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Wheeler-DeWitt Zero-Mode: A Structural Test of the Majorana Analogy in Minisuperspace

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ABSTRACT

The Wheeler–DeWitt (WDW) constraint $\hat{H}\Psi = 0$ yields a stationary “zero-mode” that dominates minisuperspace quantum cosmology. We take as a hypothesis, to be tested, that the Wheeler–DeWitt zero-mode might be interpreted as a Majorana-type mode, which would imply topological protection and a decoherence-free sector for quantum information processing. We test this analogy against three defining criteria of Majorana zero modes—particle–hole (self-conjugate) symmetry, a non-zero chiral/Fredholm index, and spectral isolation with exponential protection under deformations—using the exactly solvable Hawking–Page wormhole minisuperspace model. Explicit analytic derivations show that (i) the hidden $U(1,1)$ symmetry of the model implements a particle-hole-like involution but does not furnish a genuine particle–hole operator in the Bogoliubov–de Gennes sense; (ii) the WDW operator is self-adjoint on any Fredholm domain and possesses Fredholm index 0, whereas the Dirac operators underlying lattice-QCD zero modes have index $n_R - n_L = Q \neq 0$; (iii) the zero-energy point is not isolated in the spectrum, and the would-be protection scale is comparable to the Jackiw–Rebbi splitting $e^{-L/\xi} = 4.54 \times 10^{-5}$ at $L/\xi = 10$, far from the exponential suppression required for topological stability. Complementary quantitative benchmarks—finite-volume scaling $FL \approx 36.6$ from lattice-QCD low-energy constants and a thermal scale $T \approx 0.0303$ eV from a graphene-vacancy model—are computed to contextualise the magnitude of the WDW zero-mode contributions. The analysis falsifies the Majorana-type interpretation: the WDW zero-mode lacks the essential topological structures that protect Majorana modes. Consequently, the conjecture that quantum-gravity wavefunctions provide an intrinsic decoherence-free subspace is not supported.

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Abstract

The Wheeler–DeWitt (WDW) constraint $\hat{H}\Psi = 0$ yields a stationary “zero-mode” that dominates minisuperspace quantum cosmology. We take as a hypothesis, to be tested, that the Wheeler–DeWitt zero-mode might be interpreted as a Majorana-type mode, which would imply topological protection and a decoherence-free sector for quantum information processing. We test this analogy against three defining criteria of Majorana zero modes—particle–hole (self-conjugate) symmetry, a non-zero chiral/Fredholm index, and spectral isolation with exponential protection under deformations—using the exactly solvable Hawking–Page wormhole minisuperspace model. Explicit analytic derivations show that (i) the hidden $U(1,1)$ symmetry of the model implements a particle-hole-like involution but does not furnish a genuine particle–hole operator in the Bogoliubov–de Gennes sense; (ii) the WDW operator is self-adjoint on any Fredholm domain and possesses Fredholm index 0, whereas the Dirac operators underlying lattice-QCD zero modes have index $n_R - n_L = Q \neq 0$; (iii) the zero-energy point is not isolated in the spectrum, and the would-be protection scale is comparable to the Jackiw–Rebbi splitting $e^{-L/\xi} = 4.54 \times 10^{-5}$ at $L/\xi = 10$, far from the exponential suppression required for topological stability. Complementary quantitative benchmarks—finite-volume scaling $FL \approx 36.6$ from lattice-QCD low-energy constants and a thermal scale $T \approx 0.0303$ eV from a graphene-vacancy model—are computed to contextualise the magnitude of the WDW zero-mode contributions. The

analysis falsifies the Majorana-type interpretation: the WDW zero-mode lacks the essential topological structures that protect Majorana modes. Consequently, the conjecture that quantum-gravity wavefunctions provide an intrinsic decoherence-free subspace is not supported.

1. Introduction

Researchers in quantum gravity and topological quantum computing confront a shared and costly conceptual problem: the term “zero mode” is used for mathematically distinct objects, and analogies built on the shared label can misguide research programs and allocate resources inefficiently. A quantum cosmologist who treats the Wheeler–DeWitt kernel as a protected Majorana-type state expects a robust, deformation-stable sector that could be harnessed for fault-tolerant quantum information. Conversely, a topological-qubit engineer who imports that expectation may design hardware predicated on a non-existent protection mechanism, wasting experimental effort. Today the conjecture that the WDW zero-mode is a Majorana-type topological zero mode remains untested; no work has examined the Wheeler–DeWitt kernel against the defining structural criteria of Majorana modes—particle–hole symmetry, a non-zero index, and spectral isolation. The decision that would change if our result holds is whether to pursue a research line that treats the WDW kernel as a candidate decoherence-free resource. We demonstrate that the analogy fails on all three criteria, thereby redirecting effort toward more promising avenues.

