

Thermodynamic Bottlenecks on Floquet Driving of Superconducting Matter: Quasiparticle Poisoning, Heat Extraction, and Design Rules

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Abstract

Floquet engineering—periodic modulation of a Hamiltonian to synthesize effective dynamics—is attractive for superconducting circuits and materials, but a driven superconductor is an open system: every absorbed drive photon must be dissipated into a cryogenic bath, and above-gap absorption breaks Cooper pairs. We formalize this objection as a quantitative bottleneck analysis built on a minimal quasiparticle-rate and heat-balance model, with every numerical input stated. For aluminum circuits ($T_c = 1.2$ K, $\Delta_0 = 1.76 k_B T_c$), the pair-breaking threshold is $f_{pb} = 2\Delta_0/h \approx 88$ GHz. A tolerated quasiparticle-poisoning error of 10^{-3} per 100 ns gate limits absorbed power in a qubit junction to $P_{abs}^{max} \approx 5.8 \times 10^{-19}$ W, about fourteen orders of magnitude below a typical mixing-chamber cooling power, so quasiparticle poisoning, not refrigeration, binds qubit-scale Floquet drive. For cavity-scale resonators the hierarchy reverses: one picowatt of absorbed drive yields a steady-state quasiparticle density $\approx 1.0 \times 10^{11} \text{ m}^{-3}$, exceeding the thermal density at 100 mK by a factor $\approx 9 \times 10^{31}$. A complementary field-scaling analysis gives a required drive field $E_{req} \approx 4.6 \times 10^5 \text{ V m}^{-1}$ versus an admissible $E_{max} \approx 14.1 \text{ V m}^{-1}$, heating the electronic subsystem to ≈ 1.4 K. We delimit a sub-gap Floquet window and state falsifiable design rules.

1. Introduction

Floquet driving—the application of a time-periodic modulation $H(t) = H_0 + H_1 \cos(\omega_{dr} t)$ —allows the synthesis of effective Hamiltonians, topological bands, and parametric gates inaccessible to static systems. In superconducting hardware, Floquet and parametric techniques underlie tunable couplers, driven qubit gates, and engineered resonator lattices; on the materials side, periodic drives are proposed as routes to light-induced or topologically nontrivial superconducting states [1], [3]. The hardware, however, is not a closed Floquet system: it is a condensate coupled to a cryogenic bath with a finite superconducting gap Δ_0 and a finite cooling power P_{cool} .

This paper asks a specific question, motivated by the QNFO program on thermodynamic constraints in superconducting matter and scalable quantum technologies [9], [10], [11]: **do thermodynamic bottlenecks constrain the usable strength of Floquet driving in superconducting devices, and if so, which bottleneck binds first?** Three candidate bottlenecks are considered: (i) quasiparticle generation by above-gap or multiphoton absorption, which poisons coherent dynamics; (ii) finite heat-extraction capacity at the millikelvin or helium-temperature stage; and (iii) entropy production per drive cycle, which must remain small for quasi-adiabatic behavior.

We answer with a minimal analytic model and fully explicit arithmetic (Section 4). The main findings are a hierarchy of constraints: for qubit-scale junctions, quasiparticle poisoning limits absorbed drive power to $\sim 10^{-19}$ W, far below any refrigeration limit; for cavity-scale resonators, cooling-power and quasiparticle-density constraints bind at much larger powers but still enforce strict amplitude limits; and for field-driven materials, the drive amplitude needed for materials-scale quasi-energy renormalization exceeds the thermally admissible amplitude by many orders of magnitude. A corollary design rule is that Floquet frequencies must remain below the pair-breaking

threshold $2\Delta_0/h$ (about 88 GHz for aluminum) unless pair breaking is explicitly budgeted. Throughout, we ground the model in the experimental phenomenology of superconducting microwave devices [2], [6], [7], [8] and in pairing and band-structure theory [1], [3], [4], [5].

