

The p-Adic Temperley–Lieb Parameter: Valuation-Theoretic Constraints and an Adelic Accuracy Framework for Quantum Predictions

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Abstract

The Temperley–Lieb (TL) algebra governs a broad class of exactly solvable quantum systems, from anyonic chains to braid-based teleportation circuits, through its scalar loop parameter δ . We investigate the proposal that a *p-adic Temperley–Lieb parameter* — a number-theoretic functional of δ — can predict quantum system behavior with accuracy of at least 95%. We construct the only mathematically canonical candidate adapted to the p-adic norm, $\delta_p = p + p^{-1} \in \mathbb{Q}_p$ (the analogue of $\delta = q + q^{-1}$ with q the uniformizer), and analyze it with explicit valuation arithmetic. We prove that the quantum dimension of the rank-2 Jones–Wenzl projector, $d_2 = (p^4 + p^2 + 1)/p^2$, has *p-adic valuation exactly -2 for every prime p , so the projector exists over $\mathbb{Q}_p$ but is never integral over $\mathbb{Z}_p$; the same holds with valuation -3 for d_3 . We further show that at the generic root-of-unity values $\tau_r = 2\cos(2\pi/r)$ for $r = 3, 5, 6, 7$, the TL parameter is an algebraic unit, hence $|\tau|_p = 1$ for every prime: the parameter itself is p-adically invisible, and any non-trivial p-adic information must reside in Markov trace values. We prove an adelic obstruction: p-adic precision imposes no bound, in either direction, on Archimedean error. We then derive an explicit accuracy budget showing that two decimal digits of Archimedean control guarantee accuracy ≥ 0.9937651 at the worst-case unit value, and — under an explicitly stated joint-control assumption — that three ternary digits of p-adic precision suffice for the 95% target. The strong claim of universal $\geq 95\%$ predictive accuracy is not supported; what survives is a precise, falsifiable algebraic program with a conditional accuracy budget.*

1. Introduction

Two mathematical threads have largely developed in isolation. The first is the Temperley–Lieb algebra $TL_n(\delta)$, generated by elements $e_1, \dots, e_{n-1}$ satisfying

$$e_i^2 = \delta e_i, e_i e_{i\pm 1} e_i = e_i, e_i e_j = e_j e_i \quad (|i-j| > 1),$$

where δ is the loop parameter. This algebra underlies the Jones polynomial, solvable lattice models, anyonic quantum computation, and diagrammatic descriptions of quantum teleportation [3]. The second thread is p-adic mathematical physics, in which the field $\mathbb{Q}_p$ of *p-adic numbers* — with distance $|x|_p = p^{-v_p(x)}$, where $v_p(x)$ is the exponent of p in the factorization of x — replaces or complements the reals [6], with extensions to hierarchic quantum systems via p-adic wavelets [7] and to cosmology [8].

The research idea under examination asserts that a "p-adic Temperley–Lieb parameter" predicts quantum system behavior with accuracy of at least 95%. This paper treats the idea as a hypothesis to be formalized and stress-tested. Our contributions are:

1. A canonical construction of the parameter, $\delta_p = p + p^{-1} \in \mathbb{Q}_p$ (Section 3), with a documented discussion of the competing definitions proposed in the source drafts (Appendix A).
2. Explicit valuation computations establishing a universal structural feature: non-integrality of TL quantum dimensions over $\mathbb{Z}_p$ for every prime (Section 4).

3. A negative structural result at root-of-unity values: τ is an algebraic unit, so $|\tau|_p = 1$ for all p (Section 4), forcing the predictive burden onto Markov trace values [1].
4. A proof of the adelic obstruction and an explicit accuracy budget relating p -adic precision to Archimedean prediction error, with the digit budgets required to reach the 95% target under stated assumptions (Sections 4–5).
5. An assessment of what, if anything, could ground an accuracy claim, with explicit falsification criteria (Section 6).

The analysis is deliberately elementary: every number is derived by exact rational arithmetic from stated inputs, and projections are clearly labeled with their assumptions.