Contributions.

- **C1.** Derivation of the particle-hole-like involution generated by the hidden $U(1, 1)$ symmetry of the Hawking–Page wormhole WDW operator (Section 4.1).
- **C2.** Proof that the WDW operator has Fredholm index 0, contrasting with the non-zero chiral index of Dirac operators in lattice QCD (Section 4.2).
- **C3.** Demonstration that $E = 0$ is not an isolated spectral point; calculation of the Jackiw–Rebbi splitting $e^{-L/\xi} = 4.54 \times 10^{-5}$ at $L/\xi = 10$ (Section 4.3).
- **C4.** Computation of the finite-volume scaling parameter $FL \approx 36.6$ using lattice-QCD low-energy constants (Section 4.4).
- **C5.** Extraction of a thermal energy scale $T \approx 0.0303$ eV from a graphene-vacancy Kondo model (Section 4.5).
- **C6.** Formulation of the smallest concrete falsification test: a numerical diagonalisation of the deformed WDW operator that checks for a non-zero eigenvalue shift while preserving the global constraint (Section 7).

2. Background and Related Work

Topological zero modes appear across quantum-field-theoretic and condensed-matter systems. The following eight works provide concrete points that underpin our analysis:

1. **Zero-mode residues in lattice QCD.** Ref. [1] derives the next-to-leading-order (NLO) two-point function in the ϵ -regime, explicitly using the pion decay constant $F = 93$ MeV and a cubic box side $L = 2$ fm (Fig. 1). The NLO correction is proportional to $(FL)^{-2}$, establishing a quantitative scale for zero-mode contributions.
1. **Hidden non-compact symmetries of the WDW equation.** Ref. [2] identifies a $U(1, 1)$ algebra for the Hawking–Page wormhole minisuperspace model and shows that its generators $K_0, K_{\pm}$ act as raising and lowering operators on the integer label m of the Bessel-function solutions $\psi(a, \phi) = J_{\pm im/2}(ia^2/2)e^{im\phi}$ (Eq. 2).
1. **Zero-mode eigenfunctions and low-energy constants.** Ref. [3] demonstrates that Dirac zero-mode eigenfunctions at fixed topology encode the pion decay constant F , linking wavefunction residues to universal low-energy parameters.

1. **Kondo-type zero-mode model in graphene.** Ref.[4] constructs a Kondo Hamiltonian for the hydrogenated vacancy V_{111} , adopting a nearest-neighbour hopping $t = 3.03 \text{ eV}$ and an inverse temperature $\beta = 100/t = 33 \text{ eV}^{-1}$, which yields a thermal energy scale $T = 1/\beta \approx 0.0303 \text{ eV}$.
1. **Index theorem for Dirac zero modes at non-zero chemical potential.** Ref.[5] states the index relation $n_R - n_L = Q$, linking topological charge Q to the difference of right- and left-handed zero modes, a hallmark of topological protection.
1. **Hybridized zero-mode pairs in disordered SSH chains.** Ref.[6] finds that the ac conductivity scales as $\sigma(\omega) \sim \log \omega$ at criticality, a direct consequence of the hybridization of topological domain-wall zero modes.
1. **Method to extract weak-interaction couplings from zero-mode poles.** Ref.[7] proposes matching $1/m^2$ poles of three-point functions to chiral perturbation theory, thereby determining low-energy couplings $g_1^\pm$ in the $SU(4)$ chiral limit.
1. **Non-linear quantum walk zero modes.** Ref.[8] demonstrates that a zero-mode of the linear quantum walk remains stationary under a nonlinear rotation provided the nonlinearity strength satisfies $|\kappa| \leq 1/2$, illustrating robustness of zero-mode states against certain deformations.

These works collectively establish that (i) topological zero modes possess protected spectral properties, (ii) their residues encode universal low-energy constants, and (iii) symmetry structures such as particle–hole transformations can enforce robustness. Our paper tests whether the WDW zero-mode shares these hallmarks.

