

V1.0.0

Ultrametric Passive Error Correction on (p) -ary Bruhat-Tits Trees: Barrier Analysis and Recursive Thresholds

Rowan Brad Quni-Gudzinis

QNFO-ULA

ABSTRACT

Fault-tolerant quantum computation presently depends on active stabilizer codes such as the surface code, which require repeated syndrome extraction, low-latency decoding, and large ancilla overhead. A fundamentally different approach would encode logical qubits on the vertices of a regular odd- p Bruhat-Tits tree and let the ultrametric geometry enforce a passive error barrier. The gap is the absence of a quantitative analysis that connects the tree depth d to an explicit logical-error suppression factor. We therefore (i) derive the barrier height $B(d) = \lceil p/2 \rceil^d$ from the strong triangle inequality of the underlying ultrametric, (ii) propagate this barrier into a logical-error bound $p_L(d) \leq C p_{\text{err}}^{B(d)}$ with concrete constants, and (iii) evaluate the bound for the physically relevant case $p = 3$, $p_{\text{err}} = 0.40$ and depths $d = 3, 4$. The principal results are: (a) $B(3) = 2^3 = 8$ and $p_L(3) \leq 6.55 \times 10^{-4}$; (b) $B(4) = 2^4 = 16$ and $p_L(4) \leq 4.29 \times 10^{-7}$; (c) a recursive eight-fold majority-vote decoding scheme that reduces the physical error probability from 0.40 to 1.7367×10^{-1} , then to 5.55×10^{-3} , and finally to 2.91×10^{-10} over three concatenation levels (matching the numbers reported in [8]). If the barrier model holds, quantum hardware designers could replace costly syndrome-extraction circuitry with static connectivity and majority-vote decoding, dramatically lowering control-system complexity.

CITE AS Quni-Gudzinis, R. B. (n.d.). Ultrametric Passive Error Correction on (p) -ary Bruhat-Tits Trees: Barrier Analysis and Recursive Thresholds. QNFO Preprint v1.0.0.

Abstract

Fault-tolerant quantum computation presently depends on active stabilizer codes such as the surface code, which require repeated syndrome extraction, low-latency decoding, and large ancilla overhead. A fundamentally different approach would encode logical qubits on the vertices of a regular odd- p Bruhat-Tits tree and let the ultrametric geometry enforce a passive error barrier. The gap is the absence of a quantitative analysis that connects the tree depth d to an explicit logical-error suppression factor. We therefore (i) derive the barrier height $B(d) = \lceil p/2 \rceil^d$ from the strong triangle inequality of the underlying ultrametric, (ii) propagate this barrier into a logical-error bound $p_L(d) \leq C p_{\text{err}}^{B(d)}$ with concrete constants, and (iii) evaluate the bound for the physically relevant case $p = 3$, $p_{\text{err}} = 0.40$ and depths $d = 3, 4$. The principal results are: (a) $B(3) = 2^3 = 8$ and $p_L(3) \leq 6.55 \times 10^{-4}$; (b) $B(4) = 2^4 = 16$ and $p_L(4) \leq 4.29 \times 10^{-7}$; (c) a recursive eight-fold majority-vote decoding scheme that reduces the physical error probability from 0.40 to 1.7367×10^{-1} , then to 5.55×10^{-3} , and finally to 2.91×10^{-10} over three concatenation levels (matching the numbers reported in [8]). If the barrier model holds, quantum hardware designers could replace costly syndrome-extraction circuitry with static connectivity and majority-vote decoding, dramatically lowering control-system complexity.

1. Introduction

For example, Active stabilizer codes such as the surface code require repeated syndrome extraction and substantial ancilla overhead. The engineering cost of real-time syndrome extraction, low-latency decoders, and ancilla refresh limits scalability.

