

Valuation Without $\mathbb{R}$: A Category-Theoretic Foundation for Finite Measurement

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Abstract. Measurement in physics is always finite and approximate — yet its mathematical foundations are embedded in the Archimedean continuum of real numbers and the cumulative hierarchy of set theory. We invert this dependency: the act of measurement is axiomatized as a graded distinguishability map $v : \mathcal{S}^2 \rightarrow \mathbb{N} \cup \{\infty\}$ over a poset of states satisfying the ultrametric inequality, with no dependence on $\mathbb{R}$ or set-theoretic foundations. The category $\mathbf{Val}$ of valuation spaces with non-expansive maps is self-contained — $\mathbb{R}$ appears only as a limit idealization. The effective spatial dimension d emerges as the exponent in the asymptotic growth $N(r) \sim q^{d \cdot r}$ of the distinguishability graph at resolution r , constrained by the cohomological consistency of the refinement sheaf across resolutions. Falsifiable predictions are pre-registered at Planck-scale resolution: ultrametric clustering in the distinguishability graph G_r supersedes Euclidean nearest-neighbor structure at $r > r_c$.

Keywords. valuation, ultrametric, measurement, p-adic, distinguishability, category theory, dimension emergence, sheaf cohomology

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A Category-Theoretic Foundation for Finite Measurement

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1. Introduction

Every physical measurement produces a finite output: a detector click, a pointer reading, a register of N bits. Yet the mathematical foundations of physics embed this finite act in the Archimedean continuum — the real numbers — a structure carrying uncountably many non-computable elements, power-set overhang, and a topology that is Archimedean (and therefore ONE completion among all completions of the rationals per Ostrowski's theorem). Simultaneously, the background universe of discourse is set theory (ZFC or equivalent), whose cumulative hierarchy, axiom of choice, and power-set axiom are accepted without examination as the stage on which physical theories are erected.

This paper inverts both dependencies. Rather than treating measurement as " $\mathbb{R}$ -valued with error bars," we ask: **what is the minimal structure required to describe the act of finite discrimination among states — and what does that structure, taken as primitive, imply about the apparent continuity of space and time?**

We propose that the primitive is a **valuation** — a graded distinguishability map over a poset of states satisfying the ultrametric (non-Archimedean) inequality. From this structure, the real numbers appear as a limit idealization (the Archimedean completion under one particular valuation), and the effective dimension of space emerges as a growth exponent constrained by the cohomological consistency of the measurement refinement sheaf.

1.1 The Two Dependencies and Why They Matter

Dependency 1: The real numbers ($\mathbb{R}$). The Archimedean completion of $\mathbb{Q}$ is ONE completion among ALL completions — per Ostrowski's theorem, the valuations of $\mathbb{Q}$ are the trivial valuation, the Archimedean absolute value, and the p -adic absolute values for every prime p [established – Ostrowski, 1916]. Choosing $\mathbb{R}$

without justification is the **Archimedean bias** — an unexamined default that conceals viable p -adic completions and discrete structures [speculative – this paper's motivating claim].

Moreover, the power-set overhang of $\mathbb{R}$ — the non-computable reals — has no physical signature. No finite measurement protocol can discriminate two non-computable reals, so any formula whose content depends on a non-computable value is physically unfalsifiable [established — Continuum Trilogy, DOI <https://papers.qnfo.org/papers/continuum-trilogy-01-computable-continuum/>]. Only the computable Archimedean continuum $\mathbb{R}_c$ is physical; the uncountable breadth of $\mathbb{R}$ is eliminated.

Dependency 2: Set theory (ZFC). The background universe of sets — its axiom of choice, its power-set axiom, its cumulative hierarchy — is an invisible scaffolding that physics has inherited from mathematics without asking whether measurement itself requires it. Topos quantum theory [Doering & Isham, 2007] removes set theory but retains $\mathbb{R}$ -valued probabilities. Categorical quantum mechanics [Abramsky & Coecke, 2004] removes Hilbert space specifics but retains $\mathbb{C}$ as the scalar field. The question of removing BOTH has not been addressed.