2. Background and Related Work

Pairing and the condensate gap. Reference [1] argues that superconductivity in unconventional materials emerges from pairing of the coherent parts of the physical electrons, yielding BCS-like condensation regardless of the mother normal state. This matters here because the quantities controlling Floquet heating—the gap Δ_0 and the quasiparticle density of states N_0 —are exactly what such pairing theories compute; if the gap is material- and state-dependent, the bottleneck is material-dependent. Reference [4] applies a dielectric-function method to SrTiO₃ with non-parabolic conduction bands and first-principles phonon dispersion, reproducing experimental critical temperatures; non-parabolicity renormalizes N_0 entering the thermal quasiparticle formula, exemplifying why our numerical prefactors are platform-specific. Reference [3] analyzes Lifshitz transitions—topological changes of the Fermi surface—in multi-band Hubbard models as a route to topological superconductivity in complex quantum matter; a Lifshitz transition changes the number of bands at the Fermi level and can abruptly increase N_0 and soften the gap, tightening our bottleneck discontinuously. Reference [5] maps the ground-state phase diagram of population-imbalanced fermionic mixtures in quasi-one-dimensional optical lattices via an attractive Hubbard model with a Zeeman term; imbalance is a controlled way to reduce the pair-breaking threshold $2\Delta_0$, and the resulting gapless superconducting phases would show dramatically enhanced susceptibility to Floquet pair breaking.

Microwave device phenomenology. Reference [2] measured second- and third-order intermodulation distortion (IMD) of a YBa₂Cu₃O₇ thin-film resonator under three input tones, enabling spatial mapping of the nonlinear response, and observed that second- and third-order IMD relaxed in markedly different ways after removal of a static magnet. This is direct evidence that driven superconducting microwave devices harbor long-lived nonequilibrium excitations—precisely the quasiparticle population our rate equation tracks—with slow, order-dependent relaxation, supporting quasiparticle lifetimes τ_{qp} of order 10^{-4} s or longer. Reference [7] exploits the parabolic dc-voltage dip of a Superconducting Quantum Interference Filter (SQIF) around $B = 0$ to mix weak rf signals, detecting the difference frequency $f_0 = f_1 - f_2$ for tones from a few MHz to 20 GHz; two-tone mixing is the nonlinear mechanism by which sub-gap drive tones can synthesize above-gap frequencies, so [7] provides the device-level precedent for the multiphoton channel we flag as the dominant residual heating path. Reference [6] reports copper waveguide cavities with reduced surface loss coupled to transmon qubits, with coherence times approaching 0.1 ms; the design lesson—that the loss budget, not field strength, sets the usable drive—is the experimental mirror of our thermodynamic bound, and the 0.1 ms coherence scale fixes the gate time τ_g in our poisoning budget. Reference [8] demonstrates high-temperature superconducting rf coils for NMR and MRI, where low rf losses improve signal-to-noise; it also documents that superconductors at rf frequencies dissipate in proportion to drive level, the same absorbed-ac-power physics we analyze at different frequencies and tolerable dissipation scales.

Programmatic context. Reference [9] (Superconductivity Quadrangle) frames superconductivity research as an interlock of material, thermodynamic, and coherence constraints and is the source of the research question examined here. Reference [10] develops structural parallels between photosynthetic energy transduction and ambient superconductivity as driven, open, coherent systems governed by drive–dissipation balance; our per-cycle entropy calculation is the millikelvin analogue of the per-excitation entropy budgets considered there. Reference [11] develops thermodynamic and quantum-information-theoretic constraints on scalable quantum computing; our result that the binding constraint for qubit Floquet drive is quasiparticle poisoning at $\sim 10^{-19}$ W—rather than the $\sim 10^{-4}$ W cooling budget—is a concrete, computed instance of that thesis.

3. Methods

3.1 Primary model: quasiparticle rate and heat balance

We consider a superconducting condensate with gap Δ_0 driven at frequency $f_{\text{dr}} = \omega_{\text{dr}}/2\pi$, of which a fraction η_{abs} of the input power is absorbed: $P_{\text{abs}} = \eta_{\text{abs}} P_{\text{in}}$. Above-gap absorption breaks Cooper pairs; each event costs $2\Delta_0$ and produces two quasiparticles, of which we conservatively count one effective long-lived quasiparticle per event (the partner relaxes to the gap edge rapidly). The pair-breaking generation rate is

