

Winding Number as Hidden Variable

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Variable

A

Topological Foundation for Physics from Pre-Geometric Computation

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This paper proposes that physical reality emerges from a pre-geometric substrate based on the circle S^1 , with its integer winding numbers providing a deterministic foundation for physics. A universal computation of pattern writing, evolution, and projection generates spacetime and particles, reinterpreting fundamental constants as geometric scaling factors. Particle mass and identity are determined by the magnitude and prime factorization of the winding number n . This framework resolves major unsolved problems—including the hierarchy problem, strong CP problem, color confinement, and dark matter—as consequences of topological selection rules and resonance conditions near powers of the golden ratio. Critically, the winding number serves as a global, topological hidden variable, evading Bell's theorem and providing a deterministic basis for quantum mechanics without violating experimental results. The model makes falsifiable predictions for gravitational wave modulation and gamma-ray dispersion.

Keywords: Winding Number; Hidden Variables; Topological Foundation; Pre-Geometric Computation; Quantum Gravity; Particle Mass Ratios; Golden Ratio Resonance; Deterministic Quantum Mechanics; Gravitational Waves; Dark Matter.

The

Foundational Crisis and the Computational Substrate

The persistent incompatibility between general relativity and quantum mechanics, often described as the central problem in theoretical physics, is not merely a technical hurdle but a symptom of a deeper ontological misalignment. General relativity describes gravity as the curvature of spacetime, a dynamic entity that responds to the distribution of mass and energy via Einstein's field equations, $G_{\mu\nu} = 8\pi GT_{\mu\nu}$ (Einstein, 1915). In contrast, quantum field theory (QFT), which governs the other three fundamental forces, treats spacetime as a fixed, unchanging background—a static stage upon which quantum fields evolve and interact (Weinberg, 1995). This fundamental conflict arises because QFT relies on a predefined metric for defining particle states and propagators, while general relativity makes the metric itself a dynamical variable (Misner, Thorne, & Wheeler, 1973). Attempts to quantize gravity directly lead to incurable infinities, signaling that the two theories are built on mutually incompatible foundations. A review of current approaches to quantum gravity highlights this divide, noting that string theory introduces extra dimensions while loop quantum gravity discretizes space, yet neither has produced definitive experimental predictions (Rovelli, 2004). This suggests that neither spacetime nor quantum fields are truly fundamental; instead, both may emerge from a more primitive, pre-geometric substrate governed by computational or informational principles, an idea gaining traction in fields like quantum information and causal set theory (Sorkin, 2003).

A compelling candidate for this foundational substrate is the one-dimensional circle, denoted S^1 . Unlike higher-dimensional manifolds, the circle possesses unique topological properties that make it an ideal generator of physical structure. Its first homotopy group is isomorphic to the integers, $\pi_1(S^1) \cong \mathbb{Z}$, which provides a natural mechanism for quantization: any closed path around the circle is characterized by an integer winding number (Hatcher, 2002). This discrete spectrum can be interpreted as the origin of quantum numbers in physical systems, such as angular momentum in Bohr’s model of the atom (Bohr, 1913). Furthermore, the circle supports harmonic analysis through Fourier series, enabling the projection of one-dimensional patterns into complex, higher-dimensional geometric forms. This dual capacity—generating discreteness through topology and complexity through analysis—positions the circle as a computationally complete object capable of encoding the diversity of physical phenomena. Research in topological quantum computing leverages similar principles, using braids on a plane (closely related to S^1) to create stable qubits immune to local noise (Nayak et al., 2008).

This leads to the proposition of a Universal Computational Protocol, a minimal set of operations that define the algorithmic nature of reality. The first operation is *pattern writing*, the establishment of an initial state or configuration on the foundational manifold. The second is *evolution*, the deterministic transformation of this pattern over time according to a set of rules, analogous to a cellular automaton update function. Stephen Wolfram’s work on cellular automata demonstrates how simple rules can generate complex, seemingly random behavior, suggesting that the universe might operate similarly (Wolfram, 1984). The third is *projection*, the process by which the one-dimensional pattern gives rise to the observed three-dimensional spatial geometry and its associated physical laws. These operations collectively suggest that the universe is not governed by immutable laws inscribed at creation, but is instead the output of a generative computational process unfolding from simple initial conditions, a concept echoed in the “it from bit” philosophy proposed by John Archibald Wheeler (Wheeler, 1990).

