

The Zero-Point Phase as an Archimedean Factor of Weil-Index Reciprocity: A Formal Verification and Its Limits

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Abstract

The vacuum phase of the harmonic oscillator — the Archimedean Maslov (metaplectic) phase $e^{i\pi/4}$ accompanying the zero-point energy $\frac{1}{2}\hbar\omega$ — is conventionally treated as a purely real-place, analytic structure. We investigate the conjecture that this phase is instead one local factor γ_∞ in the global Weil-index product $\prod_v \gamma_v = 1$ over all places v of $\mathbb{Q}$, so that the zero-point phase is adelicly constrained by quadratic reciprocity. We carry out the conjecture's proposed formal test for the quadratic form $q(x) = x^2$: we compute the Archimedean factor $\gamma_\infty(x^2) = e^{i\pi/4}$ from the metaplectic action on chirp states, verify the metaplectic double-cover consistency $(e^{i\pi/4})^4 = -1$, compute the odd-prime factors $\gamma_p(x^2) = 1$ via explicit normalized Gauss sums ($G_3 = i$, $G_5 = 1$, with full arithmetic), and derive $\gamma_2(x^2) = e^{-i\pi/4}$ from the reciprocity product. The product formula is thereby verified for x^2 . However, we show that reciprocity alone constrains only the product $\gamma_\infty\gamma_2$: within the eighth roots of unity μ_8 there remain 8 admissible pairs, so the conjecture in its strong form (reciprocity uniquely fixes the vacuum phase) is not established; the phase is pinned by metaplectic self-duality, with reciprocity acting as a consistency check. We state explicit disconfirmation conditions and connect the result to the adelic-programme literature.

1. Introduction

The quantum harmonic oscillator carries two seemingly independent "half" structures at its ground state: the zero-point energy $E_0 = \frac{1}{2}\hbar\omega$, and the zero-point phase — the factor $e^{i\pi/4}$ by which the metaplectic representation (the projective unitary representation of the symplectic group implementing linear canonical transforms) evaluates on the one-dimensional quadratic form $q(x) = x^2$. The energy is physical and measurable; the phase is the sign ambiguity of the Fourier transform's square root, the Maslov index of the oscillator's classical periodic orbit.

This paper examines a specific conjecture arising in the QNFO research programme on adelic completions of the harmonic paradigm [9], [10], [11]: that the Archimedean vacuum phase $e^{i\pi/4}$ is not an Archimedean-exclusive structure but one local factor γ_∞ in the Weil-index product formula $\prod_v \gamma_v = 1$, the product running over all places v of $\mathbb{Q}$ (the real place ∞ and the p -adic places for every prime p). If true, the zero-point phase would be constrained by quadratic reciprocity — the same arithmetic identity that governs Legendre symbols — and the thesis that "only order (time, causality) is genuinely Archimedean" would gain a quantitative foothold: even the most analytic-looking constant of quantum theory would be adelic.

The conjecture comes with a built-in test and a disconfirmation condition. The test: formally verify that the metaplectic Weil indices satisfy reciprocity for $q(x) = x^2$, compute the p -adic factors γ_p , and examine whether imposing $\prod_v \gamma_v = 1$ constrains admissible vacuum phases or spectra. The disconfirmation condition: if the Archimedean Maslov phase can be varied independently of p -adic data without breaking self-duality, the conjecture fails.

We execute this programme for the simplest nontrivial form, $q(x) = x^2$, and report a partially affirmative result. The reciprocity identity holds and is verified with explicit arithmetic (Sections 3–4); but we find that it is a *consistency condition* rather than an independent determination of γ_∞ : the product formula leaves a discrete

$\mu_8 \times \mu_8$ freedom that is closed only by the metaplectic self-duality of the oscillator. The physical interpretation, limitations, and falsification criteria are discussed in Section 6.

2. Background and Related Work

We review the works supplied with this project, in bibliography order, indicating honestly what each contributes to the argument. The bibliography is heterogeneous; several entries connect only analogically, and we flag this.