$$\Gamma_{\text{qp}} = \frac{P_{\text{abs}}}{2\Delta_0},$$

and the steady-state quasiparticle density in a device of volume V obeys

$$\frac{dn_{\text{qp}}}{dt} = \frac{\Gamma_{\text{qp}}}{V} - \frac{n_{\text{qp}}}{\tau_{\text{qp}}} \implies n_{\text{qp}}^{\text{ss}} = \frac{\Gamma_{\text{qp}} \tau_{\text{qp}}}{V},$$

with recombination-limited lifetime τ_{qp} . The thermal baseline for an s-wave superconductor is

$$n_{\text{qp}}^{\text{th}}(T) = 2N_0 \sqrt{2\pi\Delta_0 k_B T} e^{-\Delta_0/(k_B T)},$$

with N_0 the single-spin normal-state density of states. The per-gate poisoning error is taken as $\epsilon_{\text{qp}} = \Gamma_{\text{qp}} \tau_g$, valid in the dilute regime. Heat balance requires $P_{\text{abs}} \leq P_{\text{cool}}$, and the entropy per cycle of duration t_{cyc} is

$$\Delta S_{\text{cyc}} = \frac{P_{\text{abs}} t_{\text{cyc}}}{T_{\text{bath}}}.$$

3.2 Complementary models retained as case studies

Two complementary calculations are retained from the source drafts and documented in Appendix A. (i) A driven-resonator case study: a single harmonic mode with Lindblad loss at rate $\kappa = \omega/Q$, stored energy U , dissipated power $P = \omega U/Q$, evaluated at a 4 K liquid-helium bath. (ii) A field-scaling analysis for materials: ohmic absorption $P_{\text{abs}} = \frac{1}{2}\sigma E^2 A$ balanced against electron-phonon cooling $P_{\text{cool}} = \Sigma V_f (T_e^5 - T_{ph}^5)$, with a Floquet gap estimate $\Delta_F \sim eEa$ (the electrostatic potential across one lattice period a).

3.3 Stated inputs and status

- $k_B = 1.380649 \times 10^{-23}$ J/K, $h = 6.62607015 \times 10^{-34}$ J·s, $e = 1.602176634 \times 10^{-19}$ C (SI exact).
- $T_c^{\text{Al}} = 1.2$ K; $\Delta_0 = 1.76 k_B T_c$ (BCS weak-coupling ratio; assumptions).
- $N_0 = 1.72 \times 10^{10} \text{ m}^{-3} \text{J}^{-1}$ (Al single-spin DOS; assumption).
- $\tau_{\text{qp}} = 3 \times 10^{-4}$ s (consistent with the slow relaxation phenomenology of [2]; assumption).
- $\tau_g = 1 \times 10^{-7}$ s (consistent with the ~ 0.1 ms coherence scale of [6]; assumption).
- $T_{\text{bath}} = 0.1$ K; $P_{\text{cool}} = 1.0 \times 10^{-4}$ W (typical commercial dilution-refrigerator specification; assumption; see Appendix A on the divergent choice 1×10^{-5} W).
- $V_c = 5 \times 10^{-5} \text{ m}^3$ (3D cavity volume, cf. [6]; assumption).
- Tolerated poisoning error $\epsilon_{\text{qp}}^{\text{tol}} = 10^{-3}$ (assumption).
- Illustrative absorbed power $P_{\text{abs}} = 1 \times 10^{-12}$ W (one picowatt; illustrative).
- Resonator case study: $f = 10$ GHz, $Q = 2 \times 10^4$ (loaded Q measured in [2]), $C = 1$ pF, $V = 1$ V, $T = 4$ K, $C_{\text{th}} = 1 \times 10^{-9}$ J/K (assumptions).

- Field-scaling case study: $P_{\text{DR}} = 1.0 \times 10^{-4}$ W, allocable fraction $\eta = 10^{-2}$, $A = 1.0 \times 10^{-4}$ m², $\sigma = 1.0 \times 10^{-4}$ S m⁻¹, $a = 4.0 \times 10^{-10}$ m, $T_{ph} = 0.1$ K, $\Sigma = 2.0 \times 10^9$ W m⁻³ K⁻⁵, $V_f = 1.0 \times 10^{-16}$ m³ (assumptions).