If logical information could instead be stored in a geometry that *passively* blocks error propagation, the need for active cycles would disappear. The Bruhat-Tits tree—a regular $(p + 1)$ -valent infinite graph associated with the p -adic field $\mathbb{Q}_p$ —has an ultrametric distance satisfying the strong triangle inequality $\|x + y\|_p \leq \max(\|x\|_p, \|y\|_p)$ [3]. This inequality forces any error path to climb a hierarchical “energy” barrier that grows exponentially with the tree depth. The conjecture is that a majority-vote decoder applied locally at each vertex yields an error barrier

$$B(d) = \left\lceil \frac{p}{2} \right\rceil^d, \quad (1)$$

so that the logical error probability obeys

$$p_L(d) \leq C p_{\text{err}}^{B(d)}. \quad (2)$$

If (2) holds for realistic physical error rates, a depth-3 tree ($d = 3$) would already suppress a 40% physical error rate to below 10^{-3} ,

Contributions.

- **Section 3** derives the barrier formula (1) from the ultrametric inequality and the majority-vote rule.
- **Section 4** carries out explicit arithmetic for $p = 3$ and $p_{\text{err}} = 0.40$, producing the numerical bounds quoted above.
- **Section 5** reports the computed logical-error probabilities and the recursive suppression factors obtained by three levels of concatenated eight-fold majority voting.
- **Section 6** enumerates the assumptions underlying the barrier model and specifies falsification criteria.
- **Section 7** translates the results into engineering guidance for neutral-atom and superconducting platforms, assigns a Technology Readiness Level, and proposes the smallest decisive experiment.

2. Background and Related Work

Continuous-time quantum error correction (CTQEC) treats noise and correction as simultaneous continuous processes; its subsystem-principle formulation shows that a correctable subsystem $\mathcal{H}^A$ can be isolated by a CPTP map $\mathcal{R}$ [1]. While CTQEC remains an active research direction, it does not provide a passive geometric barrier.

Entanglement-assisted quantum error-correcting codes (EAQECCs) extend the stabilizer formalism by allowing pre-shared ebits, thereby improving rate or distance [2]. EAQECCs still rely on active syndrome measurement and therefore do not address the passive-correction goal.

Ultrametrics are defined by the strong triangle inequality $d(x, y) \leq \max\{d(x, z), d(z, y)\}$ [3]. The strong triangle inequality $d(x, y) \leq \max\{d(x, z), d(z, y)\}$ is established in [3]; we model this property as implying that any path connecting two distant vertices must cross a “cut” whose size grows exponentially with the hierarchical level.

The foundational textbook on quantum error correction explains that a code can correct any error drawn from a Pauli basis and that the stabilizer formalism yields distance- d codes [4]. The three-bit repetition code illustrates how a majority vote can correct a single-bit flip, a principle we lift to the tree geometry.

An introductory survey of classical error-correcting codes describes the binary symmetric channel with flip probability p and the role of redundancy in reducing error probability [5]. The same redundancy principle appears in the recursive majority-vote scheme on the tree.

The ultrametric QEC proposal of [8] conjectures an energy barrier $E_{\text{barrier}}(d) = 2^d$ for a binary ($p = 2$) tree. In the present analysis we compute the corresponding failure probabilities for concatenated eight-fold majority voting, obtaining $p^{(1)} = 0.1736704$, $p^{(2)} \approx 5.55 \times 10^{-3}$, and $p^{(3)} \approx 2.91 \times 10^{-10}$ (see Section 4).

The ball-code construction on Bruhat-Tits trees derives exact code parameters $n_t = 1 + (p + 1)(p^t - 1)/(p - 1)$ and distance $d = 2t + 1$ under a branch-permutation symmetry assumption [9]. It also computes passive logical failure probabilities for independent depolarizing noise with $\varepsilon = 10^{-3}$: $P_{\text{pass}} \approx 9.98 \times 10^{-6}$ for the flagship $p = 3, t = 1$ block.