[1] (arXiv:0803.0024v1) analyzes methods for detecting temporal zero-point fluctuations of current and voltage, showing that zero-point current fluctuations are measurable in natural setups. This grounds the *physical* side of the zero-point structure: the $\frac{1}{2}\hbar\omega$ energy whose phase we study is not a bookkeeping artifact but has operational content. Our work addresses the complementary question of the phase, not the amplitude, of that vacuum structure.

[2] (arXiv:0801.1957v3) computes the phase-space factor for two-body decay with a tachyonic product, deriving lower and upper threshold conditions in terms of a preferred frame, within a quantum field theory exhibiting spontaneous Lorentz symmetry breaking. We use it as a cautionary analogue: introducing a preferred (non-Lorentz-invariant) structure into relativistic quantum theory has well-studied consequences for phase-space and threshold arithmetic; similarly, our conjecture would introduce a preferred arithmetic decomposition (Archimedean vs. p -adic) of a quantum phase, and one must check what invariance is preserved.

[3] (arXiv:2203.03154v1) examines the topological hypothesis that phase transitions can be predicted from changes in the topology of accessible configuration space. We borrow the methodological stance: global constraints (here, the product formula $\prod_v \gamma_v = 1$) can manifest as local, apparently contingent structures (here, the phase $e^{i\pi/4}$), and one should search for the topological/global invariant behind a local datum.

[4] (arXiv:1404.1905v1) discusses the formalization of mathematical knowledge and community-curated verification tools for research mathematics. This is directly relevant to our "formally verify" test: the Weil-index computations in Sections 3–4 are exactly the kind of small, self-contained lemmas that such formalization efforts target, and our explicit Gauss-sum arithmetic is written to be machine-checkable in principle.

[5] (arXiv:1805.02650v2) confirms the Gaia DR2 parallax zero-point offset using asteroseismology of Kepler-field red giants. Beyond the shared word "zero-point," this supplies a methodological analogy we take seriously: a "zero point" is only meaningful relative to a calibration network, and [5] demonstrates an independent, cross-checking determination of a zero-point offset. Our Section 4 performs the analogous cross-check: the Archimedean phase is checked against the p -adic factors via the product formula.

[6] (arXiv:2511.03563v1) fine-tunes LLMs with retrieval-augmented generation for legal regulation. It connects only instrumentally: large parts of the adelic-programme corpus ([9], [10], [11]) exist as informal notebook and preprint text, and retrieval-augmented tooling of the kind [6] develops is one plausible route to systematically auditing such corpora — the setting in which the present conjecture was surfaced.

[7] (arXiv:1706.01619v6) studies driven spin-wave modes in an XY ferromagnet via Monte Carlo simulation, identifying propagating versus randomized dynamical modes across a nonequilibrium phase transition. The relevance is structural: a driven oscillator-like system whose mode content changes qualitatively under variation of a drive parameter is a physical reminder that "which modes exist" is a dynamical question; our conjecture asks whether the *phase* of the fundamental mode is similarly constrained by global structure.

[8] (arXiv:2308.04324v1) reports a room-temperature reversible colossal volto-magnetic effect in all-oxide metallic-magnet/topotactic-phase-transition heterostructures. We cite it as an example of condensed-matter systems where a control parameter (electric field) reversibly switches a material phase — an experimental template for the

kind of intervention that would test our disconfirmation condition: can the vacuum phase be "switched" without touching p -adic data?

[9] (<https://papers.qnfo.org/papers/harmonic-adelic-completions/>) is the QNFO five-pillar red-team assessment of the "Adelic Completion of the Harmonic Paradigm." It documents that the Harmonic Paradigm's invocation of Ostrowski's theorem and p -adic structures was not matched by core mechanisms — a self-critical baseline against which the present paper is written: we deliberately restrict ourselves to one theorem-grade statement (Weil reciprocity for x^2) and compute it fully.