1.2 The Thesis

Measurement can be formalized as a valuation space: a set of states $\mathcal{S}$ equipped with a graded distinguishability map $v : \mathcal{S}^2 \rightarrow \mathbb{N} \cup \{\infty\}$ satisfying (1) identity of indiscernibles, (2) symmetry, and (3) the ultrametric inequality — with no dependence on $\mathbb{R}$ or set-theoretic foundations. The category Val of such spaces is self-contained; $\mathbb{R}$ appears only as a limit idealization. The effective spatial dimension d emerges as the asymptotic growth exponent of the distinguishability graph, constrained by the cohomological consistency of the refinement sheaf.

2. The Distinguishability Poset

2.1 Primitive Notions

We begin with two primitives:

1. **A set $\mathcal{S}$ of states.** "State" is deliberately unspecific — it could be a

physical configuration, an experimental outcome, a logical proposition, or a quantum density matrix. The only property that matters is that two states can be compared for distinguishability.

1. **A distinguishability ordering $\prec_d$.** The relation $a \prec_d b$ means: "the

measurement apparatus that distinguishes a from its complement also distinguishes b from its complement." That is, b is AT LEAST as distinguishable from its complement as a is from its complement.

2.2 Axioms of the Distinguishability Poset

(D1) Reflexivity: $a \prec_d a$ — every state is at least as distinguishable from its complement as itself (trivially).

(D2) Transitivity: If $a \prec_d b$ and $b \prec_d c$, then $a \prec_d c$ — distinguishability is transitively ordered. If apparatus A can tell b from its complement and apparatus B can tell c from its complement, then the discrimination power stacks.

(D3) Partial — not total: Two states may be incomparable under $\prec_d$. This captures the empirical fact that no single measurement protocol discriminates all state pairs simultaneously — complementary observables are the quantum witness to this partiality.

The pair $(\mathcal{S}, \prec_d)$ is a **distinguishability poset**. It is a thin category: objects are states, a morphism $a \rightarrow b$ exists iff $a \prec_d b$, and composition is transitivity.

2.3 Relation to Operational Theories

In the operational probabilistic theories (OPTs) framework [Hardy, 2001], a measurement is an effect-valued map from states to $[0, 1]$. The distinguishability poset generalizes this to a purely ordinal setting: instead of "state a produces outcome x with probability p ," we have "states a and b are distinguishable at level r ." The probability structure is recovered as the normalized counting measure on the equivalence classes of the valuation (see §4).

3. The Valuation Space

3.1 Definition

A **valuation** on a distinguishability poset $(\mathcal{S}, \prec_d)$ is a function

$$v : \mathcal{S} \times \mathcal{S} \rightarrow \mathbb{N} \cup \{\infty\}$$

where $v(a, b)$ is the **coarsest measurement resolution** at which a and b become distinguishable. Higher values of v mean MORE similar (harder to distinguish); lower values mean MORE different (easier to distinguish). $v(a, b) = \infty$ means a and b are indistinguishable at ALL finite resolutions — they are operationally identical.

The valuation satisfies three axioms:

(V1) Identity of Indiscernibles:

$$v(a, b) = \infty \iff a \equiv b \text{ (indistinguishable at all finite resolutions)}$$

(V2) Symmetry:

$$v(a, b) = v(b, a) \quad \text{for all } a, b \in \mathcal{S}$$

(V3) Ultrametric Inequality (strong triangle):

$$v(a, c) \geq \min(v(a, b), v(b, c)) \quad \text{for all } a, b, c \in \mathcal{S}$$

The pair $(\mathcal{S}, v)$ satisfying V1–V3 is a **valuation space**.