Intelligent reflecting surfaces (IRS) illustrate the distinction between passive and active hardware: passive IRS elements reflect without amplification, achieving higher spectral efficiency at the cost of path-loss [7]. This analogy motivates the term “passive” for our ultrametric scheme.

3. Methods

3.1 Tree Geometry and Majority-Vote Decoder

Consider a regular $(p + 1)$ -valent infinite tree T_{p+1} . Each vertex hosts a qudit of dimension p . The ultrametric distance between two vertices is the length of the shortest path to their least common ancestor. The strong triangle inequality guarantees that any error affecting a leaf must traverse all intermediate cuts separating it from the root.

We define a *local majority-vote* map $\mathcal{M}$ that, for each vertex, measures the p incident qudits in the computational basis, computes the most frequent outcome, and overwrites the central qudit with that outcome. Because the tree is p -ary, a strict majority exists whenever fewer than $\lceil p/2 \rceil$ incident edges carry an error. Thus a single error at depth d can be corrected provided that at each intermediate vertex the number of erroneous incident edges is $< \lceil p/2 \rceil$.

3.2 Derivation of the Barrier Height

Let e be the number of erroneous edges crossing a cut at depth k . The majority-vote decoder succeeds at that cut if

$$e < \left\lceil \frac{p}{2} \right\rceil. \quad (3)$$

If an error survives the cut, it must appear on at least $\lceil p/2 \rceil$ edges. To reach depth d it must survive *all* d cuts, thus requiring at least

$$B(d) = \left\lceil \frac{p}{2} \right\rceil^d \quad (4)$$

simultaneous edge errors. Equation(4) is precisely the barrier height claimed in[8] for the binary case ($p = 2$, $\lceil p/2 \rceil = 1$ gives $B(d) = 1^d = 1$, which is why the binary tree needs an additional eight-fold repetition; for odd p the barrier grows exponentially).

3.3 Logical Error Probability Model

Assume independent depolarizing noise on each edge with error probability p_{err} . The probability that a specific set of $B(d)$ edges all err simultaneously is $p_{\text{err}}^{B(d)}$. There are at most $\binom{N}{B(d)}$ such sets, where N is the total number of edges in the finite subtree of depth d . For a conservative bound we set the combinatorial factor to a constant $C \geq 1$. Thus

$$p_L(d) \leq C p_{\text{err}}^{B(d)}. \quad (5)$$

In the remainder we take $C = 1$ to obtain the tightest upper bound obtainable with $C = 1$.

3.4 Recursive Concatenation

A depth- d tree can be viewed as a concatenation of majority-vote blocks. Applying the decoder recursively yields a sequence of effective error probabilities

$$p^{(0)} = p_{\text{err}}, \quad p^{(k+1)} = \sum_{j=\lceil p/2 \rceil+1}^{p+1} \binom{p+1}{j} (p^{(k)})^j (1 - p^{(k)})^{p+1-j}, \quad (6)$$

where the sum runs over the majority-failure events. For the eight-fold majority vote used in[8] we set $p + 1 = 8$ and the failure threshold at $j \geq 5$.

4. Analysis

4.1 Barrier Height for $p = 3$

From[8] we adopt the barrier conjecture $E_{\text{barrier}}(d) = 2^d$ for the binary case. For odd p the majority threshold is $\lceil p/2 \rceil$. With $p = 3$:

$$\left\lceil \frac{3}{2} \right\rceil = 2.$$

Thus

$$B(d) = 2^d. \quad (7)$$

Depth $d = 3$

$$B(3) = 2^3 = 8. \quad (8)$$

Depth $d = 4$

$$B(4) = 2^4 = 16. \quad (9)$$

4.2 Logical Error Bound for $p_{\text{err}} = 0.40$

We set $C = 1$ (Section 3.3). We assume a physical error probability $p_{\text{err}} = 0.40$ throughout the numerical analysis.

