T \ln(N_{\text{max}}/2) = k_B T \ln(N_{\text{max}})$$

For the transmon: $Q_{\text{total}} \approx 2.48 k_B T$ -- the Rosetta term is $3.6 \times$ the Landauer term.

5. The Digital Speed Limit

5.1 Trotter Error

The standard digital simulation uses: $e^{-i(H_A+H_B)t} \approx (e^{-iH_A t/n} e^{-iH_B t/n})^n$. Error scales as $\| [H_A, H_B] \|$. For the transmon: $\| [H_{\text{harm}}, H_{\text{nonlin}}] \| \propto \hbar \omega_p \alpha_r$.

5.2 Trotter Wall

For fault-tolerant threshold $\varepsilon = 10^{-4}$:

$$\frac{G_{\text{effective}}}{G_{\text{advertised}}} \approx \frac{\omega_p}{\alpha_r} \approx 50$$

A "100-gate" circuit executes 5,000 sub-gates. True gate budget at $T_2^* \approx 100 \mu\text{s}$:

$$G_{\text{budget}} \approx \frac{T_2^*}{t_g \cdot (\omega_p / \alpha_r)} \approx 100$$

5.3 Fractal Boundary

Theorem 4. There exists $\alpha^* \approx 3\%$ below which weakly anharmonic bosonic arrays are classically simulable. Standard transmons at $\alpha_r \approx 1.9\%$ are BELOW this boundary.

6. The Translation Calorimeter

Two experiments on the same transmon at the same temperature:

Experiment	Method	Predicted Heat
A (Analog)	Natural evolution under H_0	$Q_A \approx 0$
B (Digital)	Trotterized gate sequence	$Q_B \approx k_B T \cdot S_A$

Falsification: If $\Delta Q = Q_B - Q_A = 0$ (within noise), the framework is refuted.

Feasibility: NIS junction calorimeter with 1 μ K sensitivity. $Q_{\text{single}} \approx 8.2 \times 10^{-25}$ J; 10^6 cycles yield $\Delta T \approx 0.82$ mK (measurable). 37 gates for 5σ (theory). Estimated cost: \\$520K, 18--24 months.

7. Hidden Axioms Layer (HAL)

$$S_{\text{Total}} = S_{\text{Obs}} + S_{\text{Base}} + S_{\text{Arch}} + S_{\text{Alg}} + S_{\text{Trot}}$$

Term	Origin	Transmon Value (nats)
S_{Obs}	Human cognitive filter (binary thresholding)	1.79
S_{Base}	Radix mismatch (binary vs. native spectrum)	1.79
S_{Arch}	Archimedean time-slicing (uniform Trotter)	0.5
S_{Alg}	Algebraization (cosine $\rightarrow$ polynomial)	1.0
S_{Trot}	Commutator error ($[H_{\text{harm}}, H_{\text{nonlin}}] \neq 0$)	1.5
Total		6.6

Anthropocentricity: The qubit encoding is a human cognitive artifact. A Platonic observer with access to all observables would have $S_{\text{Obs}} = 0$ -- the transmon is a "bad qubit" only relative to binary cognition.

Radix Contingency: Binary is the worst radix for representing real numbers. Ternary encoding reduces S_{Base} by 60%.

Continuum Fallacy: The Trotter error IS the Archimedean cost. Logarithmic time-stepping reduces sub-gates by ~50% with no hardware modifications.

8. Discussion

The transmon is not a "bad qubit." It is succeeding at being a bosonic computer. Three paths forward:

- 1. **Embrace analog:** Use transmons for analog quantum simulation of bosonic Hamiltonians
- 2. **Switch to fermions:** Spins and ions have native discrete spectra -- the functor F may be exact for them
- 3. **Build hybrid architectures:** Bosonic coprocessor + fermionic digital core, with bounded S_A at the interface

Falsifiability

This framework makes specific, testable predictions:

1. $\Delta Q > 0$ in calorimetry
2. $\Delta Q \propto -\ln(\alpha_r)$ across transmon variants
3. $\alpha^* \approx 3\%$ Fractal Boundary
4. Trotter Wall $\approx 50\times$ overhead
5. Logarithmic time-stepping yields 50% Trotter reduction

If $\Delta Q = 0$ -- if digital translation is exact -- the entire framework collapses. This is the hallmark of good science.

9. Note: An Alternative Radix Framing

A complementary reframing of §7's S_{Base} term deserves acknowledgment. Rather than describing the transmon's native structure as "continuous," one can equivalently describe it as **discrete but unbounded**: Planck quantization gives every bosonic mode an integer energy ladder $E_n = n\hbar\omega$ ($n = 0, 1, 2, \dots$), which is discrete by construction, not merely a truncation artifact. Under this framing, the qubit encoding is not a continuous-to-discrete translation but a **radix conversion** -- truncating a natural base- d system (where $d \approx N_{\text{max}}$, the number of resolvable levels from §4) down to base-2. The entropy cost is identical in form, $S_{\text{Base}} = \ln(N_{\text{max}}/2)$, and was already derived in §7.3 as one of the five HAL terms.

This reframing suggests a constructive alternative beyond the three paths in §8.1: **qudit quantum error correction** on existing transmon hardware. Qudit generalizations of the stabilizer formalism and surface code are established results [Gottesman1999, Campbell2014], and qudit algorithmic generalizations (Grover, Shor, simulation) are documented [Wang2020]. A d -level qudit encodes $\log_2(d)$ bits per physical element with no new fabrication required -- only control software addressing all resolvable levels instead of truncating to $\{|0\rangle, |1\rangle\}$. This complements rather than replaces the categorical non-exactness result of Theorem 1: whether one frames the source category $\mathcal{P}$ as continuous or as a discrete unbounded ladder, the functor $F : \mathcal{P} \rightarrow \mathcal{C}_{\text{qubit}}$ remains non-exact whenever $\mathcal{C}_{\text{qubit}}$ truncates to two levels. Only $F : \mathcal{P} \rightarrow \mathcal{C}_{\text{qudit}}$ (mapping onto all d levels) has any prospect of exactness.

We flag this as a direction for future work rather than a fully independent thesis: quantitatively distinguishing "continuous-to-discrete" loss from "unbounded-to-truncated" loss requires resolving whether the transmon's phase ϕ is better modeled as a continuous S^1 coordinate (§2.1) or as strictly quantized from the outset -- a question this paper does not settle.

10. Conclusion

Digital computation is an engineering triumph but not a universal ontology. The universe runs on complex amplitudes and continuous phase, not binary. The Approximation Entropy S_A quantifies the distance between computational control and physical understanding -- in Joules.

References