3.2 Why the Ultrametric Inequality?

The standard triangle inequality of metric spaces is:

$$d(a, c) \leq d(a, b) + d(b, c)$$

This is Archimedean — it allows distances to accumulate additively. In contrast, the ultrametric inequality:

$$v(a, c) \geq \min(v(a, b), v(b, c))$$

is **non-Archimedean**. It says: the resolution needed to tell a from c is bounded below by the coarser of the two pairwise resolutions. If you can tell a from b at resolution r , and b from c at resolution r , you can tell a from c at resolution r (or coarser).

This is the natural structure when distinguishability is **hierarchical and tree-structured** rather than continuous and incremental. Every measurement apparatus partitions the state space into finitely many equivalence classes; the refinement of one measurement by another produces a tree, not a continuum.

Operationally: If you need 10 bits to distinguish a from b , and 15 bits to distinguish b from c , you need at most 15 bits to distinguish a from c — because the 15-bit measurement of (b, c) already distinguishes (a, c) transitively through b . You do NOT need $10 + 15 = 25$ bits — the distinguishability does not accumulate additively.

3.3 The Induced Ultrametric

The valuation v induces an ultrametric distance on the states:

$$d_v(a, b) = q^{-v(a, b)}$$

for some base $q > 1$. When q is prime, d_v is precisely the p -adic metric on $\mathbb{Q}_p$. The valuation space $(\mathcal{S}, v)$ is equivalent to the ultrametric space $(\mathcal{S}, d_v)$ — the choice of v over d_v is a matter of convenience (v takes integer values; d_v takes values in $(0, 1]$).

For $q = 2$ (binary distinguishability, the natural choice per the Landauer bound: $kT \ln 2$ per bit), $d_v(a, b) = 2^{-v(a, b)}$.

4. Category Val

4.1 Definition

The category **Val** has:

- **Objects:** Valuation spaces $(\mathcal{S}, v)$ satisfying V1–V3.
- **Morphisms:** Non-expansive maps $f : (\mathcal{S}, v) \rightarrow (\mathcal{S}', v')$ satisfying

$$v'(f(a), f(b)) \geq v(a, b) \quad \text{for all } a, b \in \mathcal{S}$$

A non-expansive map does not INCREASE distinguishability — the image of two states under f is at least as hard to distinguish (at least as similar) as the originals. Equivalently: f is a **1-Lipschitz map** in the ultrametric d_v .

4.2 Categorical Structure

Terminal object: The one-point space $(\{*\}, v(*, *) = \infty)$. Every state maps trivially to $*$.

Products: The product of $(\mathcal{S}_1, v_1)$ and $(\mathcal{S}_2, v_2)$ is

$$(\mathcal{S}_1 \times \mathcal{S}_2, \quad v((a_1, a_2), (b_1, b_2)) = \min(v_1(a_1, b_1), v_2(a_2, b_2)))$$

The coproduct (disjoint union) is the maximum: $v(a, b) = v_i(a, b)$ if both are in component i , ∞ otherwise.

No dependence on $\mathbb{R}$, $\mathbb{C}$, or Set: The category **Val** is defined entirely in terms of $\mathbb{N}$ -valued valuations and non-expansive maps. The real numbers appear only if we CHOOSE to complete the rationals under the Archimedean valuation — but that is a choice, not a necessity. **Val** is agnostic to which completions, if any, are physically realized.

4.3 Relation to Other Categories

Met (metric spaces): Val is a subcategory of ultrametric spaces — but Val's objects are DISTINGUISHABILITY structures, not arbitrary metric spaces. The morphisms (non-expansive maps) preserve distinguishability but not necessarily distance.

Hilb (Hilbert spaces over $\mathbb{C}$): Standard quantum mechanics lives in Hilb. Val replaces $\mathbb{C}$ -valued inner products with $\mathbb{N}$ -valued valuations. The transition from Hilb to Val is the transition from "probability amplitudes" to "distinguishability depth."

Top (topos of sets): Val does not require a background topos — it IS a category, and its internal logic is the logic of finite discrimination. The topos approach [Doering & Isham, 2007] embeds quantum theory in a presheaf topos; Val provides a simpler, lower-level foundation that does not require choosing a topos first.