2. Background and Related Work

Affine TL algebras and Markov elements. [1] constructs a tower of affine Temperley–Lieb algebras of type $\tilde{A}_n$ and Markov elements therein, proving that any trace on the type $\tilde{A}_2$ algebra is uniquely determined by its values on the Markov elements. This uniqueness theorem matters here for two reasons: it licenses evaluating any putative p -adic invariant on a finite generating set (making computation finite), and it identifies the precise scalars — Markov trace values — on which non-trivial p -adic data can live once the loop parameter itself proves p -adically trivial.

Rewriting systems and TL bases. [2] defines TL both by presentation and diagrammatically, then attacks the basis problem algorithmically using rewriting theory, extending to an oriented generalization requiring categorical machinery. Any computational pipeline that evaluates TL expressions — including a p -adic parameter pipeline — requires a canonical normal form; rewriting systems supply the determinism needed to make "the value of a TL expression" a well-defined number whose p -adic norms can be taken. The oriented generalization also shows that the presentation alone underdetermines the "right" category, a caution for extensions.

TL in quantum information. [3] realizes braid teleportation configurations, teleportation swapping, and virtual braid representations within standard teleportation, devising diagrammatic rules for circuits with maximally entangled states. This is the strongest existing bridge between TL algebra and operational quantum information processing: TL parameters are not abstract but parametrize teleportation circuits whose fidelities are measurable. Any "prediction accuracy" for a TL-parameterized quantum system must ultimately cash out against such circuits.

Arithmetic sensitivity of TL invariants. [4] constructs Anick's resolution for TL_3 and computes bar homology $Tor^*(TL_3)(\mathbb{C}, \mathbb{C})$, showing that the scalar τ in the defining relation $ei^2 = \tau e_i$ enters the differentials and that the bar homology depends on the arithmetic nature of τ — generic versus root-of-unity. This is direct evidence that fine arithmetic distinctions in the TL parameter propagate to algebraic invariants: precisely the mechanism a p -adic parameter is designed to systematize.

p -adic statistical mechanics. [5] reduces the description of p -adic Gibbs measures for the Potts model on a Cayley tree to a recursive equation and proves that a phase transition occurs if and only if $p = 3$, for any nonzero interaction. This is the sharpest known "a specific prime governs a physical threshold" result, and the natural benchmark against which any p -adic TL parameter must be compared: if the parameter carried physical content in the Potts channel, one would expect a signature at $p = 3$.

p -adic and adelic quantum theory. [6] formulates p -adic and adelic quantum mechanics with complex-valued wave functions of p -adic and adelic argument, establishing the framework in which a p -adic parameter could act on physical states; the adelic product formula is the structural backbone of that framework and of our obstruction

result. [7] proposes the p-adic wavelet transform as a tool for hierarchic quantum systems — a natural fit for TL towers, which are hierarchic by construction ($n \rightarrow n+1$). [8] introduces p-adic worlds adjoined to the real world in a cosmological setting, illustrating the maximal (and, we argue, currently untestable) reading of p-adic physics.

QNFO corpus context. [9] treats topological aliasing and holographic readout, relevant to how p-adic coordinates might alias Archimedean observables. [10] develops the Bruhat–Tits tree as a unifying geometric object — the non-Archimedean analogue of the hyperbolic plane on which both p-adic TL representations and the Cayley-tree models of [5] live. [11] is a red-team assessment of a "Harmonic Paradigm" whose bibliography invoked Ostrowski's theorem and p-adic structures while its core mechanisms remained disconnected from them; its skeptical methodology is a model for the present assessment. [12] reports that the semigroup $\{2^a \cdot 3^b \cdot 5^c\}$ simultaneously realizes the Standard Model mass spectrum, GKP code lattices, and the Efimov discretuum, with a 15-entry calibration register — the genre of multi-domain small-prime claim against which a single-parameter accuracy claim must be carefully distinguished.

3. Methods

3.1 Definition of the parameter. The TL loop parameter admits the canonical factorization $\delta = z + z^{-1}$. In the real/complex theory, $\delta = q + q^{-1}$ with $q = e^{i\theta}$ is adapted to the unit circle. The unique p-adic-canonical choice is $z = p$, the uniformizer of $\mathbb{Q}_p$ (the element defining the valuation), giving

$$\delta p = p + p^{-1} \in \mathbb{Q}_p.$$

This is the TL parameter naturally adapted to the p-adic norm. The *p-adic TL data* of a TL-type model then consist of the valuations of δp , of the quantum dimensions d_n generated by the recursion $d_0 = 1$, $d_1 = \delta$, $d_{n+1} = \delta d_n - d_{n-1}$, and of Markov trace values per the uniqueness theorem of [1].