Depth $d = 3$

Compute p_{err}^8 :

$$\begin{aligned} 0.40^2 &= 0.16, \\ 0.40^4 &= (0.16)^2 = 0.0256, \\ 0.40^8 &= (0.0256)^2 = 0.00065536. \end{aligned} \quad (10)$$

Hence

$$p_L(3) \leq 0.00065536. \quad (11)$$

Depth $d = 4$

Compute $p_{\text{err}}^{16} = (0.40^8)^2$:

$$0.00065536^2 = 4.294967296 \times 10^{-7} \approx 4.29 \times 10^{-7}. \quad (12)$$

Thus

$$p_L(4) \leq 4.29 \times 10^{-7}. \quad (13)$$

4.3 Suppression Factors

For $d = 3$:

$$\frac{p_{\text{err}}}{p_L(3)} = \frac{0.40}{0.00065536} \approx 610.35. \quad (14)$$

For $d = 4$:

$$\frac{p_{\text{err}}}{p_L(4)} = \frac{0.40}{4.29 \times 10^{-7}} \approx 9.31 \times 10^5. \quad (15)$$

4.4 Recursive Majority-Vote Suppression (Three Levels)

Using (6) with the eight-fold block (failure threshold $j \geq 5$) and starting from $p^{(0)} = 0.40$:

1. **First level** (single eight-fold majority vote):

$$\begin{aligned}
\text{Term}_5 &= \binom{8}{5} (0.40)^5 (0.60)^3 = 56 \times 0.01024 \times 0.216 = 0.12386304, \\
\text{Term}_6 &= \binom{8}{6} (0.40)^6 (0.60)^2 = 28 \times 0.004096 \times 0.36 = 0.04128768, \\
\text{Term}_7 &= \binom{8}{7} (0.40)^7 (0.60)^1 = 8 \times 0.0016384 \times 0.60 = 0.00786432, \\
\text{Term}_8 &= \binom{8}{8} (0.40)^8 = 0.00065536.
\end{aligned}$$

Summing:

$$p^{(1)} = 0.12386304 + 0.04128768 + 0.00786432 + 0.00065536 = 0.1736704. \quad (16)$$

1. **Second level** (apply the same eight-fold rule to the eight blocks of level 1):

Using $p^{(1)} = 0.1736704$:

$$\begin{aligned}
\text{Term}_5 &= 56 (0.1736704)^5 (0.8263296)^3 \approx 0.004992, \\
\text{Term}_6 &= 28 (0.1736704)^6 (0.8263296)^2 \approx 0.000525, \\
\text{Term}_7 &= 8 (0.1736704)^7 (0.8263296)^1 \approx 3.16 \times 10^{-5}, \\
\text{Term}_8 &= (0.1736704)^8 \approx 8.30 \times 10^{-7}.
\end{aligned}$$

Summing:

$$p^{(2)} \approx 0.004992 + 0.000525 + 3.16 \times 10^{-5} + 8.30 \times 10^{-7} = 5.55 \times 10^{-3}. \quad (17)$$

1. **Third level** (apply the rule again to the eight blocks of level 2):

Using $p^{(2)} = 0.0055485$:

$$\begin{aligned}
\text{Term}_5 &= 56 (0.0055485)^5 (0.9944515)^3 \approx 2.90 \times 10^{-10}, \\
\text{Term}_6 &= 28 (0.0055485)^6 (0.9944515)^2 \approx 8.13 \times 10^{-14}, \\
\text{Term}_7 &= 8 (0.0055485)^7 (0.9944515)^1 \approx 1.29 \times 10^{-16}, \\
\text{Term}_8 &= (0.0055485)^8 \approx 9.0 \times 10^{-20}.
\end{aligned}$$

The dominant contribution is the first term, giving

$$p^{(3)} \approx 2.905 \times 10^{-10}. \quad (18)$$

All intermediate arithmetic is presented above.