5. Measurement as Refinement

5.1 The Refinement Operator

An act of measurement at resolution r does not produce a real number — it produces a **coarse-graining** of the state space:

$$\mathcal{S} \mapsto \mathcal{S}_r = \mathcal{S} / \sim_r$$

where $a \sim_r b \Leftrightarrow v(a, b) \geq r$ (i.e., indistinguishable at resolution r). The quotient $\mathcal{S}_r$ is the set of equivalence classes at resolution r .

The refinement operator satisfies:

$$\mathcal{S}_{r+1} \xrightarrow{\pi_{r+1}} \mathcal{S}_r$$

where π_{r+1} maps each finer equivalence class to the coarser class containing it. This is a natural transformation in Val.

5.2 The Refinement Sheaf

The refinement maps form a **sheaf** over the poset $\mathbb{N}$ (reverse-ordered: $r+1 \rightarrow r$ because finer resolution maps TO coarser). The assignment

$$r \mapsto \mathcal{S}_r$$

with restriction maps $\pi_{r+1} : \mathcal{S}_{r+1} \rightarrow \mathcal{S}_r$ is a presheaf on $\mathbb{N}^{op}$.

Sheaf condition: For any cover of resolution r by finer resolutions $\{r+1, r+2, \dots\}$, the states in $\mathcal{S}_r$ are determined by their refinements: if $a, b \in \mathcal{S}_r$ have the same refinement at all finer resolutions, they are the same state at resolution r . This is automatically satisfied because the cover is just the singleton $\{r+1\}$ ($\mathbb{N}$ is a linear order).

5.3 Information-Theoretic Interpretation

At resolution r , the measurement partitions the state space into $|\mathcal{S}_r|$ equivalence classes. The information content of the measurement is:

$$I(r) = \log_2 |\mathcal{S}_r| \quad \text{bits}$$

By the Landauer bound [Landauer, 1961], each bit of distinguishable information costs $kT \ln 2$ in dissipated energy. The measurement at resolution r is a **channel** from $\mathcal{S}$ to $\mathcal{S}_r$ with capacity $I(r)$.

If the valuation space has a constant branching factor q (each refinement step splits each equivalence class into q subclasses), then:

$$|\mathcal{S}_r| = q^{d \cdot r}$$

where d is the effective dimension (see §7). The information content is:

$$I(r) = d \cdot r \cdot \log_2 q$$

For binary distinguishability ($q = 2$): $I(r) = d \cdot r$ bits.

6. Relation to Known Structures

6.1 p -Adic Valuation

The p -adic valuation v_p on $\mathbb{Q}$ is the canonical example of an ultrametric valuation:

$$v_p\left(\frac{a}{b}\right) = \text{ord}_p(a) - \text{ord}_p(b)$$

where $\text{ord}_p(n)$ is the exponent of p in the prime factorization of n .

The valuation space $(\mathbb{Q}, v_p)$ satisfies V1–V3 exactly. The p -adic numbers $\mathbb{Q}_p$ are the Cauchy completion of $\mathbb{Q}$ under the metric $d_p(x, y) = p^{-v_p(x, y)}$. The distinguishability graph G_r of $(\mathbb{Q}, v_p)$ is the projection of the Bruhat–Tits tree at depth r .

Key difference: p -adic physics [Vladimirov & Volovich, 1994] starts from $\mathbb{Q}_p$ as a pre-existing number system. The valuation-first approach starts from v and DERIVES $\mathbb{Q}_p$ as a completion — making the valuation the primitive and the continuum the derived structure.

6.2 Bruhat–Tits Tree

The Bruhat–Tits tree T_p is the infinite $(p + 1)$ -regular tree associated with $\text{PGL}(2, \mathbb{Q}_p)$. Its vertices at depth r classify p -adic disks of radius p^{-r} . The boundary at infinity is $\mathbb{P}^1(\mathbb{Q}_p)$.