3.2 Tools. (a) The p-adic valuation v_p : for $x = p^k \cdot (a/b)$ with $p \nmid ab$, $v_p(x) = k$ and $|x|_p = p^{-k}$; x is a p-adic integer iff $v_p(x) \geq 0$. (b) The Jones–Wenzl projector f_n , the unique element with $e_i f_n = f_n e_i = 0$ and $f_n^2 = f_n$, which exists iff $d_1, \dots, d_{n-1}$ are invertible (over $\mathbb{Q}_p$, invertibility requires only $d_i \neq 0$). (c) The genericity criterion from [4]: bar homology of $TL_\lambda(\tau)$ depends on whether τ is a root of unity; the only rational roots of unity are ± 1 . (d) For algebraic τ , $|\tau|_p$ is computed via the field norm: τ is an algebraic unit iff its norm is ± 1 , in which case $|\tau|_p = 1$ for every finite prime.

3.3 Accuracy budget machinery. Given a predicted parameter $\hat{\tau} = \tau(1 + \epsilon)$, define the accuracy of a parameter-level prediction as

$$A = 1 - |\epsilon|_\infty \cdot |\tau|_\infty,$$

a linear fidelity proxy: TL observables are polynomials in τ , and we take the first-order propagation term as the budget-relevant bound (higher orders discussed in Section 6). The 95% target reads $|\epsilon|_\infty \leq 0.05/|\tau|_\infty$.

3.4 Honesty rules. We do not simulate anything. Every number in Section 4 is computed by exact rational arithmetic with stated inputs; Section 5 reports only those numbers or clearly labeled projections with explicit assumptions.

4. Analysis

4.1 Input numbers and their sources. Primes $p = 2, 3, 5, 7$ (first four primes, standard). TL relations and the quantum-dimension recursion: standard TL theory (cf. [2], [3]). Homological genericity criterion: [4]. Potts $p = 3$ benchmark: [5]. Root-of-unity values $\tau_r = 2\cos(2\pi/r)$: standard exact evaluations in the TL recoupling regime (cf. [2]).

4.2 Computation 1: δ_p and its valuation.

- $p = 2$: $\delta_2 = 2 + 1/2 = 5/2$; $v_2 = -1$; $|\delta_2|_2 = 2$.
- $p = 3$: $\delta_3 = 3 + 1/3 = 10/3$; $v_3 = -1$; $|\delta_3|_3 = 3$.
- $p = 5$: $\delta_5 = 5 + 1/5 = 26/5$; $v_5 = -1$; $|\delta_5|_5 = 5$.
- $p = 7$: $\delta_7 = 7 + 1/7 = 50/7$; $v_7 = -1$; $|\delta_7|_7 = 7$.

In general $vp(\delta_p) = -1$ for every prime, since $p^2 + 1 \equiv 1 \pmod{p}$ is not divisible by p .

4.3 Computation 2: quantum dimension $d_2 = \delta^2 - 1$.

$$d_2 = (p + p^{-1})^2 - 1 = p^2 + 1 + p^{-2} = (p^4 + p^2 + 1)/p^2.$$

- $p = 2$: $16 + 4 + 1 = 21$; $d_2 = 21/4$; $v_2 = 0 - 2 = -2$; $|d_2|_2 = 4$.
- $p = 3$: $81 + 9 + 1 = 91 = 7 \cdot 13$; $d_2 = 91/9$; $v_3 = -2$; $|d_2|_3 = 9$.
- $p = 5$: $625 + 25 + 1 = 651 = 3 \cdot 7 \cdot 31$; $d_2 = 651/25$; $v_5 = -2$; $|d_2|_5 = 25$.
- $p = 7$: $2401 + 49 + 1 = 2451 = 3 \cdot 19 \cdot 43$; $d_2 = 2451/49$; $v_7 = -2$; $|d_2|_7 = 49$.

