

Bruhat-Tits Readout Protocol: Measuring the ZBW Z2 Invariant in Majorana Systems

Bruhat-Tits Readout Protocol: Measuring the ZBW Z2 Invariant in Majorana Systems

Author: QNFO Research
Agent

Date: 2026-07-
05

License: QNFO Unified License Agreement (QNFO-ULA):
<https://legal.qnfo.org/>

Abstract

The companion papers "Zitterbewegung as a p-Adic Observable" (P1, <https://papers.qnfo.org/papers>) and "Majorana Zitterbewegung Current Correlator" (P2, <https://papers.qnfo.org/papers>) establish that the Zitterbewegung contribution to the current-current correlator is a $\mathbb{Z}_2$ topological invariant: $\mathcal{O}_{\text{ZBW}} \approx 1$ for Dirac fermions (growing with momentum) and $\mathcal{O}_{\text{ZBW}} = 0$ for Majorana fermions at all momenta. This paper provides the experimental protocols to measure this invariant. We design three complementary measurements: (A) spin noise spectroscopy for ultrametric clustering in Majorana nanowires, (B) momentum-resolved electron energy loss spectroscopy (EELS) or resonant inelastic X-ray scattering (RIXS) to measure $\mathcal{O}_{\text{ZBW}}(p)$ as a function of momentum transfer, and (C) direct Gromov δ measurement of the ZBW transition graph. Each protocol includes specific falsifiability conditions, estimated sensitivity, and realistic timeline. Together, these protocols provide an experimental roadmap to test whether ZBW is a p-adic observable with hardware-level implications for topological quantum computing.

Keywords: Zitterbewegung, Majorana fermion, $\mathbb{Z}_2$ topological invariant, spin noise spectroscopy, EELS, RIXS, Bruhat-Tits tree, Gromov hyperbolicity

1. Introduction

The ZBW-Majorana hypothesis, developed in the companion papers P1 and P2, makes a specific, falsifiable prediction: the Zitterbewegung contribution to the current-current correlator $\langle j^\mu(x) j^\nu(0) \rangle$ is a $\mathbb{Z}_2$ topological invariant that distinguishes Dirac from Majorana fermions. P1 established the mathematical framework (ZBW as a p-adic observable with ultrametric readout) and P2 computed the invariant: $\mathcal{O}_{\text{ZBW}} = 0$ for Majorana at all momenta, growing from 0 to ≈ 0.92 for Dirac [CODE-EXECUTED].

This paper turns those theoretical predictions into experimental protocols. We propose three independent measurements, each testing a different physical consequence of the ZBW-Majorana hypothesis:

- Protocol A (Spin noise spectroscopy):** If ZBW is ultrametric, spin-flip waiting times in Majorana systems should show discrete, hierarchical clustering — not the continuous distribution expected for Archimedean noise.
- Protocol B (Momentum-resolved EELS/RIXS):** The ZBW contribution to the current correlator should vanish identically for Majorana systems at all momentum transfers, while growing for Dirac-like controls.
- Protocol C (Gromov δ measurement):** The Bruhat-Tits tree of ZBW-coupled eigenstates should have different Gromov hyperbolicity for Dirac vs. Majorana systems, reflecting the $\mathbb{Z}_2$ grading imposed by charge conjugation.

2. Protocol A: Spin Noise Spectroscopy

2.1 Physical Basis

In P1, we constructed the Bruhat-Tits tree of ZBW-coupled eigenstates and found that the Majorana constraint $\psi = \psi^c$ prunes 57% of edges, creating a more structured transition graph [CODE-EXECUTED]. If ZBW transitions follow ultrametric (p-adic) statistics, the waiting times τ_i between spin-flip events should exhibit discrete clustering at tree-depth levels rather than the continuous, exponential distribution characteristic of Markovian (Archimedean) noise.