In the valuation-space framework, the BT-tree is the *****distinguishability hierarchy***** of the p -adic valuation. Vertices at depth r are equivalence classes in $\mathcal{S}_r$; edges connect classes that share a boundary at finer resolution. The cross-ratio on the boundary (the p -adic absolute value) is the valuation v_p itself.

Operational interpretation: Moving down the BT-tree (increasing r) corresponds to finer measurement. The branching factor $p + 1$ counts the number of sub-classes per refinement step — the "measurement alphabet" at each resolution.

6.3 The Distinguishability Graph G_r

For a fixed resolution r , the **distinguishability graph** G_r has:

- **Vertices:** Equivalence classes $\mathcal{S}_r$ (the states distinguishable at resolution r).
- **Edges:** Between classes whose representatives a, b satisfy $v(a, b) = r$

(just barely distinguishable at this resolution).

For a q -adic valuation space (branching factor q at each step), G_r is a **cluster graph** — a disjoint union of cliques, where each clique contains the q subclassifications of a single resolution- $(r - 1)$ equivalence class. This is the ultrametric clustering property: every triangle is isosceles with the two equal sides at least as long as the third.

6.4 Information Theory

Shannon's channel capacity theorem [Shannon, 1948] bounds distinguishability by the logarithm of the number of equiprobable messages. In the valuation-space framework, the "messages" are the equivalence classes $\mathcal{S}_r$, and the channel is the measurement apparatus. The bound:

$$I(r) = \log_2 |\mathcal{S}_r| \leq \text{channel capacity}$$

is the natural information-theoretic embedding of distinguishability.

The Landauer bound [Landauer, 1961] imposes the thermodynamic cost:

$$E_{\text{meas}}(r) \geq kT \ln 2 \cdot \log_2 |\mathcal{S}_r| = kT \cdot d \cdot r \cdot \ln q$$

In dimensionless Planck units ($\hbar = c = G = k_B = 1$):

$$E_{\text{meas}}(r) \geq d \cdot r \cdot \ln q$$

7. Dimension Emergence

7.1 The Growth Function

For a valuation space with constant branching factor q , the number of distinguishable states at resolution r is:

$$N(r) = |\mathcal{S}_r| = q^{d \cdot r}$$

where d is the effective dimension. Equivalently:

$$d = \frac{\log_q N(r)}{r} \quad (\text{asymptotically as } r \rightarrow \infty)$$

This is fundamentally different from the Euclidean scaling $N(r) \sim r^d$ (continuous power-law). The valuation-space scaling is **discrete exponential** — each independent direction of distinguishability multiplies the number of discriminable states by q at each resolution step.

7.2 Sheaf Cohomology and the Consistency Constraint

The refinement maps $\pi_{r+1} : \mathcal{S}_{r+1} \rightarrow \mathcal{S}_r$ form a sheaf over $\mathbb{N}^{op}$. The consistency condition for this sheaf is that the refinement from resolution $r + 2$ to r must factor through $r + 1$:

$$\pi_{r+2} \circ \pi_{r+1} = \pi_{r+2}^{(r)} : \mathcal{S}_{r+2} \rightarrow \mathcal{S}_r$$

The **global sections** of this sheaf are states that are defined consistently at ALL resolutions (a global state). The dimension d constrains which refinement patterns are globally consistent.

Conjecture (P9 — Frontier Question): The dimension d is the rank of the first sheaf cohomology group H^1 of the refinement sheaf:

$$d = \text{rank } H^1(\mathcal{F}_{\text{ref}})$$

where $\mathcal{F}_{\text{ref}}$ is the refinement sheaf over $\mathbb{N}^{op}$. For $d = 3$ (spatial dimensions), the cohomological obstruction restricts the branching pattern to a 3-dimensional tree. For $d = 3 + 1$ (spacetime), a distinguished "causal" direction with different valuation behavior emerges from the asymmetry that measurements happen in sequence.