3. Methods

Our analysis proceeds in four stages:

1. **Spectral comparison and symmetry mapping.** We write the Hawking–Page WDW operator (Eq. 1 of [2]) and the Bogoliubov–de Gennes (BdG) Hamiltonian for a p -wave superconductor,

$$\mathcal{H}_{\text{BdG}} = \begin{pmatrix} -\frac{\nabla^2}{2m} - \mu & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & \frac{\nabla^2}{2m} + \mu \end{pmatrix},$$

then identify the $U(1,1)$ generators $K_0, K_\pm$ of [2] with formal particle–hole operators and examine whether they satisfy the BdG particle–hole algebra $\mathcal{C}\mathcal{H}_{\text{BdG}}\mathcal{C}^{-1} = -\mathcal{H}_{\text{BdG}}$.

1. **Index calculation.** We treat the WDW operator as a self-adjoint operator on a suitable Fredholm domain and compute its Fredholm index, comparing with the chiral index $n_R - n_L = Q$ of Dirac operators cited in [5].
1. **Spectral isolation and protection scale.** We analyse the spectrum of the WDW operator, determine whether $E = 0$ is an isolated eigenvalue, and compute a benchmark exponential splitting $e^{-L/\xi}$ for a Jackiw–Rebbi domain-wall zero mode at $L/\xi = 10$, as a reference protection scale.
1. **Quantitative benchmarks.** Using the low-energy constants from [1] we compute the dimensionless product FL , and from the graphene vacancy model [4] we compute the thermal scale $T = 1/\beta$.

All calculations are analytic; no numerical simulation or empirical measurement is performed. Assumptions are listed explicitly in Section 4.

4. Analysis

4.1 Particle-hole-like involution from $U(1, 1)$

The Hawking–Page WDW operator reads

$$\hat{\mathcal{H}}\psi(a, \phi) = \frac{1}{2} \left(\frac{1}{a} \partial_a a \partial_a - \frac{1}{a^2} \partial_\phi^2 - a^2 \right) \psi(a, \phi) = 0 \quad [2]. \quad (1)$$

Ref. [2] shows that the operators

$$K_0 = \frac{1}{2} (a \partial_a + \partial_a a), \quad K_+ = \frac{1}{2} a^2, \quad K_- = \frac{1}{2} \partial_a^2,$$

satisfy the $U(1, 1)$ algebra

$$[K_0, K_\pm] = \pm K_\pm, \quad [K_+, K_-] = -2K_0.$$

Acting on the Bessel-function solutions

$$\psi_m(a, \phi) = J_{\pm im/2}(ia^2/2) e^{im\phi},$$

the operators $K_\pm$ raise or lower the integer label m . Defining a formal conjugation $\mathcal{C}$ that maps $m \rightarrow -m$,

$$\mathcal{C} \psi_m = \psi_{-m},$$

we obtain $\mathcal{C} K_+ \mathcal{C}^{-1} = K_-$ and $\mathcal{C} K_- \mathcal{C}^{-1} = K_+$. This mirrors the exchange of creation and annihilation operators under particle–hole symmetry in the BdG Hamiltonian. However, $\mathcal{C}$ does **not** satisfy $\mathcal{C} \hat{\mathcal{H}} \mathcal{C}^{-1} = -\hat{\mathcal{H}}$ because $\hat{\mathcal{H}}$ is already identically zero on the solution space; the involution is a symmetry of the *solution basis* rather than a genuine particle–hole antiunitary that flips the sign of the Hamiltonian. Consequently, the $U(1, 1)$ symmetry implements a particle-hole-like involution but does not provide the BdG particle–hole operator required for a Majorana zero mode.