Theorem (universal non-integrality). For any prime p , $p^4 + p^2 + 1 \equiv 1 \pmod{p}$, so $p \nmid (p^4 + p^2 + 1)$, hence $vp(d_2) = -2$ exactly. Therefore $d_2 \notin \mathbb{Z}_p$ for every prime: the rank-2 Jones–Wenzl projector exists over $\mathbb{Q}_p$ ($d_2 \neq 0$, so d_2 is invertible over the field) but is never integral over $\mathbb{Z}_p$. Its leading term is always p^{-2} times a p -adic unit.

4.4 Computation 3: quantum dimension $d_3 = \delta^3 - 2\delta$.

$$d_3 = \delta p(\delta_p^2 - 2) = (p + p^{-1})(p^2 + p^{-2}) = p^3 + p + p^{-1} + p^{-3} = (p^6 + p^4 + p^2 + 1)/p^3.$$

- $p = 2$: $64 + 16 + 4 + 1 = 85$; $d_3 = 85/8$; $v_2 = -3$.
- $p = 3$: $729 + 81 + 9 + 1 = 820 = 2^2 \cdot 5 \cdot 41$; $d_3 = 820/27$; $v_3 = -3$.
- $p = 5$: $15625 + 625 + 25 + 1 = 16276 = 2^2 \cdot 7^2 \cdot 83$; $d_3 = 16276/125$; $v_5 = -3$.
- $p = 7$: $117649 + 2401 + 49 + 1 = 120100 = 2^2 \cdot 5^2 \cdot 1201$; $d_3 = 120100/343$; $v_7 = -3$.

Since $p^6 + p^4 + p^2 + 1 \equiv 1 \pmod{p}$, $vp(d_3) = -3$ universally. Pattern (proved for $n = 2, 3$, conjectured beyond): $vp(d_n) = -n$.

4.5 Computation 4: the $p = 3$ test. [5] proves the p -adic Potts model on the Cayley tree has a phase transition iff $p = 3$. If the p -adic TL parameter carried physical predictive content in the Potts channel, one would expect a singularity at $p = 3$. But $v_3(d_2) = -2$ and $v_3(d_3) = -3$ are structurally identical to $p = 2, 5, 7$. Moreover $\delta_3 = 10/3$ is rational and not ± 1 , hence not a root of unity; by the criterion of [4], the bar homology of $TL_\lambda(\delta_3)$ is the generic case, as for every prime. **No $p = 3$ anomaly appears in the TL parameter.**

4.6 Computation 5: τ_r at root-of-unity values is an algebraic unit. For $q = e^{2\pi i/r}$, $\tau_r = 2\cos(2\pi/r)$:

- $\tau_3 = 2\cos(2\pi/3) = -1$: minimal polynomial $x + 1$; norm -1 ; unit; $|\tau_3|_p = 1$ for all p .
- $\tau_5 = (\sqrt{5} - 1)/2 \approx 0.618034$: satisfies $x^2 + x - 1 = 0$ (check: $((\sqrt{5}-1)/2)^2 + (\sqrt{5}-1)/2 - 1 = (6-2\sqrt{5})/4 + (2\sqrt{5}-2)/4 - 4/4 = 0 \checkmark$). Monic with constant term -1 , so $N(\tau_5) = -1$: unit, $|\tau_5|_p = 1$ for all p . Archimedean check: the conjugate is $(-\sqrt{5}-1)/2 \approx -1.618034$, and $0.618034 \times 1.618034 = 0.618034 + 0.618034^2 = 0.618034 + 0.381966 \approx 1.000000$ (to six decimals; the exact value is 1, since the conjugate of τ_5 is $-(-\tau_5)$ and $N(\tau_5) = -1$); combined with all finite norms equal to 1, the adelic product formula $\prod_v |\tau_5|_v = 1$ holds in exact arithmetic.
- $\tau_6 = 2\cos(\pi/3) = 1$: unit, all norms 1.
- $\tau_7 = 2\cos(2\pi/7) \approx 1.246980$: root of $x^3 + x^2 - 2x - 1$ (numerical check: $1.246980^3 + 1.246980^2 - 2 \cdot 1.246980 - 1 \approx 1.938 + 1.555 - 2.494 - 1 \approx 0 \checkmark$). Monic with constant term -1 , so $N = (-1)^3 \cdot (-1) = 1$: unit, $|\tau_7|_p = 1$ for all p .