Status: [speculative – mathematical conjecture; see §9 for pre-registration]

7.3 Why the Exponent, Not the Base?

The dimension d appears as the EXPONENT, not the BASE. This means:

- d is about branching structure, not about the underlying cardinality.

Whether each refinement splits into $q = 2$ subclasses (binary) or $q = p$ (p -adic), the dimension is the number of independent branching directions.

- d is invariant under changes of q . If the distinguishability lattice

changes from binary ($q = 2$) to ternary ($q = 3$), the dimension can be recomputed: $d' = d \cdot \log_q(q')$ — but the NUMBER of independent directions stays the same; only the counting base shifts.

7.4 The Emergence of "3+1"

The valuation-first framework does not PREDICT $d = 3 + 1$ — it FRAMES the question correctly. The question becomes:

What consistency conditions on a valuation space force the growth exponent $d = 3$ (spatial) and a distinguished direction (temporal/causal) with different valuation behavior?

Three constraints on the answer:

1. ****The number of mutually consistent, non-degenerate 2-distinguishability**

relations** that can coexist in a single measurement network bounds d from above. If every pair of states must be simultaneously distinguishable by SOME measurement protocol, the measurement network's clique complex has a maximal dimension.

1. **The refinement sheaf must be a sheaf in the category $\mathbf{Val}$** — not just a

presheaf. The sheaf condition (gluing of local distinguishability patches) imposes cohomological constraints on the branching pattern.

1. **The causal distinction** between timelike and spacelike measurements

emerges from the asymmetry that measurements happen in sequence. The valuation operator itself is temporally directed: $v(a, b)$ compares states across measurement events, not at a single event.

8. Falsifiability and Pre-Registered Predictions

8.1 The Null-Equivalence

Framework	Prediction for $N(r)$	Status
O_N ($\mathbb{R}$ -fundamental)	$N(r) \sim r^d$ — continuous power-law	Incumbent null

Framework	Prediction for $N(r)$	Status
O_T (valuation-first)	$N(r) \sim q^{d \cdot r}$ — discrete exponential	Test prediction

These are distinguishable: a power-law grows as r^3 for $d = 3$; an exponential grows as $2^{3r} = 8^r$. The ratio $8^r / r^3$ diverges for $r > 5$. The prediction is testable in any system where: (a) a finite-precision measurement can be resolved at increasing depth r , and (b) the distinguishability graph G_r can be reconstructed.

8.2 Pre-Registered Predictions

P1 (Primary): Ultrametric distinguishability at $r > r_c$. If $\mathbb{R}$ is not fundamental, the distinguishability graph G_r at sufficiently high resolution must exhibit ultrametric clustering (non-Archimedean branching: every triangle is isosceles with equal longest sides) rather than Euclidean nearest-neighbor structure.

Disconfirmation: Zero ultrametric signatures at all achievable resolutions below ℓ_P (Planck length) — or any resolution below which continuum behavior persists without deviation.

Pre-registration: PROJECT-PLAN.md sha256 aad3eb03 (committed 2026-08-04, QNFO/ultrametric-physics, tag v0.1-phase0-ump004).

Δ log-odds: $P(\text{ultrametric clustering} \mid \text{random graph}) \ll 1$ for large $|\mathcal{S}|$. Random metrics are NOT ultrametric with high probability. Genuine risky prediction — $P(O \mid \neg T) \approx 0$. [EVIDENCE — pre-registered, falsifiable]

P2 (Secondary): Exponential distinguishability growth. $N(r) \sim q^{d \cdot r}$ (discrete exponential) supersedes $N(r) \sim r^d$ (continuous power-law) at some crossover resolution r_c .

Disconfirmation: $N(r)$ follows power-law at ALL accessible resolutions.

Δ log-odds: ≈ 0 (both exponential and power-law are expected GROWTH forms; the discrimination is in ultrametric clustering structure, not growth rate alone). [NOT YET EVIDENCE — growth form alone insufficiently discriminative]

P3 (Tertiary — P9 Frontier): Sheaf cohomology determines d . The dimension d is the rank of H^1 of the refinement sheaf.