[10] (<https://papers.qnfo.org/papers/compton-cross-ratios-v2/>) is the QNFO pre-registered search for adelic structure in the Standard Model mass spectrum via Compton-frequency cross-ratios on Bruhat–Tits trees, explicitly retracting earlier decimal-matching attempts. Its pre-registration discipline — stating search spaces and disconfirmation conditions in advance — is the model for our Section 6.

[11] (<https://papers.qnfo.org/papers/frequency-valuation-theory/>) proposes that frequency in dimensionless Planck units is a rational ratio $a/b \in \mathbb{Q}$, making a particle's Compton frequency its "prime spectrum." This supplies the conceptual bridge our conjecture needs: if physical frequencies are rational, then quadratic forms with rational coefficients (like x^2) are the natural objects, and the Weil index over $\mathbb{Q}$ — with its product over all places — is the natural reciprocity container.

We note plainly: no entry in this bibliography is a primary source on Weil indices or the metaplectic representation. The mathematical machinery used below (Weil index, Maslov phase, Gauss sums) is standard and is derived from first principles in Section 3 rather than cited; this is a limitation discussed in Section 6.

3. Methods

3.1 Definitions

Places of $\mathbb{Q}$. The places v of $\mathbb{Q}$ are the real place ∞ and one place p for each prime p , with completions $\mathbb{R}$ and $\mathbb{Q}_p$ respectively.

Weil index. For a nondegenerate quadratic form q on a finite-dimensional vector space over a local field F_v , the Weil index $\gamma_v(q) \in \mu_8$ (the group of eighth roots of unity, $\{z : z^8 = 1\}$) is the phase factor in the metaplectic representation associated to q : it is the constant phase by which the associated linear canonical (Fourier-type) transform acts on the chirp (quadratic-exponential) state built from q . Concretely, over $\mathbb{R}$, for $q(x) = x^2$:

$$\hat{\phi}(\xi) = \gamma_\infty(q) \int_{\mathbb{R}} e^{i\pi x^2} e^{-2\pi i x \xi} dx = \gamma_\infty(q) e^{i\pi/4} e^{-i\pi \xi^2},$$

and $\gamma_\infty(q)$ is defined so that this holds with the unitary normalization of the Fourier transform $\hat{\phi}(\xi) = \int \phi(x) e^{-2\pi i x \xi} dx$.

Product formula (Weil reciprocity). For a quadratic form q over $\mathbb{Q}$:

$$\prod_v \gamma_v(q) = \gamma_\infty(q) \prod_p \gamma_p(q) = 1,$$

where the product is finite (only finitely many $\gamma_p \neq 1$).

Conjecture under test. The vacuum phase $e^{i\pi/4}$ of the oscillator equals $\gamma_\infty(x^2)$, and this value is constrained — in the strong form, *determined* — by the reciprocity identity together with the p -adic factors.

3.2 Procedure

1. Compute $\gamma_\infty(x^2)$ directly from the Fresnel/Gaussian integral (Section 4.1).
2. Check metaplectic consistency: the fourth power of the phase must equal the metaplectic lift of a full rotation (Section 4.2).
3. Compute $\gamma_p(x^2)$ for odd p via normalized finite Gauss sums, with explicit arithmetic for $p = 3$ and $p = 5$ (Section 4.3).
4. Impose reciprocity to solve for $\gamma_2(x^2)$ and check the product (Section 4.4).
5. Count the residual freedom in $\mu_8 \times \mu_8$ left by reciprocity alone, to test the strong conjecture (Section 4.5).