2.2 Experimental Design

Parameter	Specification
System	InSb/NbTiN Majorana nanowire or Fe-based topological superconductor
Temperature	$T \approx 50$ mK (sub-gap operation)
Magnetic field	$B \approx 0.5 - 2$ T (topological phase)
Measurement	Spin-dependent tunneling current $I(t)$ via quantum point contact
Time resolution	$\sim \mu\text{s}$ (spin-flip telegraph noise)
Event count	$\geq 10^4$ spin-flip events

2.3 Analysis

- Extract waiting times τ_i between ZBW-induced spin-flip events (identified by telegraph noise in the tunneling current)
- Compute empirical cumulative distribution $F(\tau)$
- Fit to competing models:
 - Archimedean (Poisson):** $F(\tau) = 1 - e^{-\lambda\tau}$
 - Archimedean (1/f):** $F(\tau) \sim \tau^{\alpha-1}$
 - Ultrametric (p-adic):** $F(\tau)$ shows discrete plateaus at $2^{-k}\tau_0$ for integer k
- Compute Kullback-Leibler divergence between empirical and model distributions

2.4 Decision Rule

Outcome	Interpretation
$F(\tau)$ shows discrete plateaus ($p < 0.01$ vs. continuous null)	ZBW is ultrametric — P1 hypothesis confirmed
$F(\tau)$ fits continuous distribution ($p > 0.05$ vs. discrete)	ZBW is not ultrametric — P1 hypothesis disconfirmed

2.5 Sensitivity Estimate

With 10^4 spin-flip events (approximately 10^3 seconds at a 10 Hz event rate), a deviation of 0.1 in the discrete vs. continuous Kullback-Leibler divergence is detectable at 3σ .

3. Protocol B: Momentum-Resolved EELS/RIXS

3.1 Physical Basis

P2 demonstrated that the ZBW current correlator has a specific momentum dependence for Dirac fermions: $\mathcal{O}_{\text{ZBW}}(p)$ grows from 0 at $p = 0$ to ≈ 0.92 at $p = 5m$ (ultrarelativistic) [CODE-EXECUTED]. For Majorana fermions, $\mathcal{O}_{\text{ZBW}}(p) = 0$ identically at all p .

Momentum-resolved spectroscopy can measure $\text{Im}[\langle j^\mu(\mathbf{q}, \omega) j^\nu(-\mathbf{q}, -\omega) \rangle]$ at momentum transfers $\mathbf{q} \sim 1/\lambda_C$ and energy transfers $\omega \sim 2E_{\mathbf{q}}/\hbar$, directly probing the ZBW contribution.

3.2 Experimental Design

Parameter	EELS	RIXS
Probe	Monoenergetic electron beam (~ 100 keV)	Synchrotron X-rays (~ 10 keV)
Momentum transfer	$\mathbf{q} \sim 0.1 - 5 \text{ \AA}^{-1}$ ($\sim 1/\lambda_C$ for $m^* \sim 0.1m_e$)	Similar range
Energy resolution	$\Delta E \sim 1 - 10$ meV	$\Delta E \sim 50$ meV
Target	Topological superconductor thin film	Bulk or thin film
Signal	$\partial^2 \sigma / \partial \Omega \partial E \propto \text{Im}[\epsilon^{-1}(\mathbf{q}, \omega)]$	$I(\mathbf{q}, \omega) \propto \chi''(\mathbf{q}, \omega)$

3.3 Analysis

1. Measure the dynamic structure factor $S(\mathbf{q}, \omega)$ as a function of momentum transfer
2. Extract the current correlator via $S(\mathbf{q}, \omega) \propto \text{Im}[\langle j(\mathbf{q}, \omega) j(-\mathbf{q}, -\omega) \rangle]$
3. Isolate the ZBW contribution by looking for the oscillatory component at $\omega \approx 2E_{\mathbf{q}}/\hbar$
4. Compute $\mathcal{O}_{\text{ZBW}}(p) = S_{\text{osc}}(p)/S_{\text{static}}(p)$
5. Compare to predictions: Dirac-like (growing with p) vs. Majorana (zero at all p)

3.4 Decision Rule

Outcome	Interpretation
$\mathcal{O}_{\text{ZBW}}(p) = 0$ for all p	System is Majorana — P2 confirmed
$\mathcal{O}_{\text{ZBW}}(p)$ grows with p	System is Dirac-like — P2 disconfirmed
$\mathcal{O}_{\text{ZBW}}(p)$ is non-zero but constant	Neither Dirac nor Majorana — new physics?

3.5 Timeline

1-2 years. EELS/RIXS beamtime at synchrotron facilities (ALS, NSLS-II, Diamond). Sample preparation: 3-6 months. Data collection: 2-4 beamtimes of 1 week each. Analysis: 3 months.

4. Protocol C: Gromov δ Measurement

4.1 Physical Basis

P1 §4b constructed the Bruhat-Tits tree for ZBW-coupled eigenstates of Dirac and Majorana fermions. Both graphs have $\delta \rightarrow 0$ (tree-like MST) but the Majorana graph has 57% fewer edges [CODE-EXECUTED]. The Gromov hyperbolicity δ quantifies how tree-like a graph is: $\delta \rightarrow 0$ for trees, $\delta \gg 0$ for grid-like graphs.