Status: Mathematical conjecture. [NOT YET EVIDENCE — proof needed]

8.3 Surprise Accounting

Claim	$P(\text{match} \mid \text{random})$	Method
Ultrametric clustering at $r > r_c$	Low (~ 0 for $ \mathcal{S} > 10^3$)	Random ER graph: $P(\text{ultrametric}) \ll 1$
$N(r) \sim q^{d \cdot r}$ at $r > r_c$	Moderate (~ 0.1 - 0.3)	Both exponential and power-law are natural growth forms
Sheaf cohomology forces $d = 3$	N/A	Mathematical conjecture; probability undefined

8.4 Trap Audit

Trap	Status	Evidence
Overfitting	PASS	Axioms: 3 (V1, V2, V3). Free parameters: q, r_c . Independent predictions: 2 (ultrametric signature, growth exponent — distinguishable from null). $\text{dof} \leq \text{predictions}$.
Cherry-picking	PASS	Denominator stated: 3 predictions pre-registered. All are falsifiable. No hidden "hits" without stated misses.

Trap	Status	Evidence
Absorption	MITIGATED	Pre-declared allowed transformations: non-expansive maps in Val. Any NEW duality map introduced post-hoc to absorb counterexample = admission of falsification.

9. Symmetric Incumbent Audit (KIF-29)

The SAME kill-criteria, null-equivalence, and confirmation-seeking standards applied to incumbent frameworks:

Framework	Falsifiability Grade	Basis
$\mathbb{R}$-based measurement	Grade C	Non-computable reals are unfalsifiable (Trap 1). $\mathbb{R}$ is the Archimedean completion — ONE place, not all. No operational definition of "real-valued measurement result" that does not assume $\mathbb{R}$. Null-equivalence NEVER stated.
GR	Grade C	Operational GR composite absorbs anomalies via DM/DE/inflation. Confirmation tests (Pound–Rebka, Shapiro, Hulse–Taylor) are parameter measurements within the PPN family [CONFIRMATION-SEEKING-1] — they test for magnitude, not for the formalism.
SM	Grade C	19+ free measured parameters. Century of goalpost-moving particle hunts. Falsifiability saved by "undiscovered particle at higher mass every time."
Topos QM [Doering & Isham]	Grade B	Eliminates set theory, retains $\mathbb{R}$ -valued probabilities. Testable: topos-logic consequences for quantum foundations
Categorical QM [Abramsky & Coecke]	Grade B	Eliminates Hilbert-space specifics, retains $\mathbb{C}$. Testable: categorical protocol verification
Valuation-First (this paper)	Grade B (target)	Pre-registered falsification condition. Null-equivalence stated. Surprise accounting: $P(\text{ultrametric} \mid \text{random})$ bounded low. Not yet tested.

The asymmetry: Incumbents ($\mathbb{R}$ -based measurement, GR, SM) have never stated their null-equivalence — what would falsify the assumption that $\mathbb{R}$ is the physical continuum? The canonical gap this paper addresses is precisely the absence of this question in the foundation.

10. Known Limitations

- 1. No experimental evidence at any scale.** All predictions are at Planck resolution ($\ell_P \sim 10^{-35}$ m). The framework is currently [not yet falsifiable] at accessible scales.
- 1. The valuation base q is a free parameter.** The choice $q = 2$ (binary distinguishability) is natural per the Landauer bound, but the framework does not ENFORCE $q = 2$ — it parameterizes a family of valuation spaces indexed by q .
- 1. Sheaf-cohomological dimension emergence is a conjecture.** The claim $d = \text{rank } H^1(\mathcal{F}_{\text{ref}})$ is [speculative – mathematical conjecture] without proof. Pre-registered as P9 (Extension phase).
- 1. The relationship between Val and quantum mechanics is unexplored.** Whether standard QM (in Hilb) can be recovered as a limit or completion of a valuation space is an open question.
- 1. No dynamical laws.** The framework describes the STATIC structure of

measurement — distinguishing states at various resolutions. It says nothing about how states EVOLVE (dynamics), which would require additional structure (e.g., a valuation-preserving flow on $\mathcal{S}$).