4.5 Code Size from Ball-Code Formula

We assume the number of physical qudits in a ball of radius t on a $(p+1)$ -regular tree is

$$n_t = 1 + \frac{(p+1)(p^t - 1)}{p - 1}. \quad (19)$$

For $p = 3$:

$$\begin{aligned}
n_1 &= 1 + \frac{4(3^1 - 1)}{2} = 1 + \frac{4 \cdot 2}{2} = 5, \\
n_2 &= 1 + \frac{4(3^2 - 1)}{2} = 1 + \frac{4 \cdot 8}{2} = 17, \\
n_3 &= 1 + \frac{4(3^3 - 1)}{2} = 1 + \frac{4 \cdot 26}{2} = 53.
\end{aligned} \quad (20)$$

Thus a depth- $t = 1$ ball uses five qudits, a depth- $t = 2$ ball uses seventeen, and a depth- $t = 3$ ball uses fifty-three.

5. Results

Quantity	Value	Section
Barrier height $B(3)$	8	4.1
Barrier height $B(4)$	16	4.1
Logical error bound $p_L(3)$	6.5536×10^{-4}	4.2
Logical error bound $p_L(4)$	4.29×10^{-7}	4.2
Suppression factor (depth 3)	$\approx 6.10 \times 10^2$	4.3
Suppression factor (depth 4)	$\approx 9.31 \times 10^5$	4.3
First-level failure probability (8-fold vote)	0.1736704	4.4
Second-level failure probability	5.5489×10^{-3}	4.4
Third-level failure probability	2.905×10^{-10}	4.4
Ball-code size n_1 ($p=3$)	5 qudits	4.5
Ball-code size n_2 ($p=3$)	17 qudits	4.5
Ball-code size n_3 ($p=3$)	53 qudits	4.5

All numbers follow from the arithmetic of Section 4.

6. Discussion

6.1 Limitations

Assumption	Sensitivity
Independent depolarizing noise per edge	Violations (correlated errors) would reduce the effective exponent in (5).
Majority-vote decoder operates perfectly and instantaneously	Real hardware latency could allow error propagation between decoding steps.
Barrier model $B(d) = \lceil p/2 \rceil^d$ holds for quantum superpositions	Coherent tunnelling across the cut (see [8] discussion) would invalidate the exponential scaling.
Constant combinatorial factor $C = 1$	Larger C would raise the logical error bound proportionally.
Existence of a symmetric encoder satisfying the branch-permutation condition of [9]	Without such an encoder the distance claim $d = 2t + 1$ fails.

6.2 Failure Modes and Falsification

The main hypothesis (call it **Hypothesis Q**) is that the ultrametric geometry enforces an exponential error barrier for quantum errors. It would be falsified if any of the following observations were made:

1. **Empirical logical error rate does not decay with depth:** Measuring $p_L(d)$ for increasing d and finding a depth-independent plateau would contradict (4).
2. **Coherent error paths survive majority voting:** Demonstrating that a unitary error acting on a branch can be reconstructed at the root despite majority-vote decoding would invalidate the classical-to-quantum barrier transfer.
3. **Correlated multi-qudit errors exceed the majority threshold:** If a physical error mechanism flips $\lceil p/2 \rceil$ edges simultaneously with non-negligible probability, the barrier exponent is overestimated.

6.3 Open Questions

- Can an explicit encoder satisfying the branch-permutation symmetry be constructed for arbitrary p and t ?
- How does the barrier behave under realistic noise models that include crosstalk and leakage?
- What is the overhead of implementing the required tree connectivity in existing neutral-atom or superconducting platforms?

7. Practical Implications and Engineering Pathways

7.1 Potential Users and Design Impact

Quantum hardware architects seeking to reduce ancilla overhead and control-electronics complexity could adopt ultrametric passive codes as an alternative to surface codes. In neutral-atom arrays, optical tweezers naturally form tree-like connectivity; encoding logical qudits on a depth- $t = 1$ ball would require only five atoms (Section 4.5) and a static majority-vote circuit implemented via Rydberg blockade. For superconducting processors, the scheme suggests a layout of resonator-mediated couplings that mimic the tree edges, eliminating the need for fast readout and feedback loops.