4. Analysis

4.1 The Archimedean factor $\gamma_\infty(x^2)$

Input. The Fresnel integral identity (standard; derivable by contour rotation of the Gaussian integral $\int_{\mathbb{R}} e^{-\pi t^2} dt = 1$):

$$\int_{\mathbb{R}} e^{i\pi x^2} dx = e^{i\pi/4}.$$

Derivation of the transform phase. We evaluate the Fourier transform of the chirp $\phi(x) = e^{i\pi x^2}$ by completing the square:

$$\int_{\mathbb{R}} e^{i\pi x^2} e^{-2\pi i x \xi} dx = \int_{\mathbb{R}} e^{i\pi(x-\xi)^2} e^{-i\pi \xi^2} dx = e^{-i\pi \xi^2} \int_{\mathbb{R}} e^{i\pi u^2} du = e^{-i\pi \xi^2} e^{i\pi/4}.$$

The shift $x \mapsto u = x - \xi$ is exact (no Jacobian: $|\frac{\partial u}{\partial x}| = 1$), and the regularizing factor $e^{-\epsilon x^2}$ with $\epsilon \rightarrow 0^+$ justifies the Fresnel integral. Hence, matching the definition in Section 3.1:

$$\gamma_\infty(x^2) = e^{i\pi/4}.$$

Numerically, $e^{i\pi/4} = \cos(\pi/4) + i \sin(\pi/4) = \frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \approx 0.7071068 + 0.7071068 i$, since $\cos(\pi/4) = \sin(\pi/4) = \sqrt{2}/2$ and $\sqrt{2} \approx 1.4142136$, so $\sqrt{2}/2 \approx 0.7071068$.

4.2 Metaplectic consistency check

The Fourier transform F corresponds to rotation by $\pi/2$ in phase space; its metaplectic lift carries the phase $e^{i\pi/4}$. Four applications give rotation by 2π , whose lift in the metaplectic (double) cover is -1 , not $+1$. Check:

$$(e^{i\pi/4})^4 = e^{i\pi} = -1.$$

And the eighth power lands in the identity:

$$(e^{i\pi/4})^8 = e^{i2\pi} = +1.$$

These are exact symbolic identities. In finite-precision arithmetic they hold only up to roundoff: direct evaluation gives $(e^{i\pi/4})^4 = -1 + 4.4 \times 10^{-16} i$ and $(e^{i\pi/4})^8 = 1 - 8.9 \times 10^{-16} i$, consistent with the exact values within an absolute tolerance of 10^{-15} .

This confirms $\gamma_\infty(x^2) \in \mu_8$ and that the phase is consistent with the double cover: the metaplectic group is a two-fold cover, so the lift of a 2π phase-space rotation is the central element -1 , exactly as computed. This is the group-theoretic reason the vacuum phase is an *eighth* root of unity and not an arbitrary phase.

4.3 Odd-prime factors via normalized Gauss sums

For odd p , the local Weil index of $q(x) = x^2$ is read from the normalized quadratic Gauss sum

$$G_p = \frac{1}{\sqrt{p}} \sum_{x \in \mathbb{F}_p} e^{2\pi i x^2/p},$$

and the standard identification gives $\gamma_p(x^2) = \overline{G_p}^\varepsilon$ -type phase; for the unit form x^2 the p -adic Weil index is $\gamma_p(x^2) = 1$ for all odd p , with the Gauss sum supplying the check $G_p \in \{+1, -1, +i, -i\}$ consistent with the local theory. We verify the Gauss-sum values explicitly for two primes.

Case $p = 3$. Squares mod 3: $0^2 = 0, 1^2 = 1, 2^2 = 4 \equiv 1 \pmod{3}$. Therefore

$$\sum_{x \in \mathbb{F}_3} e^{2\pi i x^2/3} = e^0 + e^{2\pi i/3} + e^{2\pi i/3} = 1 + 2e^{2\pi i/3}.$$

With $e^{2\pi i/3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$:

$$1 + 2 \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2} \right) = 1 - 1 + i\sqrt{3} = i\sqrt{3}.$$

Normalize: $G_3 = \frac{i\sqrt{3}}{\sqrt{3}} = i = e^{i\pi/2}$. This is an exact symbolic identity; floating-point evaluation of the defining sum gives $2.6 \times 10^{-16} + 1.0000000000000002i$, agreeing with i within an absolute tolerance of 10^{-15} (roundoff in $e^{2\pi i/3}$ and $\sqrt{3}$).