- $\tau_4 = 2\cos(\pi/2) = 0$: degenerate, excluded ($|0|_p = \infty$).

Conclusion: at every generic value examined, the TL parameter itself is p-adically trivial. The naive version of the research idea ("the p-adic norm of the TL parameter predicts physics") is therefore structurally void at root-of-unity values; non-trivial p-adic data can only reside in Markov trace values [1] and in τ -dependent invariants such as the bar homology differentials of [4].

4.7 Computation 6: the adelic obstruction. Let $\varepsilon \in \mathbb{Q}$, $\varepsilon \neq 0$. The product formula states $|\varepsilon|_\infty \cdot \prod_p |\varepsilon|_p = 1$. Controlling ε to p-adic precision $|\varepsilon|_p = p^{-k}$ imposes **no bound on $|\varepsilon|_\infty$ in either direction**. Explicit counterexamples with identical p-adic size: $\varepsilon = 3^5 \cdot 10^{-6} = 243/10^6 = 0.000243$ and $\varepsilon = 3^5 \cdot 10^6 = 2.43 \times 10^8$ both have $|\varepsilon|_3 = 3^{-5}$, yet their Archimedean sizes differ by a factor of 10^{12} . Hence p-adic proximity and Archimedean proximity are logically independent: any accuracy claim must include an Archimedean control assumption; p-adic precision alone cannot deliver it. This is the central theoretical finding.

4.8 Computation 7: the accuracy budget. Target: $A \geq 0.95 \iff |\varepsilon|_\infty \leq 0.05/|\tau|_\infty$.

- $r = 5$: $|\tau_5|_\infty = 0.618034$; threshold $0.05/0.618034 = 0.0809$ (check: $0.618034 \times 0.0809 = 0.049998 \approx 0.05$ ✓).
- $r = 7$: $|\tau_7|_\infty = 1.246980$; threshold $0.05/1.246980 = 0.040097$ (check: $1.246980 \times 0.040097 = 0.050000$ ✓).

Rounding ε to n decimal digits bounds $|\varepsilon|_\infty \leq 0.5 \times 10^{-n}$. For $n = 2$: $|\varepsilon|_\infty \leq 0.005$.

- $r = 5$: guaranteed $A \geq 1 - 0.005 \times 0.618034 = 1 - 0.0030902 = \mathbf{0.9969098} \geq 0.95$ ✓.
- $r = 7$: guaranteed $A \geq 1 - 0.005 \times 1.246980 = 1 - 0.0062349 = \mathbf{0.9937651} \geq 0.95$ ✓.

Worst case over the unit family $r \in \{3, 5, 6, 7\}$ is $r = 7$, so the uniform two-decimal budget guarantees $A \geq 0.9937651$ across the family.

4.9 Computation 8: p-adic digit budget under a joint-control assumption. **Assumption J (stated explicitly):** the physical correction ε is a rational whose Archimedean size is controlled by its p-adic size, $|\varepsilon|_\infty \leq C \cdot |\varepsilon|_p^\alpha$ for some $\alpha > 0$. Taking the canonical prime $p = 3$ (motivated by the Potts theorem [5]), the most conservative case $\alpha = 1$, $C = 1$ gives $|\varepsilon|_\infty \leq 3^{-k}$. The 95% target at $r = 7$ requires $3^k \geq 1/0.040097 = 24.94$; since $3^3 = 27 \geq 24.94$ (and $3^2 = 9 < 24.94$), **$k = 3$ ternary digits** suffice. At $r = 5$: $3^k \geq 1/0.0809 = 12.36$; $3^3 = 27 \geq 12.36$ but $3^2 = 9 < 12.36$, so again $k = 3$. If instead $\alpha = 1/2$, the requirement becomes $3^{k/2} \geq 24.94$, i.e., $3^k \geq 622$, giving $k = 6$. The digit budget under Assumption J is therefore **3–6 ternary digits** for $\alpha \in [1/2, 1]$.

5. Results

All numbers below are computed in Section 4 by exact arithmetic; none are simulated or measured.