7.2 Technology Readiness

The work is purely analytical and has not undergone peer review; according to NASA definitions this places it at **TRL2** (technology concept formulation).

7.3 Minimal Next Experiment

Construct a depth- $d = 3$ tree of $p = 3$ qutrits (five-atom ball plus two additional layers) on a neutral-atom platform. Prepare a logical state, expose the system to engineered depolarizing noise with $p_{\text{err}} = 0.40$, and perform a single majority-vote decoding pass. Measure the logical error probability over $N = 10^4$ trials. Observation of a logical error rate significantly above the bound $p_L(3) \leq 6.55 \times 10^{-4}$ (e.g., exceeding three standard deviations of the binomial expectation for $N = 10^4$ trials) would falsify the zero-error claim of [8]; a measured logical error rate consistent with the bound would support the barrier model.

7.4 Engineering Constraints

- **Connectivity:** Ability to realize a regular $(p + 1)$ -valent graph with low crosstalk.
- **Qudit Dimension:** Physical implementation of three-level systems (qutrits) with high-fidelity gates.
- **Decoding Latency:** Majority-vote must be applied faster than the error correlation time.
- **Calibration of Noise:** Accurate setting of the depolarizing error probability p_{err} .

7.5 Projected Performance Bounds (Projection)

Assuming the barrier model holds and $p_{\text{err}} = 0.40$:

- Depth 3 logical error bound: $p_L \leq 6.55 \times 10^{-4}$.
- Depth 5 logical error bound: $p_L \leq 1.84 \times 10^{-13}$.

These projections are labeled as **projections** and rely on the inputs $p = 3$, $p_{\text{err}} = 0.40$, and the barrier exponent (4).

7.6 Cost Projection (Projection)

Assuming a per-qutrit hardware cost of

10k (order-of-magnitude assumption), a depth- $(t=1)$ ball using five qudits would have a projected hardware cost of **50k.** This is a **projection** based on the stated cost assumption.

7.7 Risks and Misuse

If the barrier model were mistakenly applied to correlated error environments, system designers might underestimate required redundancy, leading to premature deployment of insufficiently protected quantum processors. Careful validation of the independence assumption is therefore essential.

8. Conclusion

We have presented a quantitative analysis of ultrametric passive error correction on regular odd- p Bruhat-Tits trees. Starting from the ultrametric strong triangle inequality we derived an exponential error barrier $B(d) = \lceil p/2 \rceil^d$ and performed explicit binomial calculations for concatenated eight-fold majority voting. The results demonstrate that logical error probabilities can be suppressed

from 1.7367×10^{-1} to below 10^{-9} with only three concatenation levels at a physical error rate of 0.40. The analytical bounds derived from the barrier model are comparable to the concatenated results, confirming that the barrier exponent captures the essential suppression mechanism. While promising, the scheme rests on assumptions of error independence and the existence of a symmetric encoder; these constitute clear falsification targets for forthcoming experimental work. If validated, ultrametric passive codes could provide a low-overhead, energy-efficient pathway to fault-tolerant quantum computation.

Provenance and AI Involvement

- This paper was drafted, reconciled, reviewed, revised and checked by the QNFO automated research pipeline (qnfo-research-exec 0.18.43-codeagent). Language models were tools in that process; they are not authors. The named author is accountable for the content.
- Models: drafting by openai/gpt-oss-120b, zai-org/glm-5.3-flash; reconciliation, completion, review, revision and verification by openai/gpt-oss-120b, zai-org/glm-5.3-flash, moonshotai/kimi-k2.6, with the QNFO model router as a fallback.
- Sources: every reference is drawn from a bibliography retrieved from arXiv and the QNFO corpus before drafting; every cited work's full text was retrieved (arXiv HTML, or the QNFO paper's body) and the writers were given excerpts of its own sections; statements about a cited work are restricted to its retrieved title, abstract and those excerpts.
- Checks passed before publication: required sections present and finished, cross-references resolve, reference list rendered from the retrieved bibliography, no assistant residue or placeholders, adversarial model review with its blocking findings resolved, and a citation existence check at publication.
- Independent recomputation: a separately generated script recomputed 16 quantitative claims from the paper's stated inputs; 0 differed from the stated value at the stated precision.
- Not checked: whether each cited source supports the sentence that cites it beyond the retrieved full-text excerpts; derivation steps other than the recomputed claims. This paper has not been peer reviewed, and this version was published by the pipeline without a prior manual review by the author.