4. Analysis

4.1 The gap and the pair-breaking threshold

$$\Delta_0 = 1.76 k_B T_c = 1.76 \times (1.380649 \times 10^{-23} \text{ J/K}) \times 1.2 \text{ K}.$$

$$\begin{array}{ll} \text{Step 1:} & 1.380649 \times 10^{-23} \times 1.2 = 1.6567788 \times 10^{-23} \text{ J.} \\ & 1.6567788 \times 10^{-23} \times 1.76 = 2.91593 \times 10^{-23} \text{ J.} \end{array} \quad \text{Step 2:}$$

So $\Delta_0 = 2.91593 \times 10^{-23}$ J = 1.820×10^{-4} eV. This value yields the pair-breaking frequency

$$f_{\text{pb}} = \frac{2\Delta_0}{h} = \frac{2 \times 2.91593 \times 10^{-23} \text{ J}}{6.62607015 \times 10^{-34} \text{ J s}} \approx 8.8 \times 10^{10} \text{ Hz} \approx 88 \text{ GHz}.$$

4.2 Poisoning-limited absorbed power

The tolerated per-gate quasiparticle error is $\epsilon_{\text{qp}}^{\text{tol}} = 10^{-3}$ and the gate time is $\tau_g = 1 \times 10^{-7}$ s. The maximum allowed quasiparticle generation rate is therefore

$$\Gamma_{\text{qp}}^{\text{max}} = \frac{\epsilon_{\text{qp}}^{\text{tol}}}{\tau_g} = \frac{10^{-3}}{10^{-7}} = 10^4 \text{ s}^{-1}.$$

The corresponding absorbed power limit follows from $\Gamma_{\text{qp}} = P_{\text{abs}}/(2\Delta_0)$:

$$P_{\text{abs}}^{\text{max}} = \Gamma_{\text{qp}}^{\text{max}} 2\Delta_0 = 10^4 \times 2 \times 2.91593 \times 10^{-23} \text{ J s}^{-1} \approx 5.8 \times 10^{-19} \text{ W}.$$

This is fourteen orders of magnitude below the typical dilution-refrigerator cooling power $P_{\text{cool}} = 10^{-4}$ W, confirming that quasiparticle poisoning, not refrigeration, is the binding constraint for qubit-scale Floquet drive.

4.3 Cavity-scale quasiparticle density

For the illustrative absorbed power $P_{\text{abs}} = 1 \times 10^{-12}$ W in a cavity of volume $V_c = 5 \times 10^{-5}$ m³ and quasiparticle lifetime $\tau_{\text{qp}} = 3 \times 10^{-4}$ s, the generation rate is

$$\Gamma_{\text{qp}} = \frac{P_{\text{abs}}}{2\Delta_0} = \frac{10^{-12}}{5.83186 \times 10^{-23}} \approx 1.72 \times 10^{10} \text{ s}^{-1}.$$

The steady-state density follows from $n_{\text{qp}}^{\text{ss}} = \Gamma_{\text{qp}} \tau_{\text{qp}} / V_c$:

$$n_{\text{qp}}^{\text{ss}} = \frac{1.72 \times 10^{10} \times 3 \times 10^{-4}}{5 \times 10^{-5}} \approx 1.0 \times 10^{11} \text{ m}^{-3}.$$

The thermal quasiparticle density at $T = 0.1$ K is

$$n_{\text{qp}}^{\text{th}} = 2N_0 \sqrt{2\pi\Delta_0 k_B T} e^{-\Delta_0/(k_B T)}$$

with $N_0 = 1.72 \times 10^{10} \text{ m}^{-3} \text{ J}^{-1}$, $k_B = 1.380649 \times 10^{-23} \text{ J K}^{-1}$, $\Delta_0 = 2.91593 \times 10^{-23}$ J and $T = 0.1$ K. Evaluating step-by-step:

1. $\Delta_0/(k_B T) = \frac{2.91593 \times 10^{-23}}{1.380649 \times 10^{-23} \times 0.1} = 21.124$;
2. $e^{-21.124} = 6.71 \times 10^{-10}$;
3. $2\pi\Delta_0 k_B T = 2\pi \times 2.91593 \times 10^{-23} \times 1.380649 \times 10^{-24} = 2.53 \times 10^{-46}$;
4. $\sqrt{2.53 \times 10^{-46}} = 5.03 \times 10^{-23}$;
5. $n_{\text{qp}}^{\text{th}} = 2 \times 1.72 \times 10^{10} \times 5.03 \times 10^{-23} \times 6.71 \times 10^{-10} \approx 1.2 \times 10^{-21} \text{ m}^{-3}$.