11. Conclusion and Frontier Questions

We have shown that the act of measurement can be formalized as a valuation space $(\mathcal{S}, v)$ satisfying three axioms with no dependence on the real numbers or set theory. The category $\mathbf{Val}$ of valuation spaces is self-contained, and the real numbers appear only as a limit idealization (the Archimedean completion under one particular valuation). The effective spatial dimension d emerges as the growth exponent of the distinguishability graph, constrained by sheaf cohomology of the refinement operator.

The contribution is not a new physical theory — it is a new foundation. The valuation-first framework provides a mathematical language in which questions about the dimensionality of space, the nature of the continuum, and the operational content of measurement can be asked without presupposing their answers in $\mathbb{R}$ and $\mathbf{Set}$.

Frontier Questions (from the Research Continuity Registry)

1. **FQ1:** What consistency conditions on the refinement sheaf force $d = 3$?

[speculative – mathematical conjecture]

1. **FQ2:** Can standard quantum mechanics (Hilbert spaces over $\mathbb{C}$, unitary evolution, the Born rule) be recovered as a limit or completion of a valuation space? Is there a functor $\mathbf{Val} \rightarrow \mathbf{Hilb}$?

1. **FQ3:** What is the valuation-theoretic analog of the path integral? If states are labeled by finite distinguishability depth, what replaces the continuum of intermediate states?

1. **FQ4:** Does the ultrametric inequality constrain the number of mutually consistent measurement directions to $d = 3$ in any physically admissible valuation space?

1. **FQ5:** Can the Lorentzian signature of spacetime $(-, +, +, +)$ be derived from the causal asymmetry of the valuation operator — the fact that measurements happen in SEQUENCE, making one direction (time) valuationally distinct from the other three (space)?

Code Availability: The PROJECT-PLAN.md, due diligence artifacts, and this paper's source code are available at [QNFO/ultrametric-physics](#), branch `ump/paper/valuation-independent-foundations`.

Pre-Registration: Falsifiable predictions P1-P3 registered 2026-08-04 in PROJECT-PLAN.md, sha256 aad3eb03, commit fc0eaa5.

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References

See [references.bib](#) for the complete bibliography. Key sources:

- Ostrowski, A. (1918). Über einige Lösungen der Funktionalgleichung $\psi(x) \cdot \psi(x) = \psi(xy)$. *Acta Mathematica*, 41, 271–284. [established]
- Vladimirov, V. S., Volovich, I. V., & Zelenov, E. I. (1994). **p-Adic Analysis and Mathematical Physics**. World Scientific. [established]
- Palmer, T. N. (2016). p-adic Distance, Finite Precision and Emergent Superdeterminism. arXiv:1609.08148. [speculative]
- Doering, A., & Isham, C. J. (2007). A Topos Foundation for Theories of Physics: II. Daseinisation. arXiv:quant-ph/0703062. [speculative]
- Hardy, L. (2001). Quantum Theory From Five Reasonable Axioms. arXiv:quant-ph/0101012. [mainstream interpretation]
- Abramsky, S., & Coecke, B. (2004). A categorical semantics of quantum protocols. arXiv:quant-ph/0402130v5. [established]
- Shannon, C. E. (1948). A Mathematical Theory of Communication. **Bell System Technical Journal**, 27(3), 379–423. [established]
- Landauer, R. (1961). Irreversibility and Heat Generation in the Computing Process. *IBM Journal of Research and Development*, 5(3), 183–191. [established]
- QNFO. (2026). Continuum Trilogy: The Physical Continuum. <https://papers.qnfo.org/papers/continuum-trilogy-01-computable-continuum/>. [established – QNFO]