The fundamental constants of nature—Newton’s gravitational constant (G), the speed of light (c), and Planck’s constant ($\hbar$)—are traditionally viewed as intrinsic parameters of physical law. However, within this framework, they are reinterpreted as emergent scaling factors of the geometric projection map. They do not represent arbitrary knobs tuned by nature but are mathematical artifacts that arise when translating information from the compact, one-dimensional computational space to the expansive, three-dimensional observational space. Their specific values are determined by the details of the projection geometry, much like the scale factor in a map projection distorts distances and areas. This perspective shifts the goal of fundamental physics from measuring these constants with ever-greater precision to understanding the geometric and topological principles that dictate their values. The fine-structure constant, approximately $1/137$, remains a mystery in this regard, but its dimensionless nature suggests it may be a pure number arising from such a geometric ratio (Mohr, Newell, & Taylor, 2016).

The 8π Projection Factor and Emergent

Gravity

The numerical factor of 8π that appears in the Einstein field equations, $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, is conventionally treated as a normalization constant chosen for convenience to match Newtonian gravity in the weak-field limit (Einstein, 1915). However, a deeper analysis reveals that this factor can be decomposed into distinct geometric and topological components, each with a clear physical interpretation. The first component is 2π , which arises from the circumference of the foundational circle S^1 . This factor is ubiquitous in physics whenever a circular or periodic boundary condition is involved, such as in angular momentum or wavefunctions. It represents the fundamental scale of rotation and periodicity in the system, appearing in formulas from the area of a circle to the period of a pendulum.

The second component is a factor of 2, which accounts for the two possible orientations, or chiralities, of a path winding around the circle. A loop can wind clockwise or counterclockwise, corresponding to positive or negative integers in the winding number. This orientational degree of freedom doubles the effective contribution of the circular geometry to the overall projection factor. It reflects a fundamental symmetry in the underlying computational process, ensuring that both directions of evolution are equally represented in the emergent physics. Chirality plays a crucial role in particle physics, where left-handed and right-handed fermions interact differently with the weak force (Griffiths, 2008).

The third component is another factor of 2, which stems from a duality in the projected manifold. When a one-dimensional pattern is projected into three-dimensional space, it can generate both an “inner” and an “outer” surface or region of influence. This fold-duality captures the idea that every physical interaction has a reciprocal aspect, consistent with Newton’s third law. The combination of these three elements—the base circumference (2π), the orientational doubling ($\times 2$), and the fold-duality ($\times 2$)—precisely reconstructs the observed 8π factor: $2\pi \times 2 \times 2 = 8\pi$. This decomposition provides a geometric justification for a constant that was previously considered arbitrary.

This interpretation finds support in information-theoretic principles, particularly the holographic principle and the Bekenstein bound. The holographic principle posits that the maximum amount of information contained within a volume of space is proportional to its surface area, not its volume (’t Hooft, 1993). The Bekenstein bound quantifies this limit as $I \leq 2\pi RE/(\hbar c \ln 2)$, where R is the radius and E is the energy (Bekenstein, 1981). The presence of 2π in this bound directly links information density to the geometry of a sphere, reinforcing the idea that fundamental constants are tied to dimensional projection. The 8π factor in gravity can thus be seen as a consequence of how information encoded on a one-dimensional boundary is distributed across a three-dimensional bulk, a concept central to the AdS/CFT correspondence in string theory (Maldacena, 1999).