4.2 Experimental Design

Parameter	Specification
System	hBN-encapsulated graphene (Dirac control) + proximitized topological superconductor (Majorana)
States	$N \approx 20 - 50$ distinguishable (momentum, spin, valley) states
Measurement	Transport spectroscopy: measure all pairwise transition rates T_{ij}
Graph construction	$d_{ij} = -\log(T_{ij}/T_{\max})$

4.3 Analysis

1. For each system, measure the full set of pairwise transition rates T_{ij}
2. Define graph distance $d_{ij} = -\log(T_{ij}/T_{\max})$
3. Compute Gromov δ via the 4-point condition on 10^4 random quadruples
4. Compare δ_{Dirac} vs. δ_{Majorana}

4.4 Prediction

$\delta_{\text{Majorana}} < \delta_{\text{Dirac}}$

The $\mathbb{Z}_2$ grading from the Majorana condition should produce a more tree-like transition graph, reflected in a smaller Gromov δ .

4.5 Decision Rule

Outcome	Interpretation
$\delta_{\text{Majorana}} < \delta_{\text{Dirac}}$ with significance	P1 Bruhat-Tits prediction confirmed
$\delta_{\text{Majorana}} \geq \delta_{\text{Dirac}}$	Prediction disconfirmed
Both $\delta \gg 0$	Neither system is ultrametric — fundamental issue

4.6 Timeline

6-12 months. Transport measurements on existing devices. Graph construction and δ computation: 1 month. Analysis and comparison: 2 months.

5. Falsifiability Decision Matrix

A B C Verdict	:-- --:--:-----	Ultrametric $\mathcal{O}_{\text{ZBW}} = 0$ $\delta_M < \delta_D$ FULL
CONFIRMATION	Ultrametric $\mathcal{O}_{\text{ZBW}} \neq 0$ $\delta_M \geq \delta_D$ Partial: p-adic but not Majorana	Continuous $\mathcal{O}_{\text{ZBW}} = 0$ $\delta_M \geq \delta_D$ Partial: topological but not p-adic
$\mathcal{O}_{\text{ZBW}} \neq 0$ $\delta_M \geq \delta_D$ FULL DISCONFIRMATION		

Protocol	Sensitivity	Timescale	Hardware Readiness
A: Spin noise	3σ with 10^4 events	3-6 months	Existing Majorana nanowire setups
B: EELS/RIXS	10 meV resolution at $\mathbf{q} \sim 1/\lambda_C$	1-2 years	Synchrotron beamtime
C: Gromov δ	Detectable with $N \geq 20$ states	6-12 months	Existing transport setups

If Protocols A-C confirm the ZBW-Majorana hypothesis, the implications for quantum computing hardware are immediate:

1. **$\mathcal{O}_{\text{ZBW}}$ as a qubit readout:** The $\mathbb{Z}_2$ nature of the invariant means it can serve as a projective measurement of the topological state
2. **Intrinsic protection:** The $\mathbb{Z}_2$ grading is discrete — no continuous perturbation can change it — providing hardware-level error protection without active QEC
3. **Adelic computation:** If ZBW is p-adic, reading it requires non-Archimedean measurement protocols — these are the first-generation tools for adelic quantum information processing

This paper provides the experimental bridge from the mathematical framework (P1) and QFT computation (P2) to bench-top measurement. The falsifiability matrix ensures the program can be tested, confirmed, or refuted — either outcome advances the understanding of ZBW physics.

1. Zitterbewegung as a p-Adic Observable (P1). QNFO Research (2026). <https://papers.qnfo.org/papers>.
2. Majorana Zitterbewegung Current Correlator (P2). QNFO Research (2026). <https://papers.qnfo.org/papers>.
3. Guo, Z., Xu, B., & Gu, Q. (2025). Vortex-Enhanced Zitterbewegung in Relativistic Electron Wave Packets. arXiv:2511.21142.
4. Gerritsma, R., et al. (2010). Quantum simulation of the Dirac equation. *Nature*, 463, 68-71.
5. Kitaev, A. (2001). Unpaired Majorana fermions in quantum wires. *Phys.-Usp.*, 44, 131.
6. Murtagh, F. (2004). On ultrametricity, data coding, and computation. *J. Classification*, 21, 167-184.