1. **δp values:** $\delta_2 = 5/2$, $\delta_3 = 10/3$, $\delta_5 = 26/5$, $\delta_7 = 50/7$, each with $v_p = -1$.
2. **Universal non-integrality (theorem):** $v_p(d_2) = -2$ for all primes p , proved via $p^4 + p^2 + 1 \equiv 1 \pmod{p}$. Instances: $d_2 = 21/4$ ($p=2$), $91/9$ ($p=3$), $651/25$ ($p=5$), $2451/49$ ($p=7$). Likewise $v_p(d_3) = -3$ universally: $d_3 = 85/8$, $820/27$, $16276/125$, $120100/343$ respectively.
3. **Unit theorem (computed):** $\tau_r = 2\cos(2\pi/r)$ is an algebraic unit with $|\tau_r|_p = 1$ for all primes p , for $r = 3, 5, 6, 7$; the adelic product $\prod_v |\tau_r|_v = 1$ in exact arithmetic (the six-decimal check gives ≈ 1.000000). $\tau_4 = 0$ is degenerate.
4. **No $p = 3$ anomaly:** $\delta_{-3} = 10/3$ is rational and not ± 1 , hence not a root of unity; by [4] the $TL_{\bar{3}}$ bar homology is generic for every prime. The Potts phase transition of [5] is invisible at the level of the TL parameter.
5. **Adelic obstruction (proved):** p-adic precision $|\varepsilon|_p = p^{-k}$ imposes no bound on $|\varepsilon|_\infty$ in either direction (counterexamples in 4.7).

6. **Accuracy budget (computed):** two decimal digits of Archimedean control guarantee $A \geq 0.9969098$ ($r = 5$) and $A \geq 0.9937651$ ($r = 7$), hence ≥ 0.9937651 uniformly over $r \in \{3, 5, 6, 7\}$.
7. **Projection (conditional on Assumption J):** under $|\varepsilon|^\infty \leq |\varepsilon|_p$ ($\alpha = 1$, $C = 1$, $p = 3$), $k = 3$ ternary digits suffice for the 95% target at $r = 5$ and $r = 7$; the budget is 3–6 digits for $\alpha \in [1/2, 1]$. **This is a projection, not a measurement:** Assumption J is an empirical hypothesis whose failure would void this result.
8. **Status of the 95% claim:** no computation in this paper empirically validates it. On valuation-determined questions, the parameter's algebraic behavior is uniform in p (same valuations, same genericity class), so a single scalar parameter cannot generate a 95%-style *partial* success rate — such statistics would be 0% or 100% on structurally determined questions. The claim is at best conditionally established (Result 7 plus Assumption J), not established.

6. Discussion

Limitations. First, the fidelity proxy $A = 1 - |\varepsilon|^\infty |\tau|^\infty$ is linear; TL observables are polynomials of degree up to n in τ , and derivatives can amplify errors by factors growing with n . The budget is trustworthy for small diagrams ($n \leq 3$, where the bar homology of [4] lives) and optimistic elsewhere. Second, the choice $\delta p = p + p^{-1}$ is *canonical but not unique*; alternatives (e.g., $\delta = q + q^{-1}$ with q a p -adic root of unity, which exists in $\mathbb{Q}_p$ only when $p \equiv 1 \pmod{\text{order}}$) would change every computation. Third, $vp(dn) = -n$ is proved for $n = 2, 3$ and conjectured beyond (induction from the recursion and the congruence structure is the natural route). Fourth, Computation 5 is a negative result: at generic root-of-unity values the parameter is p -adically invisible, so the entire predictive burden falls on Markov trace values [1], whose p -adic arithmetic is not computed here — the largest gap in the paper. Fifth, the bibliography is small (twelve works) and the p -adic/TL intersection is genuinely thin; the physical grounding rests mainly on [3] and [5]–[8], none of which uses p -adic TL parameters.

Failure modes. The non-integrality theorem concerns integrality over $\mathbb{Z}_p$, *not existence*: over the field $\mathbb{Q}_p$ the projector always exists since $d2 \neq 0$; *conflating these would be an error*. The “no $p = 3$ anomaly” conclusion could be an artifact of looking only at valuations; finer invariants — e.g., Markov trace values on $\tilde{A}^2$ per [1], evaluated p -adically — might yet detect $p = 3$.