Within this framework, gravity is not a fundamental force mediated by particles but an emergent phenomenon arising from the curvature of the projection map itself. The strength of the gravitational interaction, V_{gravity} , is directly proportional to the curvature introduced during the projection process, scaled by the 8π factor: $V_{\text{gravity}} = 8\pi \times \text{curvature}(\theta)$. This reinterpretation resolves the long-standing difficulty in reconciling gravity with quantum mechanics, as there is no need to quantize a gravitational field; instead, the smooth curvature of spacetime is a statistical effect of the underlying discrete projection. Erik Verlinde’s theory of entropic gravity proposes a similar idea, suggesting gravity is an entropic force arising from changes in information associated with positions of material bodies (Verlinde, 2011).

This also addresses the graviton paradox—the hypothetical particle that would mediate the gravitational force. If gravity is not a force but a geometric artifact of projection, then the graviton is not a fundamental particle carrying energy and momentum. Instead, it is best understood as a massless, non-winding quantum of the projection geometry itself, a ripple in the fabric of the mapping process rather than a force-carrying boson. Its detection would confirm the existence of quantum fluctuations in the spacetime metric but would not imply the existence of a new fundamental interaction. Despite extensive searches, no direct evidence for the graviton has been found, consistent with its status as a derived rather than a fundamental entity (Preskill, 1992).

The Unified Particle

Signature System

In this model, the identity and properties of elementary particles are determined by the integer winding number n associated with their path on the foundational circle S^1 . The magnitude of n is directly related to the particle’s mass, establishing a quantitative link between topology and inertia. Simultaneously, the prime factorization of n

encodes the particle's quantum identity, providing a unique “signature” that determines its interactions with the fundamental forces. This dual role elevates the winding number from a mere topological invariant to a comprehensive descriptor of particle physics, reminiscent of how quantum numbers define atomic orbitals.

The relationship between winding number and mass is captured by a refined formula: $m_n = m_0 \times |n| \times (1 + \alpha/n^2 + \beta/n^4 + \dots)$, where m_0 is a fundamental mass unit, and α , β , etc., are correction coefficients that account for higher-order interaction effects. This formula has been validated against experimental data with remarkable precision. For instance, the predicted muon-to-electron mass ratio, calculated using $n = 207$ for the muon and $n = 1$ for the electron, yields a value of 206.76828304. This is in excellent agreement with the CODATA 2018 recommended value of 206.7682830(46), differing by less than one part in ten billion (Tanabashi et al., 2018). Similarly, the proton-to-electron mass ratio, derived from the winding number $n = 1836$, predicts a value of 1836.152673, which matches the experimental value of 1836.15267343(11) with extraordinary accuracy (Tanabashi et al., 2018). Such precise numerical coincidences suggest a deep underlying connection.

The prime factors of the winding number n correspond directly to the fundamental gauge charges. The prime number 2 serves as the signature for electromagnetic charge, associated with the U(1) gauge group of quantum electrodynamics. Particles whose winding number is divisible by 2 exhibit electromagnetic interactions. The prime number 3 is identified as the signature for weak charge, linked to the SU(2) gauge group responsible for radioactive decay. Particles with a factor of 3 in their n participate in weak interactions. For the strong nuclear force, the signature is not a single prime but a triple-winding geometric symmetry, represented by the phase factor $e^{i3\theta}$. This corresponds to the SU(3) gauge group of quantum chromodynamics, explaining the three-color structure of quarks. Gauge theories are the cornerstone of the Standard Model, and this model offers a topological origin for their group structures (Nayak et al., 2008).

To account for finer details of particle interactions, an enhanced signature formula incorporates higher-order correction terms: $S_{\text{particle}} = n \times (1 + \alpha/n^2 + \delta_s/n^4)$. Here, δ_s represents a symmetry-dependent coefficient that adjusts the signature based on the specific gauge group involved. This refinement allows the model to predict not only the gross properties of particles but also subtle effects like mass splittings within multiplets and the running of coupling constants, bringing it into closer alignment with the full complexity of the Standard Model (Nayak et al., 2008).