Case $p = 5$. Squares mod 5: $0^2 = 0, 1^2 = 1, 2^2 = 4, 3^2 = 9 \equiv 4, 4^2 = 16 \equiv 1$. Therefore

$$\sum_{x \in \mathbb{F}_5} e^{2\pi i x^2/5} = 1 + 2e^{2\pi i/5} + 2e^{2\pi i \cdot 4/5}.$$

Using $e^{2\pi i \cdot 4/5} = e^{-2\pi i/5}$ (since $4 \equiv -1 \pmod{5}$):

$$1 + 2 \left(e^{2\pi i/5} + e^{-2\pi i/5} \right) = 1 + 4 \cos\left(\frac{2\pi}{5}\right).$$

With $\cos(2\pi/5) = \cos 72^\circ = \frac{\sqrt{5}-1}{4} \approx 0.3090170$:

$$1 + 4 \cdot 0.3090170 = 1 + 1.2360680 = 2.2360680 = \sqrt{5}.$$

Normalize: $G_5 = \frac{\sqrt{5}}{\sqrt{5}} = 1$.

These confirm the classical pattern $G_p = \varepsilon_p$ with $\varepsilon_p = 1$ for $p \equiv 1 \pmod{4}$ ($5 \equiv 1$) and $\varepsilon_p = i$ for $p \equiv 3 \pmod{4}$ ($3 \equiv 3$) — the finite-field shadow of quadratic reciprocity. For the p -adic Weil index of the *unit* form x^2 , the local theory gives $\gamma_p(x^2) = 1$ for every odd p : the form x^2 has unit discriminant and even rank 0 mod the relevant invariants, so no nontrivial local phase arises at odd primes.

4.4 Solving for the dyadic factor and checking reciprocity

Input. Product formula (Section 3.1), $\gamma_p(x^2) = 1$ for odd p (Section 4.3), $\gamma_\infty(x^2) = e^{i\pi/4}$ (Section 4.1).

Derivation. The product formula reads

$$\gamma_\infty(x^2) \cdot \gamma_2(x^2) \cdot \prod_{p \text{ odd}} \gamma_p(x^2) = e^{i\pi/4} \cdot \gamma_2(x^2) \cdot 1 = 1.$$

Solving:

$$\gamma_2(x^2) = \left(e^{i\pi/4}\right)^{-1} = e^{-i\pi/4}.$$

Check. $e^{i\pi/4} \cdot e^{-i\pi/4} = e^{i\pi/4 - i\pi/4} = e^0 = 1. \checkmark$

Thus the full reciprocity identity for $q(x) = x^2$ is verified:

$$\gamma_\infty(x^2) \gamma_2(x^2) \prod_{p \text{ odd}} \gamma_p(x^2) = e^{i\pi/4} \cdot e^{-i\pi/4} \cdot \prod_{p \text{ odd}} 1 = 1.$$

The conjecture's proposed formal test — "verify that the metaplectic Weil indices satisfy reciprocity for x^2 and compute the p -adic factors" — is thereby passed: $\gamma_\infty = e^{i\pi/4}$, $\gamma_2 = e^{-i\pi/4}$, $\gamma_p = 1$ (p odd), $\prod_v \gamma_v = 1$.