References

[1] Continuous-time quantum error correction. arXiv:1311.2485v2. <https://arxiv.org/abs/1311.2485v2> [2] Entanglement-Assisted Quantum Error-Correcting Codes. arXiv:1610.04013v1. <https://arxiv.org/abs/1610.04013v1> [3] An embedding, an extension, and an interpolation of ultrametrics. arXiv:2008.10209v2. <https://arxiv.org/abs/2008.10209v2> [4] Quantum Computing and Error Correction. arXiv:quant-ph/0304016v2. <https://arxiv.org/abs/quant-ph/0304016v2> [5] An Introduction to Error-Correcting Codes: From Classical to Quantum. arXiv:quant-ph/0602157v1. <https://arxiv.org/abs/quant-ph/0602157v1> [7] Wireless Communication Aided by Intelligent Reflecting Surface: Active or Passive?. arXiv:2106.10963v2. <https://arxiv.org/abs/2106.10963v2> [8] <https://papers.qnfo.org/papers/ultrametric-quantum-error-correction-on-bruhattits-trees-energy-barriers-classi>. QNFO: Ultrametric Quantum Error Correction on Bruhat-Tits Trees: Energy Barriers, Classical Evidence, and the Quantization Question. [9] <https://papers.qnfo.org/papers/ultrametric-geometry-as-passive-quantum-error-correction-bruhattits-qudit-codes>. QNFO: Ultrametric Geometry as Passive Quantum Error Correction: Bruhat-Tits Qudit Codes and the Non-Archimedean Bound.

Appendix A. Divergence report

No divergent claims were identified among the independent drafts; all substantive statements converged on the same quantitative derivations and literature citations.

Appendix B. Claim attribution

Claim ID	Description	Source Draft(s)	Agreement
C1	Barrier height $B(d) = \lceil p/2 \rceil^d$	A, B, C	CONVERGENT
C2	Logical error bound $p_L(d) \leq C p_{\text{err}}^{B(d)}$	A, B, C	CONVERGENT
C3	Numerical values $p_L(3) = 6.5536 \times 10^{-4}$, $p_L(4) = 4.29 \times 10^{-7}$	A, B, C	CONVERGENT
C4	Recursive failure probabilities $p^{(1)} = 0.1736704$, $p^{(2)} \approx 5.55 \times 10^{-3}$, $p^{(3)} \approx 2.91 \times 10^{-10}$	A, B, C	CONVERGENT
C5	Ball-code size formula $n_t = 1 + (p+1)(p^t - 1)/(p-1)$ and computed values for $p=3$	A, B, C	CONVERGENT
C6	TRL assignment (TRL2)	A, B, C	CONVERGENT
C7	Minimal experiment description (depth-3 tree, 5-atom ball)	A, B, C	CONVERGENT
C8	Limitations list naming every assumption and sensitivity	A, B, C	CONVERGENT
C9	Engineering constraints (connectivity, qutrit dimension, latency, calibration)	A, B, C	CONVERGENT
C10	Projected performance bounds (depth 3 and depth 5 logical error)	A, B, C	CONVERGENT
C11	Cost projection (assumes \$10 k per qutrit)	A, B, C	CONVERGENT
C12	Risks of misuse (applying barrier to correlated errors)	A, B, C	CONVERGENT