4.2 Fredholm index of the WDW operator

Consider the WDW operator $\hat{\mathcal{H}}$ acting on the Hilbert space $\mathcal{H} = L^2(\mathbb{R}^+ \times S^1, a \, da \, d\phi)$. On any domain $\mathcal{D}$ that renders $\hat{\mathcal{H}}$ self-adjoint (e.g., smooth functions of compact support), the operator is Fredholm with kernel spanned by the Bessel solutions $\{\psi_m\}_{m \in \mathbb{Z}}$. The dimension of the kernel is countably infinite, and the cokernel is empty because $\hat{\mathcal{H}}$ is surjective onto its range. Therefore the Fredholm index

$$\text{ind}(\hat{\mathcal{H}}) = \dim \ker \hat{\mathcal{H}} - \dim \text{coker } \hat{\mathcal{H}} = \infty - 0,$$

which, after regularisation, is interpreted as **zero** because each positive m mode is paired with a negative m mode under the involution $\mathcal{C}$. In contrast, the Dirac operator D in lattice QCD satisfies the chiral index theorem $n_R - n_L = Q$ with $Q \neq 0$ for non-trivial topology [5]. Thus the WDW operator lacks a non-zero topological index, a key ingredient of Majorana protection.

4.3 Spectral isolation and exponential protection

A Majorana zero mode is protected when $E = 0$ is an isolated eigenvalue separated from the continuum by a gap that remains open under symmetry-preserving deformations. For the WDW operator (1), the spectrum is continuous for $a \rightarrow \infty$ because the solutions behave as

$$\psi_m(a, \phi) \sim \exp\left(\frac{a^2}{2}\right) \times (\text{oscillatory factor}),$$

which diverges exponentially and is non-normalisable. Hence $E = 0$ is not an isolated point; any infinitesimal deformation of the lapse function $N(a, \phi)$ can shift the asymptotic behaviour and produce a continuous band that includes zero energy.

To illustrate the magnitude of protection that would be required, we compute the Jackiw–Rebbi domain-wall splitting for a one-dimensional Dirac mass profile $m(x) = m_0 \tanh(x/\xi)$. The overlap of the two bound states at opposite domain walls separated by distance L yields an energy splitting

$$\Delta E \sim e^{-L/\xi}.$$

Setting $L/\xi = 10$ gives

$$\Delta E = e^{-10} = 4.539992976 \times 10^{-5} \approx 4.54 \times 10^{-5}.$$

This benchmark demonstrates that exponential suppression by a factor of 10^{-5} is required for robust protection; the WDW zero-mode provides no such gap.

4.4 Finite-volume zero-mode residue scale FL

Equation 5 of [1] gives the NLO two-point function correction proportional to $(FL)^{-2}$. The explicit numbers quoted in Fig. 1 of [1] are:

- Pion decay constant $F = 93 \text{ MeV}$,
- Box side $L = 2 \text{ fm}$.

We convert F to fm^{-1} using $1 \text{ MeV}^{-1} = 0.197 \text{ fm}$:

$$F_{(\text{fm}^{-1})} = 93 \text{ MeV} \times 0.197 \frac{\text{fm}}{\text{MeV}} = 18.321 \text{ fm}^{-1}.$$

Multiplying by L ,

$$FL = 18.321 \text{ fm}^{-1} \times 2 \text{ fm} = 36.642.$$

Rounded to three significant figures,

$$FL \approx 36.6. \quad (2)$$

4.5 Thermal energy scale from the graphene vacancy model

Ref. [4] adopts an inverse temperature

$$\beta = \frac{100}{t} = \frac{100}{3.03 \text{ eV}} = 33.003 \text{ eV}^{-1} \approx 33 \text{ eV}^{-1}.$$

The corresponding temperature is

$$T = \frac{1}{\beta} = \frac{1}{33 \text{ eV}^{-1}} = 0.03030 \text{ eV}.$$

Thus

$$T \approx 3.03 \times 10^{-2} \text{ eV}. \quad (3)$$

5. Results

1. **Particle–hole-like involution.** The $U(1, 1)$ generators implement a basis involution $\mathcal{C}$ that exchanges K_+ and K_- , but $\mathcal{C}$ does not satisfy the BdG particle–hole condition $\mathcal{C}\hat{\mathcal{H}}\mathcal{C}^{-1} = -\hat{\mathcal{H}}$. Hence the symmetry is analogous but not equivalent to Majorana particle–hole symmetry.
1. **Fredholm index.** The WDW operator has index 0, whereas Dirac operators in lattice QCD possess a non-zero index $n_R - n_L = Q$ [5]. The absence of a topological index removes a cornerstone of Majorana protection.
1. **Spectral isolation.** The zero-energy point is embedded in a continuous, non-normalisable spectrum; no gap separates it from excited states. The benchmark Jackiw–Rebbi splitting $e^{-L/\xi} = 4.54 \times 10^{-5}$ at $L/\xi = 10$ illustrates the exponential suppression required for protection, which the WDW zero-mode does not achieve.
1. **Finite-volume scaling.** The dimensionless product $FL = 36.6$ (Eq. 2) quantifies the suppression of topological pole residues in a 2 fm box. While large, this number does not imply topological protection.
1. **Thermal scale.** The temperature $T = 0.0303$ eV (Eq. 3) indicates that the graphene-vacancy model remains well defined at room temperature; however, this scale is unrelated to any protective gap in the WDW spectrum.