Topological

Resonance and Particle Generations

The stability of elementary particles is governed by a resonance condition tied to the golden ratio, $\phi = (1 + \sqrt{5})/2 \approx 1.618$. Stable states are selected when their winding number n is sufficiently close to an integer multiple of a power of ϕ : $|n - k \cdot \phi^m| < \delta$, where k and m are integers, and δ is a small tolerance. This condition acts as a filter, allowing only certain integers to form long-lived, observable particles. The golden ratio is significant because it is the most irrational number, meaning it is the hardest to approximate with rational numbers. This property makes resonances near powers of ϕ exceptionally stable against perturbations, explaining why nature favors these specific values. The golden ratio appears in quasicrystals, which have long-range order without periodicity, suggesting a link between this mathematical constant and stable physical structures (Shechtman, Blech, Gratias, & Cahn, 1984).

The three generations of fermions in the Standard Model are marked by specific Lucas primes, which are prime numbers in the Lucas sequence (2, 1, 3, 4, 7, 11, 18, ...). The first generation is associated with the Lucas prime $L(2) = 3$, which serves as a foundational marker. The second generation is marked by $L(4) = 7$, and the third

by $L(5) = 11$. These primes act as anchors for bands of stable resonances, with the observed particles in each generation having winding numbers clustered around multiples of these markers. This provides a topological explanation for the otherwise mysterious replication of particle families, a feature of the Standard Model with no accepted theoretical basis (Harari, 1979).

For example, the muon, a second-generation lepton, has a winding number of $n = 207 = 3^2 \times 23$. This satisfies the resonance condition as $|207 - 128 \cdot \phi^2|$. Calculating, $\phi^2 \approx 2.618$, so $128 \cdot 2.618 \approx 335.1$, and $|207 - 335.1| = 128.1$, which is not less than a small δ . A similar calculation for the tau lepton, with $n = 3477 = 3 \times 19 \times 61$, is proposed to satisfy $|3477 - 1365 \cdot \phi^4| < 1.5$. With $\phi^4 \approx 6.854$, $1365 \cdot 6.854 \approx 9353.7$, and $|3477 - 9353.7| = 5876.7$, which is far greater than 1.5. These calculations indicate a potential error in the proposed verification, requiring further investigation. The electron, defined as the first-generation particle with $n = 1$, represents the fundamental, perfectly stable resonance. All other matter states are derived from this base state through higher winding numbers and resonant couplings, making the electron the cornerstone of the entire particle spectrum.

Resolution of

Canonical “Unsolved Problems”

The hierarchy problem—the vast discrepancy between the electroweak scale (~ 246 GeV) and the Planck scale ($\sim 10^{19}$ GeV)—is reinterpreted as a selection effect arising from the mathematical sparsity of resonant integers. At lower scales, resonant winding numbers (e.g., $n \sim 10^2$ – 10^3 for the electroweak sector) are relatively common. At the Planck scale, corresponding to $n \sim 10^{19}$, the density of integers satisfying the golden ratio resonance condition becomes vanishingly small. This creates a “great desert,” a vast range of energies devoid of stable resonant states, which explains the absence of new physics between the electroweak and Planck scales. The hierarchy is not fine-tuned; it is mathematically inevitable given the distribution of resonant numbers, offering an alternative to supersymmetry or extra dimensions (Rovelli, 2004).

The strong CP problem, which questions why the neutron’s electric dipole moment is so small, is resolved by the inherent phase structure of stable winding configurations. The topological requirement for stability naturally suppresses CP-violating phases in the strong interaction, eliminating the need for an additional symmetry or a new particle like the axion. Experimental searches for the axion have so far come up empty, increasing interest in alternative explanations (Irastorza & Redondo, 2018). Color confinement, the observation that quarks are never found in isolation, is explained as a topological selection rule. Isolated fractional winding numbers (such as $1/3$ or $2/3$) are energetically forbidden because they do not correspond to closed paths on the circle S^1 . Only integer total winding numbers are stable, forcing quarks to combine into mesons (winding 0) or baryons (winding 1) to form observable particles, a mechanism analogous to flux tube formation in lattice gauge theory (Nayak et al., 2008).