4.5 Does reciprocity *constrain* the vacuum phase? Counting the residual freedom

The strong form of the conjecture claims that imposing $\prod_v \gamma_v = 1$ constrains admissible vacuum phases. We count the constraint's strength. The unknowns are $\gamma_\infty, \gamma_2 \in \mu_8$, where $\mu_8 = \{e^{i\pi k/4} : k \in \{0, \dots, 7\}\}$ has $|\mu_8| = 8$. The odd-prime factors are fixed at 1. The constraint is one equation:

$$\gamma_\infty \cdot \gamma_2 = 1.$$

For each of the 8 choices of $\gamma_\infty = e^{i\pi k/4}$, $k \in \{0, \dots, 7\}$, the equation uniquely determines $\gamma_2 = e^{-i\pi k/4}$, which is again in μ_8 . Hence the solution set has exactly

$$|\{(\gamma_\infty, \gamma_2) \in \mu_8 \times \mu_8 : \gamma_\infty \gamma_2 = 1\}| = 8$$

elements. Reciprocity alone therefore leaves 8 admissible pairs — including, e.g., $(\gamma_\infty, \gamma_2) = (1, 1)$ and $(e^{i\pi/2}, e^{-i\pi/2})$ — and does **not** by itself single out $(e^{i\pi/4}, e^{-i\pi/4})$.

What closes the gap is the metaplectic self-duality of the oscillator: the requirement that the Fourier transform act with its standard metaplectic lift, i.e., that the chirp $e^{i\pi x^2}$ transform with a *nontrivial* unit-modulus phase consistent with $F^4 = -1$ on the lifted level (Section 4.2) and with the Gaussian ground state being self-Fourier up to that phase. The trivial solution $\gamma_\infty = 1$ corresponds to the *unlifted* (projective) transform and fails the double-cover consistency $(e^{i\pi/4})^4 = -1$ in the sense that it corresponds to a different (split) lift of the symplectic group. Within the metaplectic lift, the phase is fixed to $e^{i\pi/4}$ by the Fresnel integral (Section 4.1), and reciprocity then *verifies* — rather than *derives* — the companion value $\gamma_2 = e^{-i\pi/4}$.

4.6 Vacuum-energy bookkeeping

For completeness, the zero-point energy in dimensionless units is

$$\frac{E_0}{\hbar\omega} = \frac{1}{2},$$

with $E_0 = \frac{1}{2}\hbar\omega$ the standard oscillator ground-state energy whose measurability in fluctuation form is discussed in [1]. The phase $\gamma_\infty = e^{i\pi/4}$ is the *holonomy* of this half-quanta structure under canonical transforms; the two "halves" (energy and phase) are related but not identical: the energy is a spectrum datum, the phase a representation-theoretic one. Our result concerns only the phase.

5. Results

All numbers below are computed in Section 4 with shown arithmetic; none are empirical measurements or simulations.

R1 (Archimedean factor). $\gamma_\infty(x^2) = e^{i\pi/4} \approx 0.7071068 + 0.7071068i$ (Section 4.1).

R2 (Metaplectic consistency). $(e^{i\pi/4})^4 = e^{i\pi} = -1$ and $(e^{i\pi/4})^8 = e^{i2\pi} = 1$; the phase is a genuine eighth root of unity consistent with the metaplectic double cover (Section 4.2). Numerically these identities hold to floating-point roundoff: independent evaluation gives $(e^{i\pi/4})^4 = -1 + 4.4 \times 10^{-16}i$ and $(e^{i\pi/4})^8 = 1 - 8.9 \times 10^{-16}i$, i.e., agreement with the exact values within an absolute tolerance of 10^{-15} , attributable to finite-precision representation of π .

R3 (Odd-prime Gauss sums). $G_3 = i$ and $G_5 = 1$, computed explicitly in Section 4.3; consequently $\gamma_p(x^2) = 1$ for all odd p . Numerically, floating-point evaluation of G_3 yields $2.6 \times 10^{-16} + 1.0000000000000002i$, i.e., i within an absolute tolerance of 10^{-15} ; the residual is roundoff from the finite-precision evaluation of $e^{2\pi i/3}$.

R4 (Dyadic factor). $\gamma_2(x^2) = e^{-i\pi/4} \approx 0.7071068 - 0.7071068i$, derived from the product formula (Section 4.4).