The ratio of driven to thermal density is therefore

$$\frac{n_{\text{qp}}^{\text{ss}}}{n_{\text{qp}}^{\text{th}}} \approx \frac{1.0 \times 10^{11}}{1.2 \times 10^{-21}} \approx 8.6 \times 10^{31} \approx 9 \times 10^{31}.$$

4.4 Field-scaling case study

The allocated drive power is $P_{\text{DR}} = 10^{-4} \text{ W}$ with an allocable fraction $\eta = 10^{-2}$, giving $P_{\text{alloc}} = \eta P_{\text{DR}} = 10^{-6} \text{ W}$. Ohmic absorption in a film of area $A = 10^{-4} \text{ m}^2$ and conductivity $\sigma = 10^{-4} \text{ S m}^{-1}$ obeys

$$P_{\text{abs}} = \frac{1}{2} \sigma E^2 A,$$

so the admissible field amplitude is

$$E_{\text{max}} = \sqrt{\frac{2P_{\text{alloc}}}{\sigma A}} = \sqrt{\frac{2 \times 10^{-6}}{10^{-4} \times 10^{-4}}} = \sqrt{200} \approx 14.1 \text{ V m}^{-1}.$$

To achieve a Floquet-engineered gap comparable to the static gap we set $\Delta_F \approx eEa$, with lattice constant $a = 4 \times 10^{-10} \text{ m}$. Solving for the required field gives

$$E_{\text{req}} = \frac{\Delta_0}{ea} = \frac{2.91593 \times 10^{-23}}{1.602176634 \times 10^{-19} \times 4 \times 10^{-10}} \approx 4.6 \times 10^5 \text{ V m}^{-1}.$$

Thus $E_{\text{req}} \gg E_{\text{max}}$, a mismatch of roughly three orders of magnitude.

4.5 Electron temperature from electron-phonon cooling

Electron-phonon cooling follows $P_{\text{abs}} = \Sigma V_f (T_e^5 - T_{\text{ph}}^5)$ with $\Sigma = 2.0 \times 10^9 \text{ W m}^{-3} \text{ K}^{-5}$, $V_f = 10^{-16} \text{ m}^3$, $T_{\text{ph}} = 0.1 \text{ K}$ and $P_{\text{abs}} = 10^{-6} \text{ W}$. Rearranging gives

$$T_e^5 = \frac{P_{\text{abs}}}{\Sigma V_f} + T_{\text{ph}}^5 = \frac{10^{-6}}{2.0 \times 10^9 \times 10^{-16}} + 0.1^5 = 5 \times 10^{-5} \approx 5,$$

so $T_e \approx 5^{1/5} \approx 1.38 \text{ K}$. The electronic subsystem therefore heats only to the kelvin scale, far below any temperature relevant to Floquet operation.

5. Results

- Pair-breaking threshold: $f_{\text{pb}} \approx 88 \text{ GHz}$.
- Poisoning-limited absorbed power for a qubit junction: $P_{\text{abs}}^{\text{max}} \approx 5.8 \times 10^{-19} \text{ W}$.
- Steady-state quasiparticle density in a $5 \times 10^{-5} \text{ m}^3$ cavity driven at 1 pW: $n_{\text{qp}}^{\text{ss}} \approx 1.0 \times 10^{11} \text{ m}^{-3}$, exceeding the thermal density by a factor $\sim 9 \times 10^{31}$.
- Admissible drive field: $E_{\text{max}} \approx 14.1 \text{ V m}^{-1}$.

- Required field for Floquet gap comparable to Δ_0 : $E_{\text{req}} \approx 4.6 \times 10^5 \text{ V m}^{-1}$.
- Electron temperature under allocated power: $T_e \approx 1.4 \text{ K}$.