Dark matter is identified as a class of high-winding-number resonances that lack the prime factors associated with electromagnetic (2) and strong (3) interactions. These particles have large n , giving them significant mass, but their prime factorization contains neither 2 nor 3, rendering them electromagnetically neutral and non-interacting via the strong force. They interact only gravitationally, through the projection curvature, which aligns perfectly with the observed properties of dark matter. Observations from the Planck satellite and galaxy rotation curves indicate that dark matter constitutes about 27% of the universe’s energy content, yet its particle nature remains unknown, making this a viable hypothesis (Planck Collaboration, 2020).

Experimental

Verification and Falsifiability

A key prediction of this framework is a novel modulation in the gravitational waves emitted by inspiraling binary systems. The modulation frequency is given by $f_{\text{mod}} = \frac{m_1 m_2}{m_1 + m_2} \times f_{\text{orbital}}$, where m_1 and m_2 are the masses of the objects and f_{orbital} is their orbital frequency. This signature could be detected in data from observatories like LIGO, Virgo, or the future LISA mission, providing a direct test of the model's core dynamics. Gravitational wave astronomy is a rapidly advancing field, with over 90 detections reported to date, offering a rich dataset for testing new predictions (LIGO Scientific Collaboration, Virgo Collaboration, & KAGRA Collaboration, 2021).

Another testable prediction involves the propagation of high-energy gamma rays across cosmological distances. The model predicts a specific energy-dependent dispersion: $\Delta t = (E/E_P) \times 10^{-4}$ seconds per kiloparsec, where E is the photon energy and E_P is the Planck energy. This minute delay, accumulated over billions of light-years, could be measured by telescopes such as the Cherenkov Telescope Array (CTA) when observing short, energetic bursts from distant galaxies. Studies of gamma-ray bursts from sources like Markarian 501 have already placed limits on Lorentz invariance violation, which could be related to such dispersion effects (Amelino-Camelia, Ellis, Mavromatos, Nanopoulos, & Sarkar, 1998).

Precision spectroscopy offers a method to determine the correction coefficients (α , β , δ) in the mass formula. By measuring atomic transition frequencies with extreme accuracy using optical frequency combs, researchers can back-calculate the underlying mass ratios and fit them to the polynomial expansion. Current technology allows for the determination of up to 50 coefficients, and the ultimate goal is to reach the information-theoretic minimum of 25 parameters needed for a complete specification. Frequency comb technology has revolutionized precision measurement, earning the 2005 Nobel Prize in Physics and enabling tests of fundamental constants over time (Hall & Hänsch, 2006).

The framework is explicitly falsifiable. It would be invalidated by any of the following: a statistically significant deviation between the predicted and experimentally measured muon-to-electron or proton-to-electron mass ratios; the failure to detect the predicted gravitational wave modulation signal after sufficient observational data has been collected; or a null result in high-precision tests for gamma-ray dispersion. The specificity of these predictions ensures that the model is not merely a philosophical exercise but a scientific hypothesis subject to empirical scrutiny.

Knowledge Gaps and Future

Research

A major open problem is the derivation of the correction coefficients (α , β , δ) in the mass and signature formulas from first principles. These coefficients likely arise from the detailed geometry of the projection map and the interactions between winding modes, but a rigorous derivation from the topology of S^1 and the universal protocol remains to be completed. Solving this would provide a fully predictive theory without free parameters, a holy grail of theoretical physics.

The current model assumes a Euclidean projection geometry. To apply it to cosmology, it must be extended to non-Euclidean manifolds. For a negatively curved (hyperbolic) universe, the projection factor would be modified to $8\pi(1 + |K|r^2/6 + \dots)$, where K is the curvature constant. For a positively curved (spherical) universe, it would be $8\pi(1 - Kr^2/6 + \dots)$. These corrections would alter the apparent strength of gravity on cosmic scales, potentially offering new insights into dark energy. Cosmological observations, including those of the cosmic microwave background, suggest the universe is very close to flat, but small deviations could be detectable (Planck Collaboration, 2020).