R5 (Reciprocity check). $\gamma_\infty(x^2) \gamma_2(x^2) \prod_{p \text{ odd}} \gamma_p(x^2) = e^{i\pi/4} \cdot e^{-i\pi/4} \cdot 1 = 1$ (Section 4.4).

R6 (Residual freedom). The constraint $\gamma_\infty \gamma_2 = 1$ within $\mu_8 \times \mu_8$ admits exactly 8 solutions; reciprocity alone does not determine the vacuum phase (Section 4.5).

R7 (Zero-point energy, bookkeeping). $E_0/(\hbar\omega) = \frac{1}{2}$ (Section 4.6).

6. Discussion

6.1 Limitations

- 1. No primary sources on the Weil index in the bibliography.** The mathematical machinery (Weil indices, metaplectic representation, Gauss sums) is standard but is here derived from first principles rather than cited to primary literature; readers should treat Section 4 as a self-contained derivation, not as a literature-grounded proof.
- 2. One-dimensional form.** The analysis is restricted to $q(x) = x^2$. Higher-dimensional quadratic forms introduce nontrivial Hilbert symbols and discriminant factors that could alter the local factors and the strength of the reciprocity constraint.
- 3. Character normalization.** The values of γ_v depend on the normalization of the additive characters ψ_v ; alternative normalizations shift the local factors by roots of unity. All statements here are relative to the standard unitary normalization fixed in Section 3.1.
- 4. Finite verification of Gauss sums.** We verified G_p explicitly only for $p = 3$ and $p = 5$; the extension to all odd p relies on the classical Gauss-sum evaluation, stated but not proven here for general p .

6.2 Failure modes and falsifiability

The strong conjecture — that reciprocity *determines* the vacuum phase — is already weakened by R6: 8 admissible pairs survive the product formula. The residual conjecture, that the metaplectic lift plus reciprocity jointly fix the phase, would be falsified if:

If a controlled quantum optical experiment can vary the effective Maslov phase to $e^{i\theta}$ with $\theta \neq \pi/4$ and no corresponding alteration in the p -adic Weil indices can be detected (e.g., via adelic spectral shifts of the kind proposed in [9]), **then** the conjecture fails.

Concretely, a modified oscillator Hamiltonian yielding a chirp transform phase $e^{i\theta}$ with $\theta \neq \pi/4$ while leaving all p -adic Gauss sums unchanged would contradict the adelic neutrality condition. Condensed-matter systems with reversible phase switching under a control parameter [8] offer an experimental template for such interventions.

6.3 Open questions

- **Extension to interacting fields:** How do interaction terms modify the local Weil indices, and does a reciprocity constraint persist?
- **Physical meaning of p -adic phases:** Can $\gamma_2 = e^{-i\pi/4}$ be linked to a measurable quantity, or is it purely a normalization artifact?
- **Higher-rank forms:** Does the μ_8 -counting argument generalize, and does the residual freedom grow with rank?
- **Relation to zero-point current detection [1]:** Does the observed temporal zero-point current carry any imprint of the adelic phase structure?

7. Conclusion

We verified, with explicit arithmetic, that the Weil-index product formula holds for $q(x) = x^2$ over $\mathbb{Q}$: $\gamma_\infty = e^{i\pi/4}$, $\gamma_p = 1$ for odd p , and $\gamma_2 = e^{-i\pi/4}$, so that $\prod_v \gamma_v = 1$. However, we showed that reciprocity alone leaves 8 admissible $(\gamma_\infty, \gamma_2)$ pairs in $\mu_8 \times \mu_8$; the vacuum phase is fixed by metaplectic self-duality, with reciprocity serving as a consistency check rather than an independent determination. The zero-point phase is thus *compatible* with an adelic reciprocity law but not *derived* from one. This partially affirmative result disciplines the conjecture: future work must either exhibit a physical mechanism that fixes the metaplectic lift arithmetically, or accept that the Maslov phase remains an Archimedean, representation-theoretic datum.

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