These quantitative results substantiate the hierarchy of bottlenecks described in the Introduction.

6. Discussion

The analysis rests on several simplifying assumptions. First, we treat quasiparticle recombination as a single exponential lifetime τ_{qp} , ignoring possible density-dependent effects that could shorten the effective lifetime at higher drive powers. Second, the heat-balance model assumes that all absorbed power is transferred to the electron system and then removed by a single electron-phonon channel; in real devices phonon bottlenecks or Kapitza resistance may dominate, potentially lowering the effective P_{cool} . Third, the field-scaling estimate uses a uniform conductivity and neglects skin-depth effects that become important at GHz frequencies. Fourth, the Floquet gap estimate $\Delta_F \approx eEa$ is a crude order-of-magnitude bound; more sophisticated band-structure calculations could yield different scaling.

Failure modes. If the quasiparticle lifetime τ_{qp} were orders of magnitude shorter than $3 \times 10^{-4} \text{ s}$, the poisoning-limited power would increase, potentially making cooling the dominant bottleneck. Conversely, if the cooling power P_{cool} were lower than 10^{-4} W (e.g., 10^{-5} W as mentioned in the text), the cooling constraint would bind even for qubit-scale devices. The present conclusions would be falsified by experimental observation of stable Floquet operation at absorbed powers $\gtrsim 10^{-15} \text{ W}$ without measurable quasiparticle poisoning, or by direct measurement of τ_{qp} far below 10^{-4} s under drive.

Open questions. A full kinetic treatment of quasiparticle generation, including multiphoton processes and sub-gap absorption pathways, would refine the poisoning budget. Incorporating realistic cavity field distributions and material-specific conductivity tensors could tighten the field-scaling limits. Finally, exploring engineered phononic environments to boost P_{cool} may shift the hierarchy of constraints for large-scale Floquet-driven superconducting metamaterials.

7. Conclusion

We have presented a transparent, arithmetic-driven analysis of thermodynamic bottlenecks for Floquet driving of superconducting devices. For qubit-scale junctions, quasiparticle poisoning imposes an absorbed-power ceiling of $\sim 10^{-19} \text{ W}$, far tighter than refrigeration limits. For cavity-scale resonators, the driven quasiparticle density overwhelms the thermal background by many orders of magnitude, and the admissible drive field is three orders of magnitude below that required to open a Floquet gap comparable to the static superconducting gap. These quantitative bounds define a sub-gap Floquet window and provide concrete, falsifiable design rules for future superconducting quantum technologies.

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Appendix A. Divergence report

The cooling-power parameter P_{cool} appears with two conflicting values in the source drafts: $1 \times 10^{-4} \text{ W}$ (adopted in the main text) and $1 \times 10^{-5} \text{ W}$ (mentioned as a divergent choice). We have chosen the larger, more optimistic value $1 \times 10^{-4} \text{ W}$ for all calculations, documenting the alternative in this appendix.

Appendix B. Claim attribution

Claim ID	Statement	Source Draft(s)	Agreement
C1	Pair-breaking threshold $f_{\text{pb}} \approx 88 \text{ GHz}$	A, B, C	CONVERGENT
C2	Poisoning-limited absorbed power $P_{\text{abs}}^{\text{max}} \approx 5.8 \times 10^{-19} \text{ W}$	A, B, C	CONVERGENT
C3	Cavity steady-state quasiparticle density $\approx 1.0 \times 10^{11} \text{ m}^{-3}$	A, B, C	CONVERGENT
C4	Ratio to thermal density $\sim 9 \times 10^{31}$	A, B, C	CONVERGENT
C5	Admissible field $E_{\text{max}} \approx 14.1 \text{ V m}^{-1}$	A, B, C	CONVERGENT
C6	Required field $E_{\text{req}} \approx 4.6 \times 10^5 \text{ V m}^{-1}$	A, B, C	CONVERGENT
C7	Electron temperature under allocated power $T_e \approx 1.4 \text{ K}$	A, B, C	CONVERGENT
C8	Cooling-power divergence $P_{\text{cool}} = 10^{-4} \text{ W}$ vs 10^{-5} W	A, B	DIVERGENT