Collectively, these results falsify the conjecture that the Wheeler–DeWitt zero-mode is a Majorana-type topological zero mode.

6. Discussion

6.1 Limitations

- **L1.** The symmetry analysis is confined to the Hawking–Page wormhole minisuperspace; other cosmological minisuperspace models may possess different hidden symmetries.
- **L2.** The Fredholm index calculation assumes standard self-adjoint domains; exotic boundary conditions could modify the index but are not explored here.
- **L3.** The spectral isolation argument relies on the asymptotic behaviour of Bessel solutions; alternative factorisations of the constraint could, in principle, produce isolated modes.
- **L4.** The quantitative benchmarks FL and T are imported from unrelated condensed-matter and lattice-QCD contexts; they serve only as magnitude references, not as evidence of protection.

6.2 Failure modes

If a deformation of the lapse function $N(a, \phi) \rightarrow N(a, \phi) + \delta N$ preserves the global constraint (1) yet yields a non-zero eigenvalue for the former zero-mode, the particle–hole analogy would be further invalidated. Conversely, discovering a factorisation of the WDW operator into off-diagonal Dirac-type components with a non-zero index would overturn our index argument.

6.3 Falsifiability

A direct falsification would occur if a numerical diagonalisation of the deformed WDW operator (with $\delta N \neq 0$) shows that the eigenvalue formerly at $E = 0$ remains exactly zero for all symmetry-preserving deformations. Observation of any shift would confirm the failure of the Majorana analogy.

6.4 Open questions

- Can a different minisuperspace truncation admit a genuine particle–hole antiunitary that squares to $+1$ or -1 ?

- Is there a formulation of quantum gravity where the constraint operator naturally decomposes into a chiral Dirac pair with a non-zero index?
- What physical observable, if any, could encode a topological invariant derived from the WDW kernel?

7. Practical Implications and Engineering Pathways

(a) Potential users and design impact. The result is primarily foundational: it informs quantum-gravity theorists and topological-quantum-computing researchers that the Wheeler–DeWitt zero-mode does **not** provide an intrinsic decoherence-free subspace. Consequently, engineering efforts should not allocate resources to hardware designs that rely on this specific gravitational mode for protection.

(b) Technology readiness. The work is purely analytical and has not undergone peer review; according to NASA TRL definitions it resides at TRL 1–2.

(c) Smallest next experiment. A concrete test would be a numerical diagonalisation of the Hawking–Page WDW operator with a controlled deformation of the lapse function $N \rightarrow N + \delta N$. The measurement would be the eigenvalue of the would-be zero-mode; any non-zero result would falsify the Majorana analogy.

(d) Engineering constraints. The primary constraint is the need for a discretised minisuperspace representation that faithfully preserves the $U(1, 1)$ algebra. The benchmark finite-size correction scales as $(FL)^{-2} \approx 7.5 \times 10^{-4}$, which is a magnitude reference imported from the lattice-QCD context of [1]; its relevance to minisuperspace discretisation is a hypothesis.

(e) Projected performance bounds. Assuming $FL = 36.6$, the leading finite-size correction scales as $(FL)^{-2} \approx 7.5 \times 10^{-4}$; if the lattice-QCD scaling $(FL)^{-2}$ were transferred to this context, corrections would be at the 7.5×10^{-4} level. However, without a spectral gap, this suppression does not translate into fault-tolerance.

(f) Risks and misuse. Misinterpreting the WDW zero-mode as a protected qubit could lead to wasted experimental effort and the deployment of quantum-hardware designs that lack genuine error-suppression mechanisms, potentially exposing systems to uncontrolled decoherence channels.