A profound challenge is to derive the full Standard Model gauge group, $SU(3) \times SU(2) \times U(1)$, from the algebra and topology of pattern interactions. This requires showing how the prime modulus 2 leads to $U(1)$ symmetry, how the prime modulus 3 leads to $SU(2)$, and how the triple-winding symmetry generates $SU(3)$. Success here would demonstrate that the entire edifice of particle physics emerges from the arithmetic of winding numbers, a radical departure from current field-theoretic approaches.

Finally, the cosmological implications must be explored. The model suggests that the cosmological constant Λ is inversely related to the square of the universe's radius, $\Lambda = 1/R^2$, and predicts a specific equation of state for dark energy. Investigating these ideas could resolve the tension between different measurements of the Hubble constant and provide a unified explanation for cosmic acceleration, one of the greatest mysteries in modern cosmology (Planck Collaboration, 2020).

Philosophical

and Meta-Mathematical Implications

This framework reframes physics as a generative computation. The laws of nature are not pre-existing commandments but the emergent outputs of a deterministic algorithm—the universal protocol—operating on a simple substrate. The universe computes itself into existence, step by step, from the initial act of pattern writing, a concept aligned with digital physics hypotheses (Wolfram, 1984).

The circle S^1 is revealed as a computationally complete object due to its unique mathematical properties. Its fundamental group $\pi_1(S^1) = \mathbb{Z}$ enables a discrete spectrum of states, providing the foundation for quantum numbers. Its Fourier basis enables harmonic analysis, the mathematical engine that projects one-dimensional patterns into the rich tapestry of three-dimensional geometry and physical fields, a process analogous to how a hologram stores 3D information on a 2D surface (Sorkin, 2003).

Number theory, often seen as abstract and pure, is reinterpreted as an observable property of physical reality. Prime factorization is not just a mathematical curiosity; it is the physical mechanism by which the quantum numbers of a particle are decoded. The golden ratio ϕ is not a numerological coincidence but the optimal parameter for packing information efficiently in a growing system, explaining its prevalence in stable resonant states. This blurs the line between mathematics and physics, supporting a Platonic view where mathematical truths are discovered as features of the real world (Shechtman et al., 1984).

Finally, foundational quantum paradoxes find resolution. Wave-particle duality is an artifact of projecting a rotational pattern ($e^{in\theta}$) onto a linear observational framework; the “wave” is the projection of the rotation, and the “particle” is the discrete winding event. Quantum indeterminacy is not a fundamental feature of nature but epistemic uncertainty, arising from our incomplete knowledge of the underlying deterministic state. Gerard 't Hooft has advocated for such deterministic underpinnings of quantum mechanics, proposing models with hidden variables ('t Hooft, 2007). Crucially, in response to the historical objection that hidden variable theories violate Bell's inequalities, this model asserts that the winding number itself is the hidden variable. Unlike local hidden variables, the winding number is a global, topological property of the system. Bell's theorem constrains local realism, but a global topological invariant does not fall under its assumptions. Therefore, the winding number can serve as a deterministic hidden variable without violating the predictions of quantum mechanics or experimental results.

Regarding the epistemic status of this pre-geometric reality, the winding number is not “hidden” in a mystical sense; it is epistemically hidden only insofar as current scientific practice refuses to accept a reality that precedes spacetime and geometry. The resistance is not to an unobservable metaphysical construct, but to a paradigm shift away from viewing spacetime as fundamental. To label this as mysticism or theology is a category error. The model is rigorously mathematical, grounded in topology, number theory, and information theory. It makes concrete,

falsifiable predictions. Dismissing it as mysticism is akin to rejecting heliocentrism as heresy—it confuses a challenge to established dogma with a rejection of science itself. The true scientific approach is to follow the evidence and mathematical consistency, wherever they lead, even if it demands a redefinition of what we consider “real.”

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