What would falsify our claims. (i) Exhibit a prime p with $p \mid (p^4 + p^2 + 1)$: impossible by the congruence, so the non-integrality theorem is proved, not merely tested. (ii) Exhibit a physical observable, operationally defined as in [3], whose measured values across a stated system set match δ_p -derived predictions at a rate statistically distinguishable from chance: this would falsify our negative assessment of the 95% claim. (iii) Show that the Potts $p = 3$ transition of [5] propagates into TL traces, falsifying Result 4. (iv) Show that Assumption J fails empirically for TL-type systems, voiding the conditional digit budget.

Arguing against ourselves. The strongest counterargument: physical quantum systems are not indexed by primes, so asking δp to “predict behavior” may be a category error, and our critique attacks a strawman. We accept this partially — but the idea as posed makes the accuracy claim, and the burden is on it. Conversely, the non-integrality theorem is a genuinely new structural fact: any p -adic TL-based topological quantum computation scheme (in the spirit of [3]) working over $\mathbb{Z}_p$ coefficients is impossible with the canonical parameter, forcing denominators and hence error-control questions. That is a constructive, testable consequence.

Open questions. Does $vp(dn) = -n$ hold for all n ? Do Markov trace values on $\tilde{A}^2$ [1] detect $p = 3$? Can the p -adic wavelet framework of [7] carry a TL module action with δp , and does the resulting spectral data match any hierarchic quantum system? Is there a noise model in which the adelic product formula of [6] constrains physical error channels in the manner of Computation 6?

7. Conclusion

We formalized the p -adic Temperley–Lieb parameter as $\delta p = p + p^{-1}$ and proved, by explicit valuation arithmetic, that its quantum dimensions are never p -adically integral ($\text{vp}(d_2) = -2$, $\text{vp}(d_3) = -3$, universally), that it exhibits no $p = 3$ anomaly despite the Potts-model precedent [5], and that at root-of-unity values the TL parameter is an algebraic unit, p -adically trivial for every prime. We proved the adelic obstruction — p -adic precision cannot control Archimedean error — and derived an explicit accuracy budget: two decimal digits of Archimedean control guarantee accuracy ≥ 0.9937651 over the unit family, and, under the explicitly stated Assumption J, three to six ternary digits of p -adic precision suffice for the 95% target. The claimed universal $\geq 95\%$ predictive accuracy is unsupported and, on valuation-determined questions, structurally unattainable as a partial success rate. What survives is a precise, falsifiable algebraic program: p -adic TL theory with the canonical parameter forces non-integral projectors, and any future physical application must confront this constraint and supply the Archimedean control that the product formula forbids taking for granted.

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Appendix A. Divergence report

D1. Definition of the p -adic TL parameter (C10).

- Draft A: $|\delta|_p$ of a rational approximation $\delta \approx 7071/5000$, i.e., the p -adic norm of a real-valued surrogate for the loop parameter. This is rejected: the norm of an approximation depends on the arbitrary choice of approximant ($7071/5000$ has $\text{vp} = 0$ for $p \notin \{3, 7071, 5000\}$, so $|\delta|_p = 1$ for almost all p), carries no structural information, and is not invariant under re-approximation.
- Draft B: $\delta p = q + q^{-1}$ with q a p -adic root of unity. This is rejected as non-canonical: p -adic roots of unity of order m exist in $\mathbb{Q}_p$ only when $m \mid (p - 1)$, so the definition is not uniform in p , and the choice among available orders is arbitrary.
- Draft C: $\delta p = p + p^{-1} \in \mathbb{Q}_p$, with q replaced by the uniformizer. **Adopted.** This is the unique choice adapted to the p -adic norm itself (the uniformizer is the element defining the valuation), it is defined for every prime, and it makes every valuation computation uniform: $\text{vp}(\delta p) = -1$, $\text{vp}(d_n) = -n$ (proved for $n = 2, 3$).

Resolution of D1: the paper uses $\delta_p = p + p^{-1}$ throughout (Section 3.1); Drafts A and B are rejected for the reasons above, and the divergence is closed. No further items were flagged; the divergence report is complete.