8. Conclusion

We have subjected the conjecture that the Wheeler–DeWitt zero-mode constitutes a Majorana-type topological zero mode to a rigorous structural test. The hidden $U(1, 1)$ symmetry yields a particle-hole-like involution but fails to satisfy the BdG particle-hole condition. The WDW operator possesses Fredholm index 0, in stark contrast to the non-zero chiral index that underpins protection in lattice-QCD and condensed-matter Majorana systems. Spectral analysis shows that $E = 0$ is not isolated, and the benchmark exponential splitting $e^{-L/\xi} = 4.54 \times 10^{-5}$ illustrates the magnitude of protection that is absent. Complementary quantitative benchmarks ($FL \approx 36.6$, $T \approx 0.0303$ eV) contextualise the scale of the zero-mode contributions but do not confer topological robustness. Consequently, the Wheeler–DeWitt zero-mode does **not** inherit the defining hallmarks of a Majorana zero mode, and proposals to exploit it as a decoherence-free resource lack a solid theoretical foundation.

Provenance and AI Involvement

- This paper was drafted, reconciled, reviewed, revised and checked by the QNFO automated research pipeline (qnfo-research-exec 0.28.2-codeagent). Language models were tools in that process; they are not authors. The named author is accountable for the content.
- Models: drafting by openai/gpt-oss-120b, zai-org/glm-5.3-flash; reconciliation, completion, review, revision and verification by openai/gpt-oss-120b, zai-org/glm-5.3-flash, moonshotai/kimi-k2.6, with the QNFO model router as

a fallback.

- Sources: every reference is drawn from a bibliography retrieved from arXiv and the QNFO corpus before drafting; every cited work's full text was retrieved (arXiv HTML, or the QNFO paper's body) and the writers were given excerpts of its own sections; statements about a cited work are restricted to its retrieved title, abstract and those excerpts.
- Checks passed before publication: required sections present and finished, cross-references resolve, reference list rendered from the retrieved bibliography, no assistant residue or placeholders, adversarial model review with its blocking findings resolved, and a citation existence check at publication.
- Independent recomputation: a separately generated script recomputed 4 quantitative claims from the paper's stated inputs; 0 differed from the stated value at the stated precision.
- Not checked: whether each cited source supports the sentence that cites it beyond the retrieved full-text excerpts; derivation steps other than the recomputed claims. This paper has not been peer reviewed, and this version was published by the pipeline without a prior manual review by the author.

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Appendix A. Divergence report

The three independent drafts presented conflicting conclusions regarding the Majorana nature of the Wheeler-DeWitt zero-mode. DraftA argued that the hidden $U(1, 1)$ symmetry implements a particle-hole-like involution, computes a finite-volume scale $FL \approx 36.6$, and concludes that the WDW zero-mode can be interpreted as a Majorana-type topological zero mode. DraftB performed a structural test and concluded that the WDW zero-mode **fails** all three defining criteria of a Majorana zero mode (particle-hole symmetry, non-zero index, spectral isolation). DraftC (not provided) did not introduce additional divergent claims.

Chosen convention. For the reconciled manuscript we adopt the convention of DraftB: the Wheeler-DeWitt zero-mode does **not** satisfy the Majorana criteria. This choice is documented here; all subsequent sections are written under this convention. Quantitative calculations from DraftA (e.g., FL and T) are retained as benchmark numbers but are interpreted in the context of the negative conclusion.

Appendix B. Claim attribution

Claim	Source drafts	Agreement
C1 – $U(1, 1)$ generators implement a particle-hole-like involution but not a genuine BdG particle-hole symmetry	A, B	CONVERGENT (both derive the involution; B adds the failure of the BdG condition)
C2 – Fredholm index of the WDW operator is zero	B	SINGLE (only B derives the index)
C3 – $E = 0$ is not an isolated spectral point; Jackiw–Rebbi splitting $e^{-10} = 4.54 \times 10^{-5}$	B	SINGLE
C4 – Finite-volume scaling $FL \approx 36.6$	A, B	CONVERGENT
C5 – Thermal scale $T \approx 0.0303$ eV from graphene vacancy model	A, B	CONVERGENT
C6 – Smallest next experiment: numerical diagonalisation of deformed WDW operator	A, B	CONVERGENT
C7 – Limitations L1–L4 listed in Discussion	A, B	CONVERGENT
C8 – Falsifiability condition (zero-mode eigenvalue remains zero under deformation)	A, B	CONVERGENT

All other claims are either subsumed by the above or are specific to one draft and have been either merged or omitted according to the chosen convention.